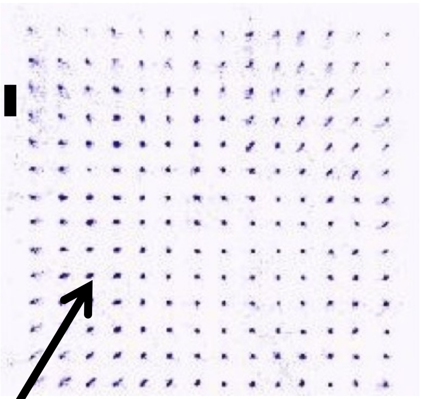
Arrays of atoms for Q simulation & computing – Lecture 2

Lecture 1: Arrays of atoms in optical tweezers
Rydberg atoms

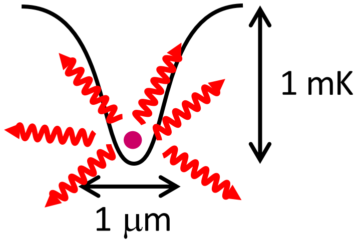
Lecture 2: Interactions between Rydberg atoms
Rydberg blockade
Quantum computing with Rydberg atoms

Lecture 3: Quantum simulation: from Rydberg interactions to spin models… and more

Combining arrays of atoms and Rydberg interactions

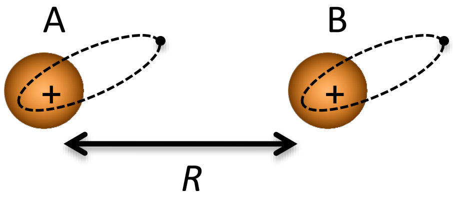


Addressable!!



+

Rydberg interactions



Van der Waals	resonant
$\frac{C_6}{R^6}$	$\frac{C_3}{R^3}$

Quantum computation

Quantum simulation:

many-body physics in synthetic matter

Quantum metrology

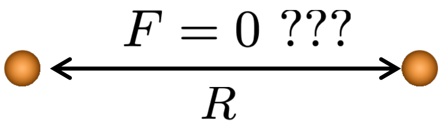
Outline – Lecture 2

1. Interactions between Rydberg atoms

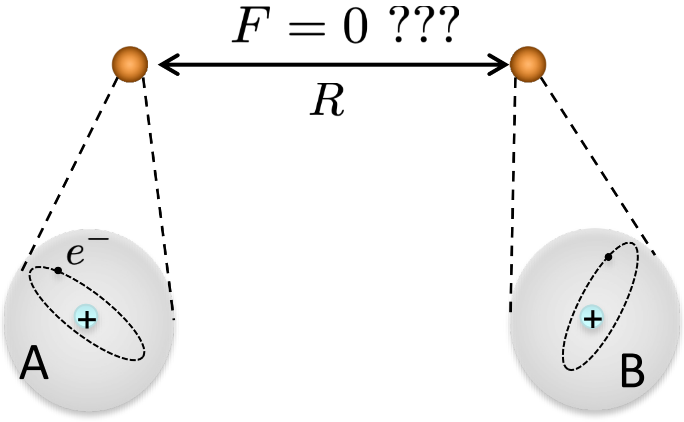
2. Rydberg Blockade

3. Quantum computing with arrays of atoms and Rydberg interactions

Dipolar Interaction between atoms



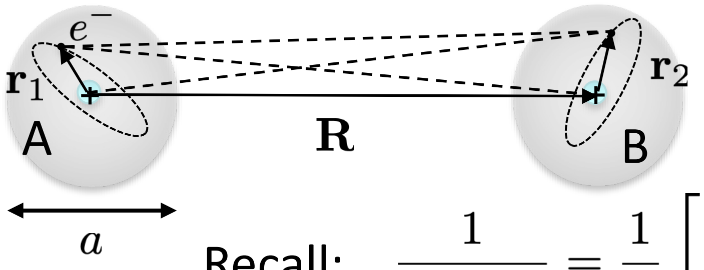
Dipolar Interaction between atoms



Dipolar Interaction between atoms

$$e^2 = \frac{q_e^2}{4\pi\epsilon_0}$$

$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2} - \frac{e^2}{|\mathbf{R}-\mathbf{r}_1|} - \frac{e^2}{|\mathbf{R}+\mathbf{r}_2|} + \frac{e^2}{|\mathbf{R}+\mathbf{r}_2-\mathbf{r}_1|} + \frac{e^2}{R}$$



Recall: $$\frac{1}{|\mathbf{R}-\mathbf{r}|} = \frac{1}{R}\left[1 - \frac{\mathbf{r}\cdot\mathbf{R}}{2R^2} - \frac{r^2}{2R^2} + \frac{3}{2}\left(\frac{\mathbf{r}\cdot\mathbf{R}}{R^2}\right)^2\right] + \mathcal{O}\left(\frac{r^4}{R^4}\right)$$

Dipole-dipole interaction: $$H_{\mathrm{dd}} = \frac{1}{4\pi\epsilon_0 R^3}\left[\mathbf{d}_1\cdot\mathbf{d}_2 - 3(\mathbf{d}_1\cdot\mathbf{u})(\mathbf{d}_2\cdot\mathbf{u})\right] \ , \quad \mathbf{u} = \frac{\mathbf{R}}{R}$$

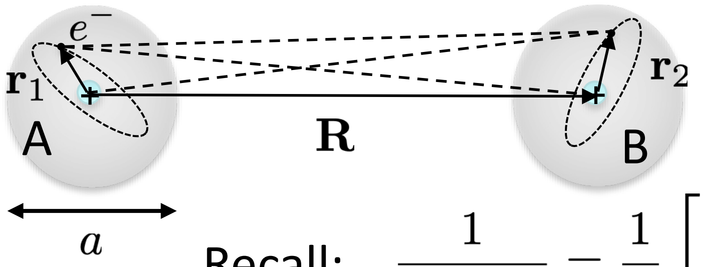
$a \ll R$

with dipoles: $\mathbf{d}_{1,2} = q\,\mathbf{r}_{1,2}$

Dipolar Interaction between atoms

$$e^2 = \frac{q_e^2}{4\pi\epsilon_0}$$

$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2} - \frac{e^2}{|\mathbf{R}-\mathbf{r}_1|} - \frac{e^2}{|\mathbf{R}+\mathbf{r}_2|} + \frac{e^2}{|\mathbf{R}+\mathbf{r}_2-\mathbf{r}_1|} + \frac{e^2}{R}$$



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Dipole-dipole interaction: $$\hat{H}_{\mathrm{dd}} = \frac{1}{4\pi\epsilon_0 R^3}\left[\hat{\mathbf{d}}_1\cdot\hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1\cdot\mathbf{u})(\hat{\mathbf{d}}_2\cdot\mathbf{u})\right] \ , \quad \mathbf{u} = \frac{\mathbf{R}}{R}$$

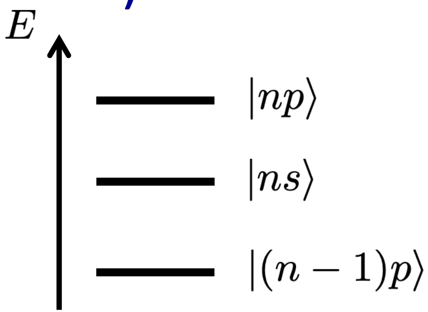
$a \ll R$

with dipoles: $\hat{\mathbf{d}}_{1,2} = q\,\hat{\mathbf{r}}_{1,2}$

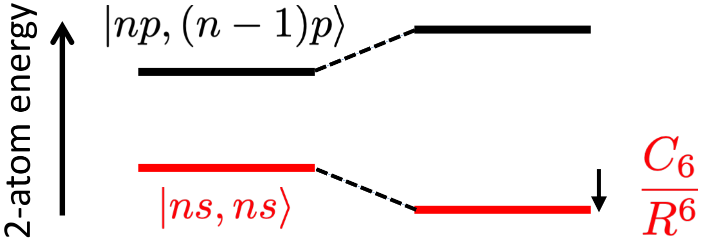
2 atom basis: $\{|n,l,m\rangle \otimes |n',l',m'\rangle\}$

Interactions between Rydberg atoms (simplified…)

$$\hat{H}_{\rm dd} = \frac{1}{4\pi\epsilon_0}\frac{\hat{d}_{Az}\hat{d}_{Bz}}{R^3}$$



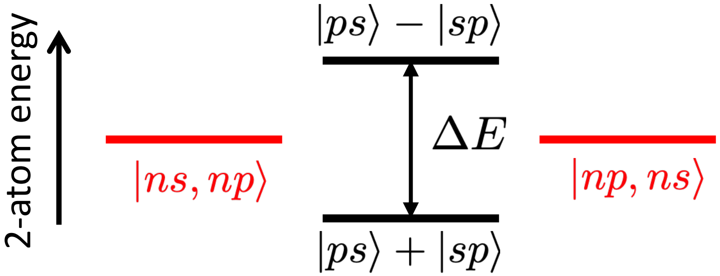
van der Waals regime



$$\Delta E^{(2)}_{ss} \approx \frac{|\langle np,(n-1)p|\hat{H}_{\rm dd}|ns,ns\rangle|^2}{E_{ss}-E_{pp}}$$

$$\propto \frac{d^4_{sp}}{E_{ss}-E_{pp}}\frac{1}{R^6} = \frac{C_6}{R^6} \qquad C_6 \propto n^{11}$$

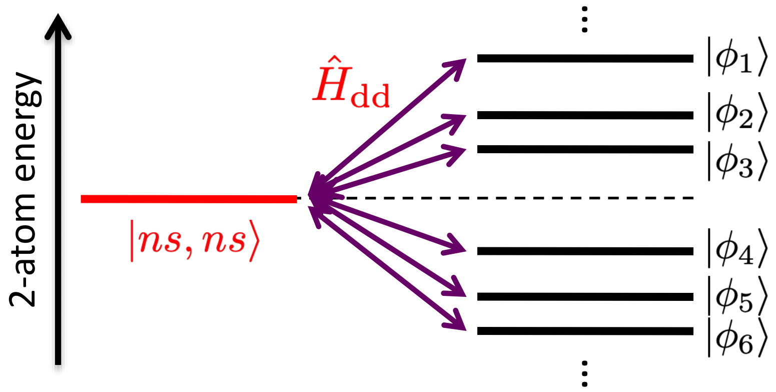
Resonant regime



$$\Delta E \propto \langle sp|\hat{H}_{\rm dd}|ps\rangle = \frac{d^2_{sp}}{R^3} \propto n^4$$

Interactions between *real* Rydberg atoms

2-atom basis: $\{|\phi_{nn'}\rangle = |n,l,m\rangle \otimes |n',l',m'\rangle\}$



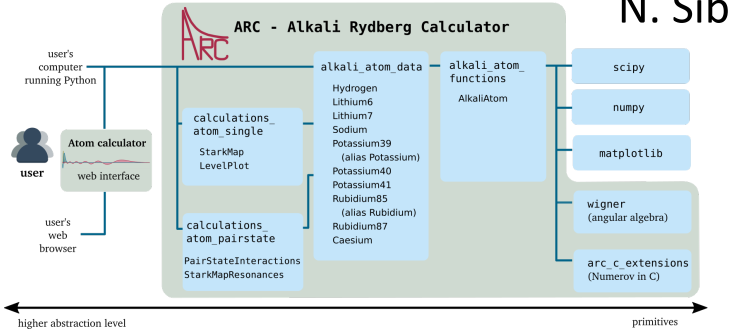
Van der Waals regime:

$$\Delta E_{ss}^{(2)} = \sum_{|\phi\rangle} \frac{|\langle\phi|\hat{H}_{\rm dd}|ss\rangle|^2}{E_{ss} - E_\phi} = \frac{C_6}{R^6}\,, \quad C_6 \propto n^{11}$$

Resonant regime:

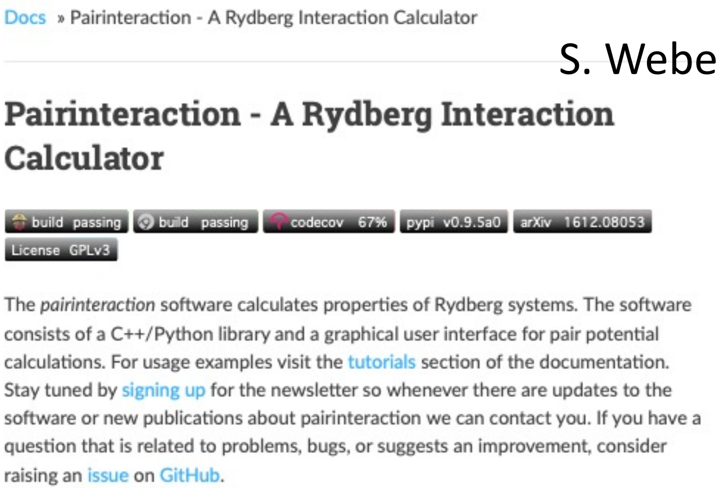
$$E_\pm = \pm\langle sp|\hat{H}_{\rm dd}|ps\rangle = \pm\frac{1}{4\pi\epsilon_0}\frac{d_{sp}^2}{R^3} \propto n^4$$

N. Sibalic



https://arc-alkali-rydberg-calculator.readthedocs.io/en/latest/

S. Weber


Docs » Pairinteraction - A Rydberg Interaction Calculator

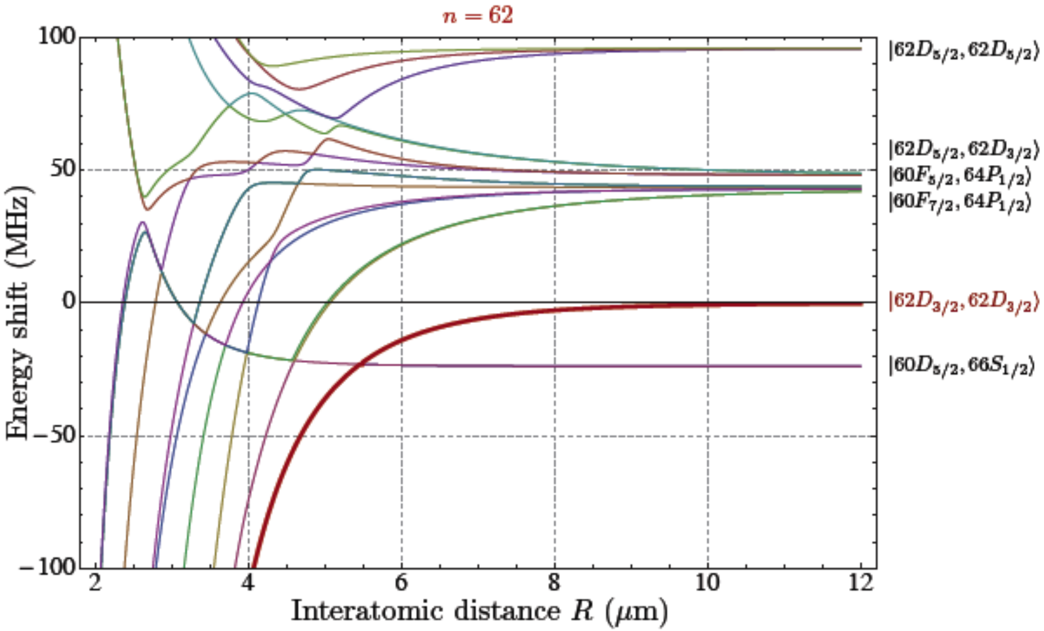
Pairinteraction - A Rydberg Interaction Calculator

build passing | build passing | codecov 67% | pypi v0.9.5a0 | arXiv 1612.08053 | License GPLv3

The *pairinteraction* software calculates properties of Rydberg systems. The software consists of a C++/Python library and a graphical user interface for pair potential calculations. For usage examples visit the tutorials section of the documentation. Stay tuned by signing up for the newsletter so whenever there are updates to the software or new publications about pairinteraction we can contact you. If you have a question that is related to problems, bugs, or suggests an improvement, consider raising an issue on GitHub.

https://pairinteraction.github.io/pairinteraction/sphinx/html/index.html

Interactions between "real" Rydberg atoms



$R = 10\ \mu\text{m} \Rightarrow V_{\text{int}}/h \sim 1 - 10\ \text{MHz}$ **⇒ timescales < µsec**

Interactions between Rydberg atoms

REVIEWS OF MODERN PHYSICS, VOLUME 82, JULY–SEPTEMBER 2010

Quantum information with Rydberg atoms

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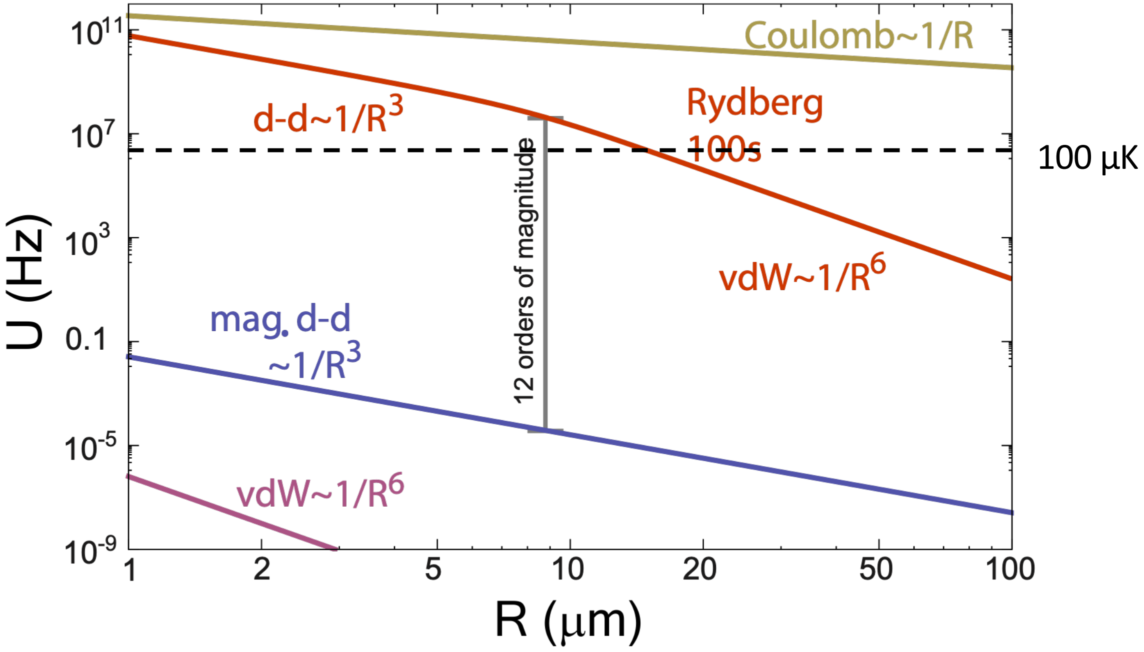
Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, Wisconsin 53706, USA

K. Mølmer



Outline – Lecture 2

1. Interactions between Rydberg atoms
2. Rydberg Blockade
3. Quantum computing with arrays of atoms and Rydberg interactions

The initial idea (~2000)

VOLUME 85, NUMBER 10 PHYSICAL REVIEW LETTERS 4 SEPTEMBER 2000

Fast Quantum Gates for Neutral Atoms

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S. L. Rolston

National Institute of Standards and Technology, Gaithersburg, Maryland 20899

R. Côté[1] and M. D. Lukin[2]

[1]*Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, Connecticut 06269-3046*
[2]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*

(Received 7 April 2000)

We propose several schemes for implementing a fast two-qubit quantum gate for neutral atoms with the gate operation time much faster than the time scales associated with the external motion of the atoms in the trapping potential. In our example, the large interaction energy required to perform fast gate operations is provided by the dipole-dipole interaction of atoms excited to low-lying Rydberg states in constant electric fields. A detailed analysis of imperfections of the gate operation is given.

Fast (< 1 μs), robust

VOLUME 87, NUMBER 3 PHYSICAL REVIEW LETTERS 16 JULY 2001

Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

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[1]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
[2]*Fachbereich Physik, Universität Kaiserslautern, D-67663 Kaiserslautern, Germany*
[3]*Physics Department, University of Connecticut, Storrs, Connecticut 06269*

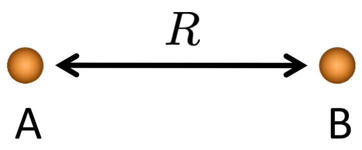
L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller

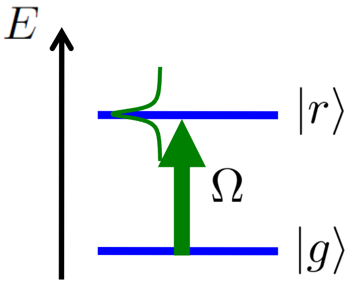
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria

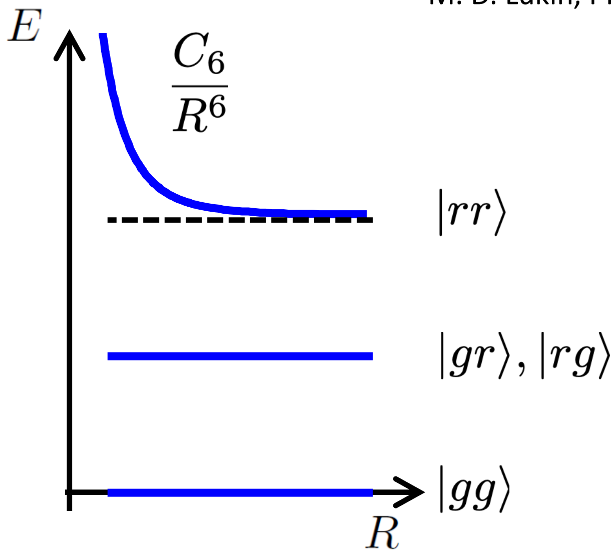
(Received 7 November 2000; published 26 June 2001)

Rydberg interaction and blockade

D. Jaksch, PRL **85**, 2208 (2000)
M. D. Lukin, PRL **87**, 037901 (2001)







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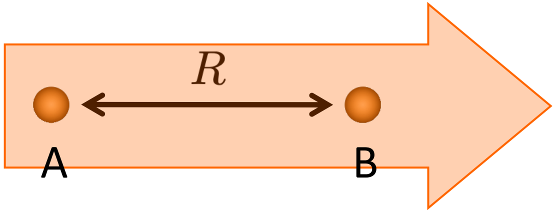

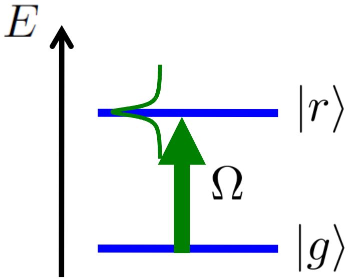

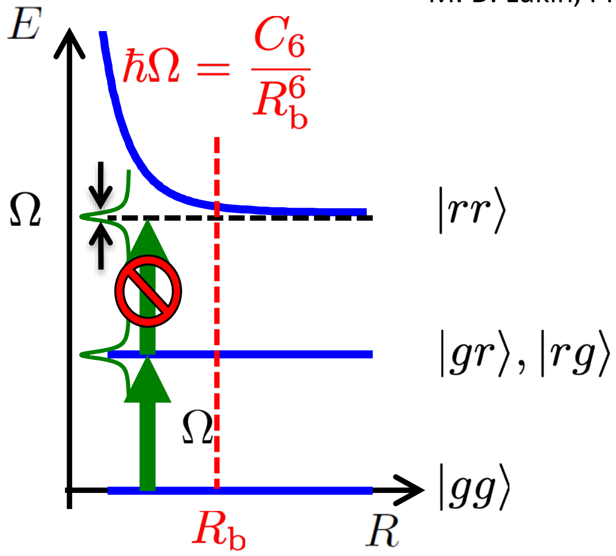

$R_\mathrm{b} \sim 2 - 10\ \mu\mathrm{m}$

If $\hbar\Omega \ll C_6/R^6$: no excitation of $|rr\rangle \Rightarrow$ **blockage**

Rydberg interaction and blockade

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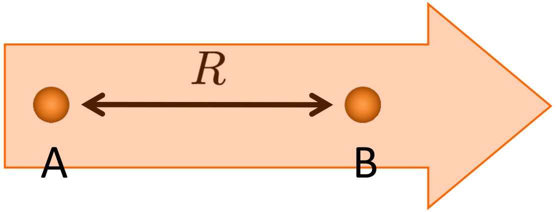

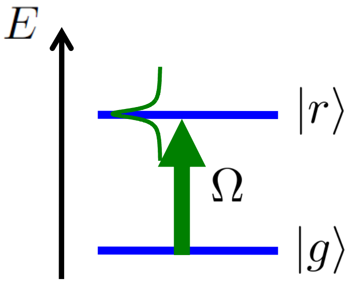

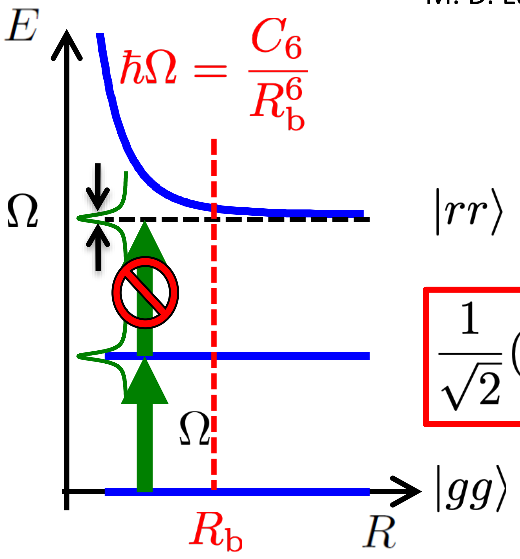

$$\frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)$$

$R_b \sim 2 - 10\ \mu\text{m}$

Blockade $\Rightarrow$ **entanglement and gates!!**

Rydberg interaction and blockade

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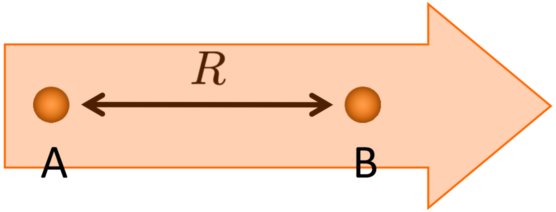
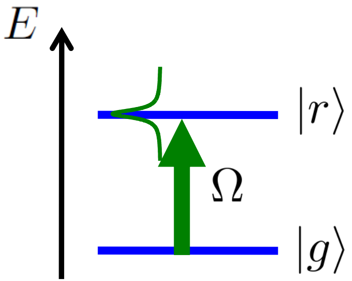
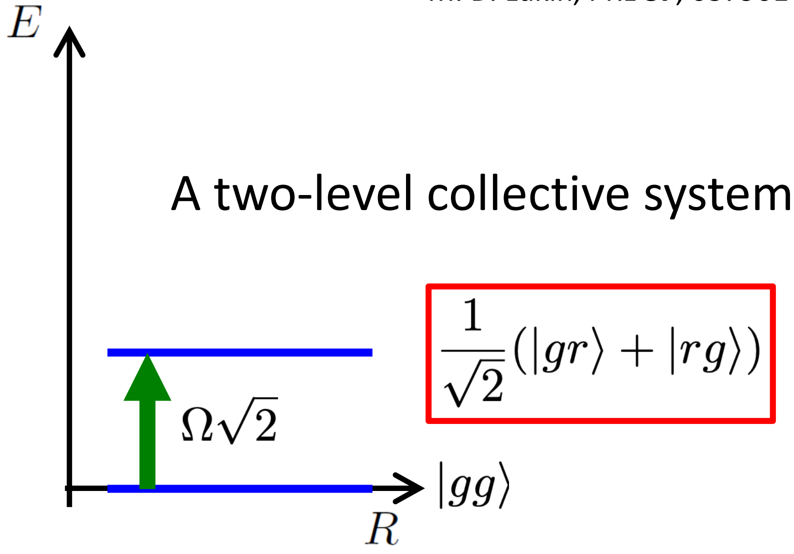




A two-level collective system!



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Rydberg interaction and blockade

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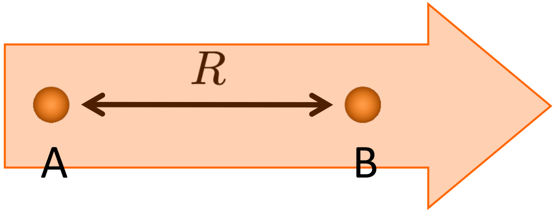
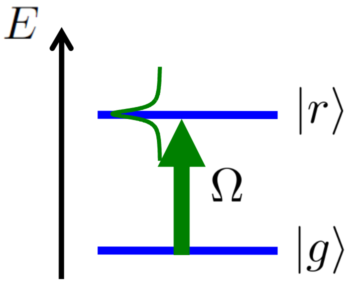
M. D. Lukin, PRL **87**, 037901 (2001)





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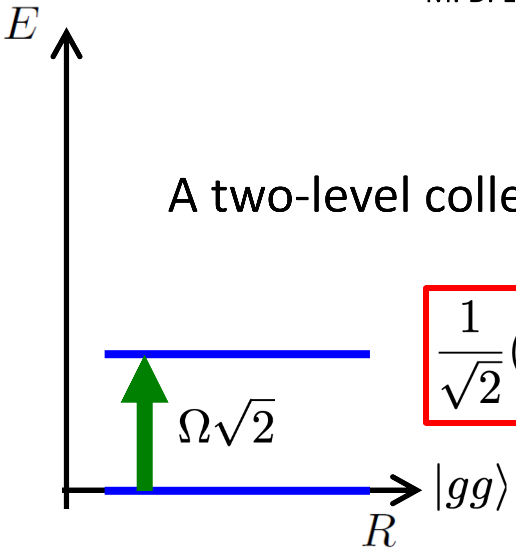


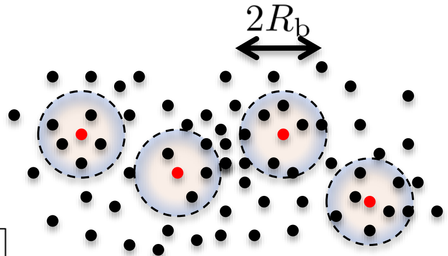
$$\frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)$$

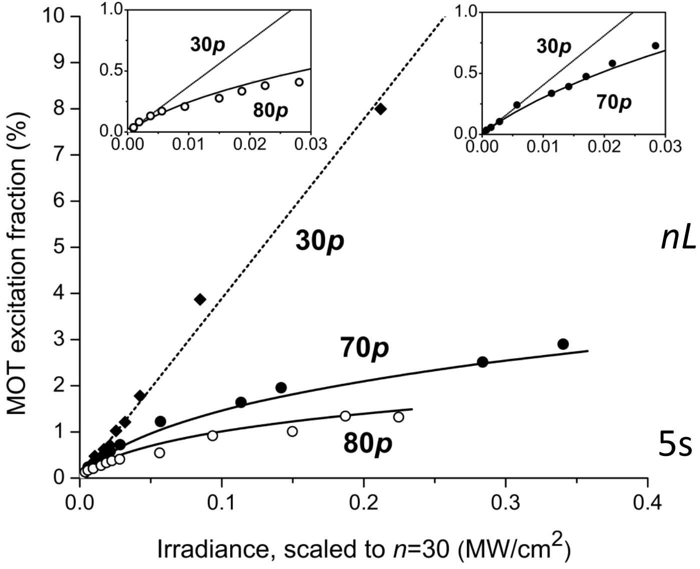
$R_{\mathrm{b}} \sim 2 - 10\ \mu\mathrm{m}$

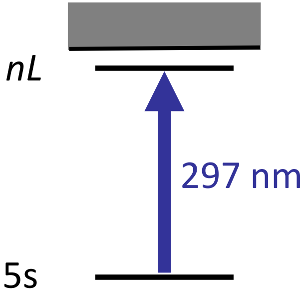
Collective oscillation between $|gg\rangle$ and $\frac{1}{\sqrt{2}}(|rg\rangle + |gr\rangle)$

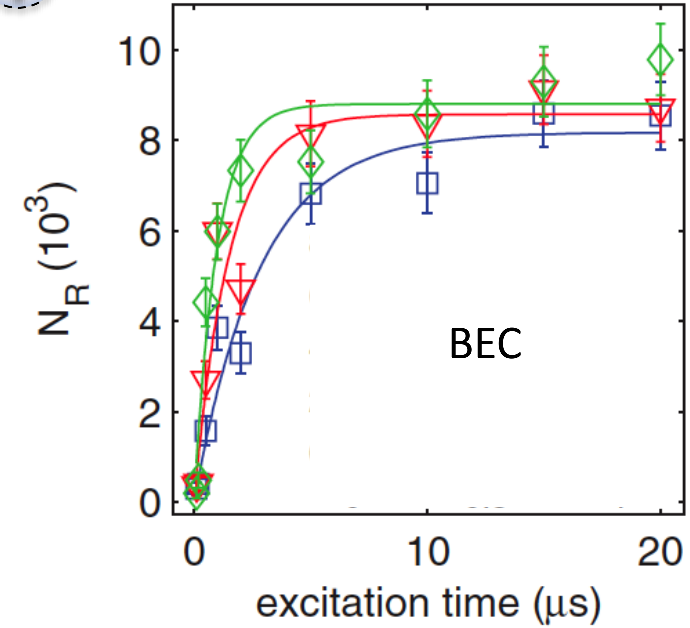
with coupling $\Omega\sqrt{2}$ (N atoms $\Rightarrow$ $\Omega\sqrt{N}$)

The first blockade experiments: atomic ensembles









Gould, PRL 2004
Weidemuller, PRL 2004
Raithel, PRL 2005

Martin, PRL 2004
Pillet, PRL 2006

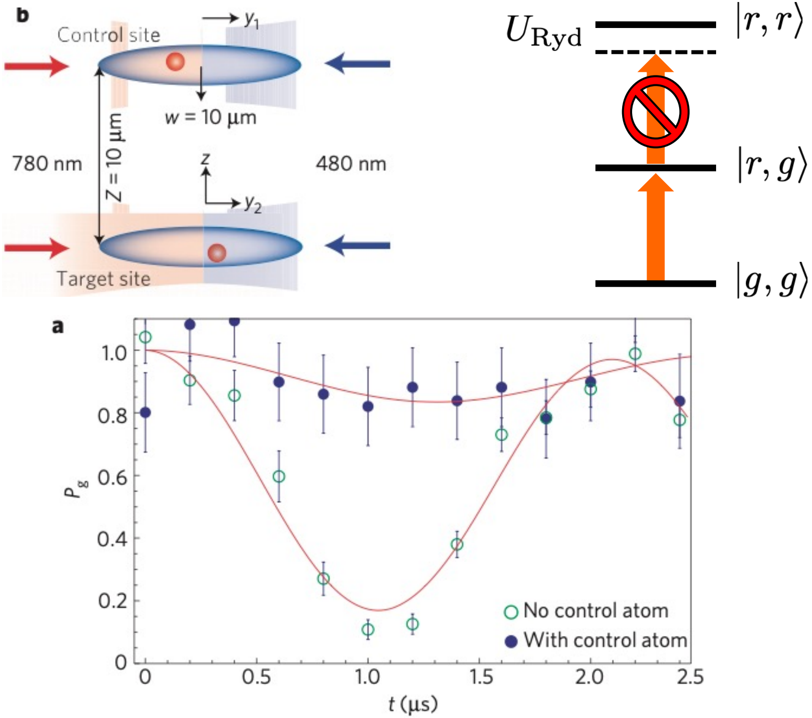
Pfau, PRL 2007

The first demonstrations of blockade with two atoms (2008-09)

nature physics
PUBLISHED ONLINE: 11 JANUARY 2009

Observation of Rydberg blockade between two atoms

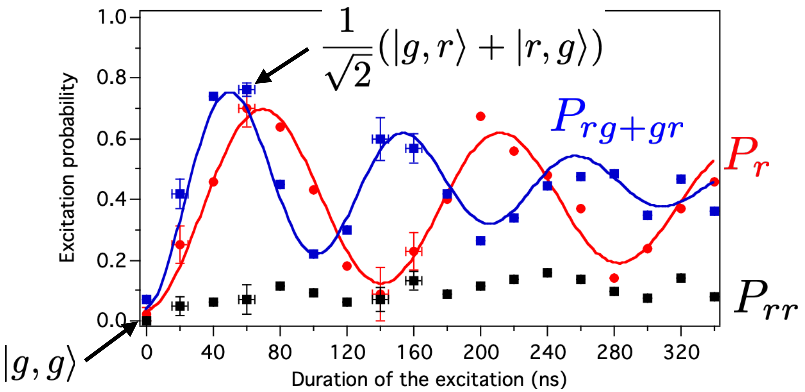
E. Urban, T. A. Johnson, T. Henage, L. Isenhower, D. D. Yavuz, T. G. Walker and M. Saffman*

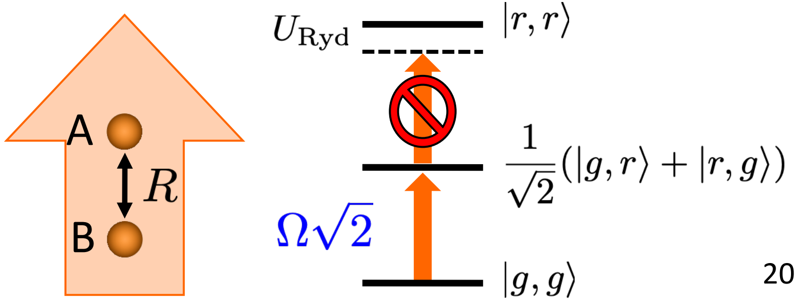


nature physics
PUBLISHED ONLINE: 11 JANUARY 2009

Observation of collective excitation of two individual atoms in the Rydberg blockade regime

Alpha Gaëtan[1], Yevhen Miroshnychenko[1], Tatjana Wilk[1], Amodsen Chotia[2], Matthieu Viteau[2], Daniel Comparat[2], Pierre Pillet[2], Antoine Browaeys[1]* and Philippe Grangier[1]





Outline – Lecture 2

1. Interactions between Rydberg atoms

2. Rydberg Blockade

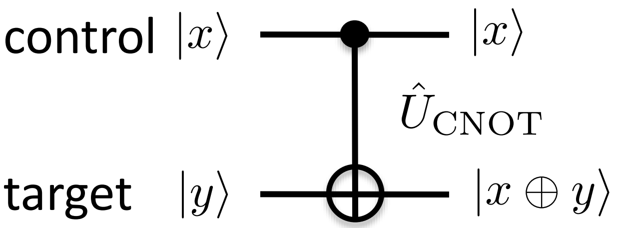
3. Quantum computing with arrays of atoms and Rydberg interactions

Quantum computing in a nutshell

Qubits: $|0\rangle, |1\rangle \Rightarrow \alpha|0\rangle + \beta|1\rangle$ **Quantum register:** ensembles of qubits

Single qubit gates: $|x\rangle$ —[$\hat{U}^{(1)}$]— $|y\rangle = \hat{U}^{(1)}|x\rangle = \alpha|0\rangle + \beta|1\rangle$ Ex: $\hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Two-qubit gates:



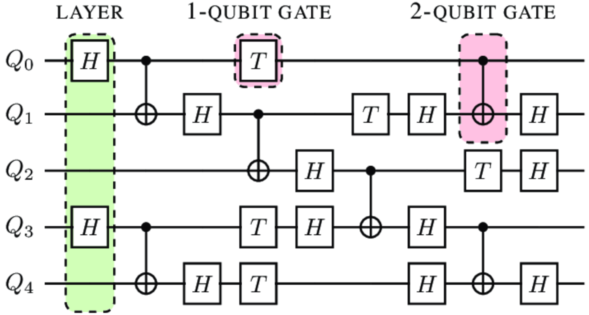
$$\hat{U}_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Universal gate set: any $\hat{U}$ can be approximated by products of

$$\hat{H}, \quad \hat{S} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, \quad \hat{U}_{\mathrm{CNOT}}$$

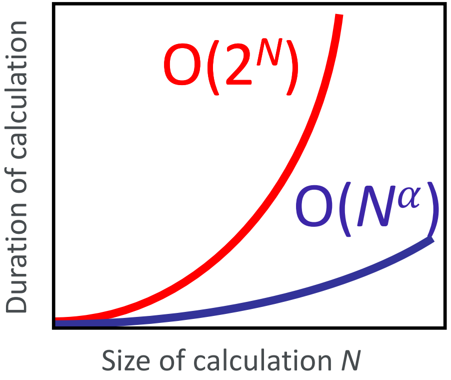
A calculation:

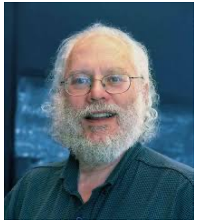
$$|\psi_{\mathrm{f}}\rangle = \hat{U}|\psi_{\mathrm{register}}\rangle$$



The promises of quantum computing

Change complexity class of calculation





P. Schor

L. Grover

Factoring

Ex: RSA-2048 en 10^9 years (Class.) $\Rightarrow$ 1 day (Quant.)

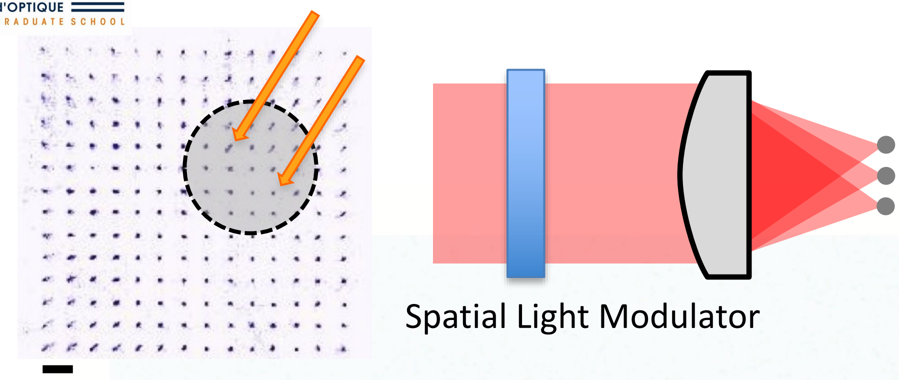
Search in list: $N/2$ (class.) $\Rightarrow \sqrt{N}$ (Quant.) (10 000 $\rightarrow$ 100)

Chemistry (fixation N_2, affinity enzymes…), **materials** (superconduct.)…

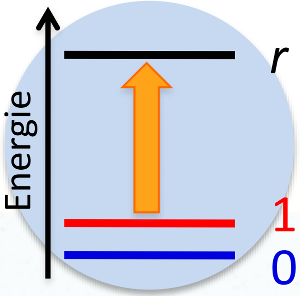
Impossible (class.) $\Rightarrow$ possible (Quant.)??

Quantum processor based on arrays of laser-cooled atoms

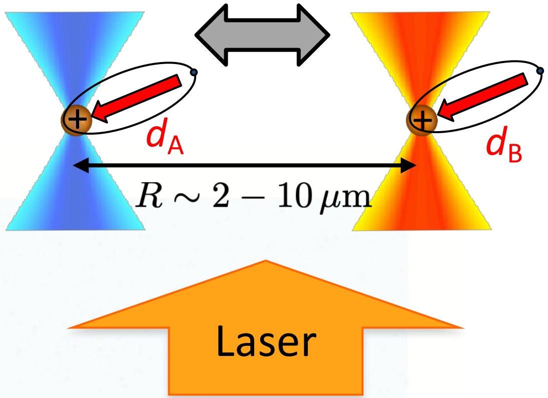
Assembled arrays of atoms



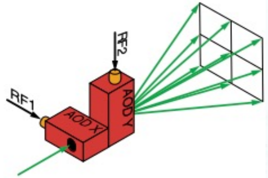
Atoms = qubits



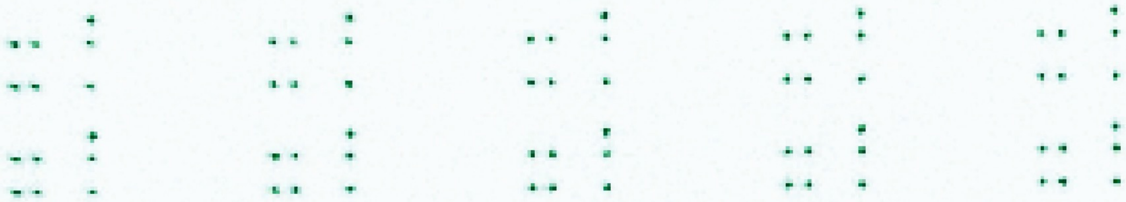
Rydberg interaction



Atoms can be moved to interact locally

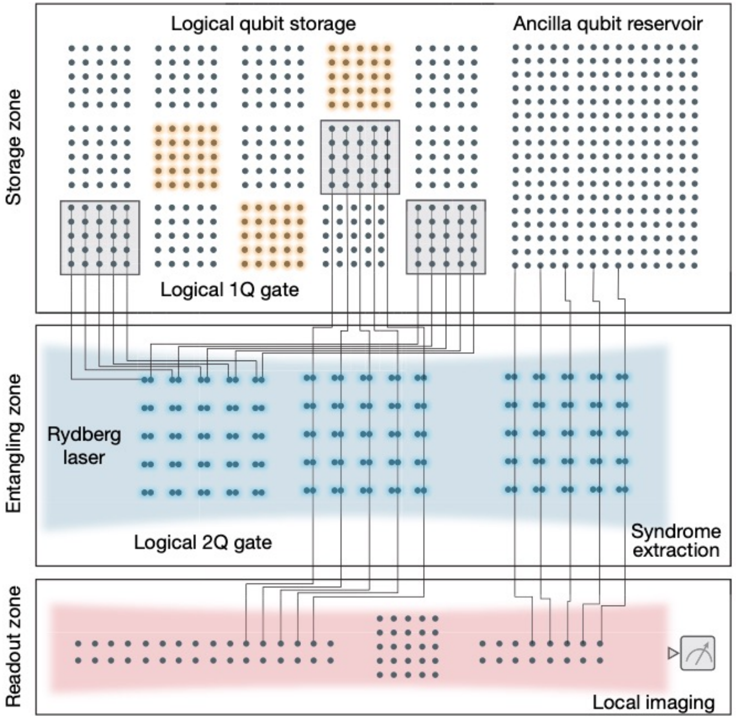


acousto-optical deflector



Lukin group

Moving the qubits around: the zoning architecture



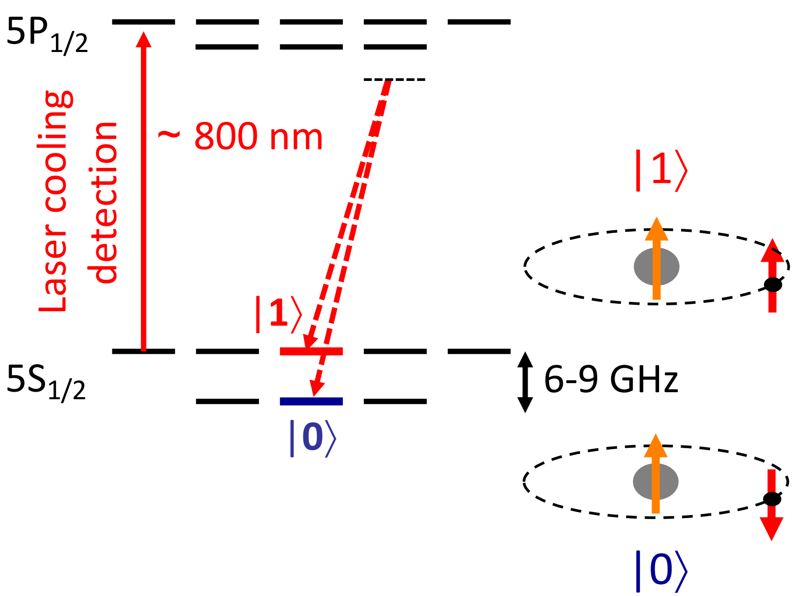
Bluvstein, Nature 2023

Useful for all-to-all connectivity and implementation of error correction

Different ways to encode the qubit

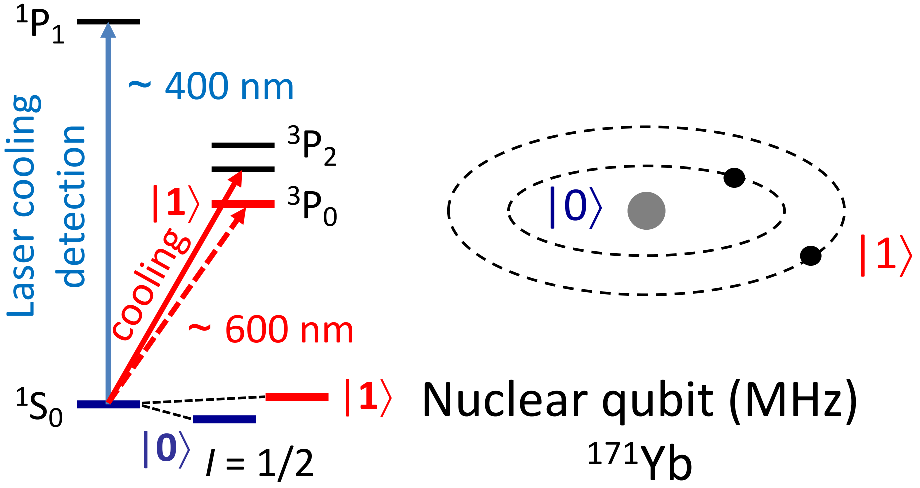
Atomic clocks ⇒ atoms = ideal qubits…

Hyperfine qubit (Rb, Cs…)



$T_1 = \infty$, $T_2 > 1$ s

Optical qubit (2-e^-: Yb, Sr…)



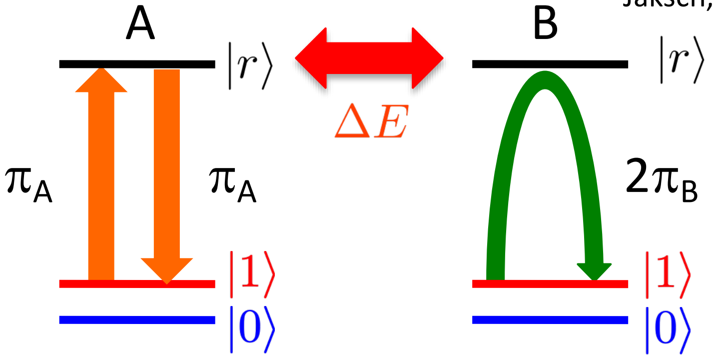
T_1, $T_2 > 10$ s

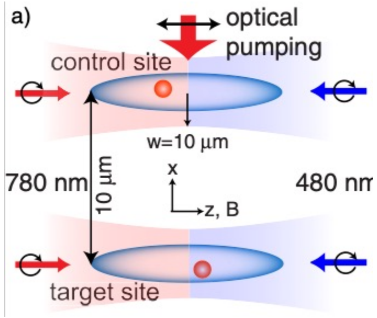
From Rydberg blockade to 2-qubit CZ gate

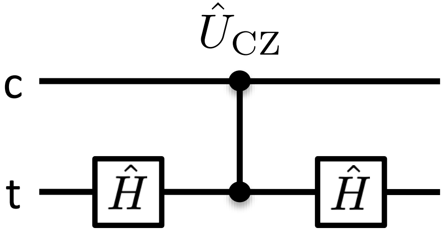
***Addressable* gate**: $\pi_A - 2\pi_B - \pi_A$ (2009-19)

Saffman, PRL 2010

Jaksch, PRL 2000



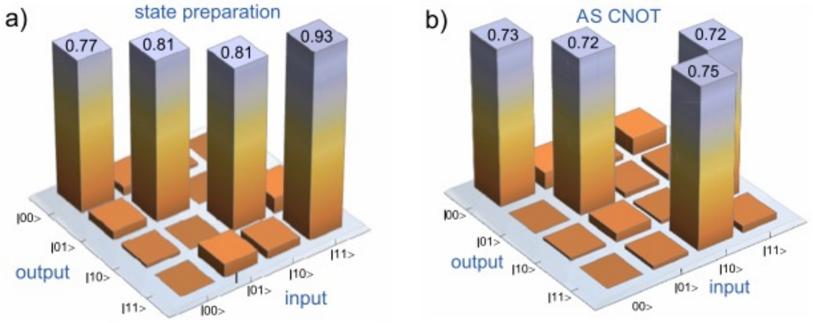




	π_A		$2\pi_B$		π_A	
00	→	00	→	00	→	00
10	→	r0	→	r0	→	-10
01	→	01	→	-01	→	-01
11	→	r1	→	r1	→	-11

CZ gate

Blockade

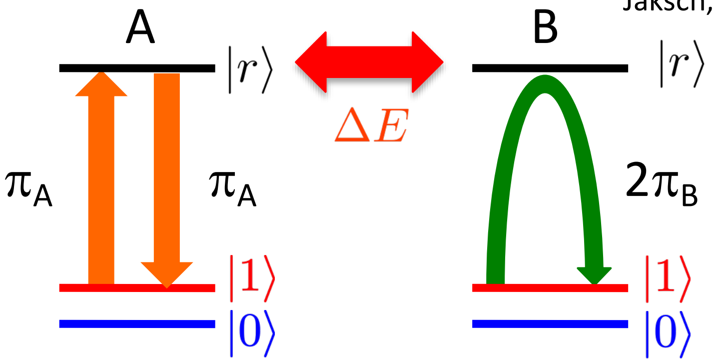


F = 73%

From Rydberg blockade to 2-qubit CZ gate

***Addressable* gate**: $\pi_A - 2\pi_B - \pi_A$ (2009-19)

Jaksch, PRL 2000

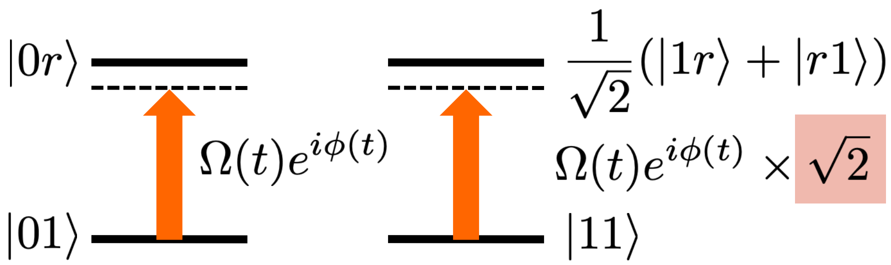


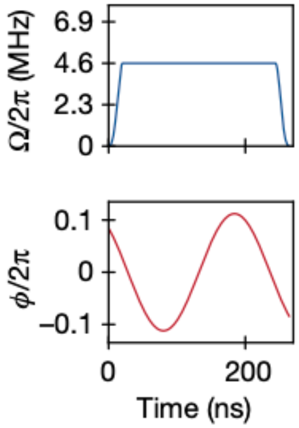
	π_A		$2\pi_B$		π_A	
00	→	00	→	00	→	00
10	→	r0	→	r0	→	-10
01	→	01	→	-01	→	-01
11	→	r1	→	r1	→	-11

CZ gate

Blockade

Optimized *global* gate (> 2019)





$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow e^{i\phi_{01}}|01\rangle$$
$$|10\rangle \rightarrow e^{i\phi_{01}}|10\rangle$$
$$|11\rangle \rightarrow e^{i\phi_{11}}|11\rangle$$

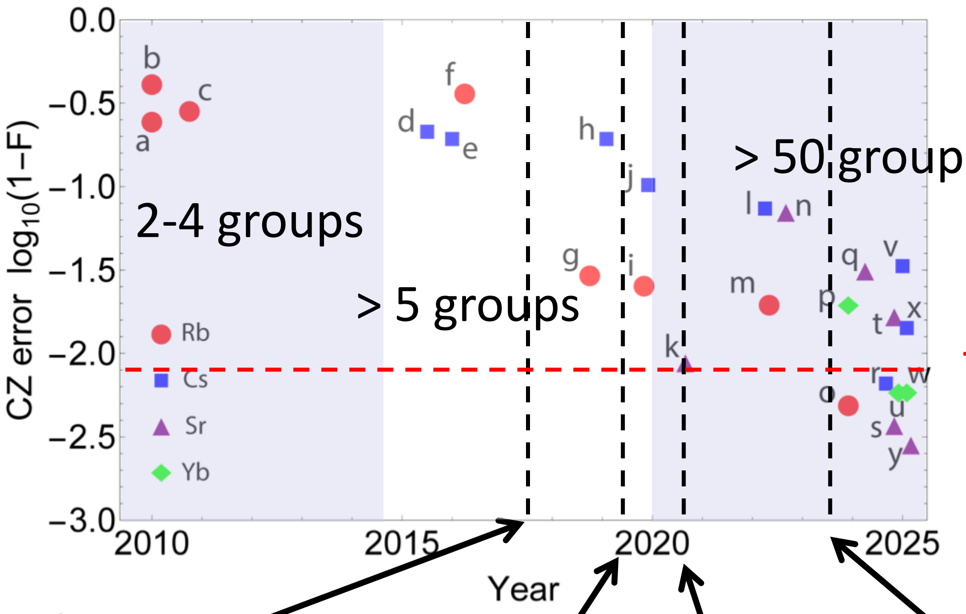
Pupillo, Quantum 2022
Evered, Nature 2023

Minimize time in Rydberg state

2026: $F \sim 99.9\%$ Rb, Sr, Yb

Fidelity improvements following the first demonstrations

arXiv:2505.11218



Threshold QEC (surface code)

Influence laser phase noise Rydberg excitation

Déléseleuc PRA 2018

Global CZ gate

Levine PRL 2019

2-*e* atom (Sr, Yb)

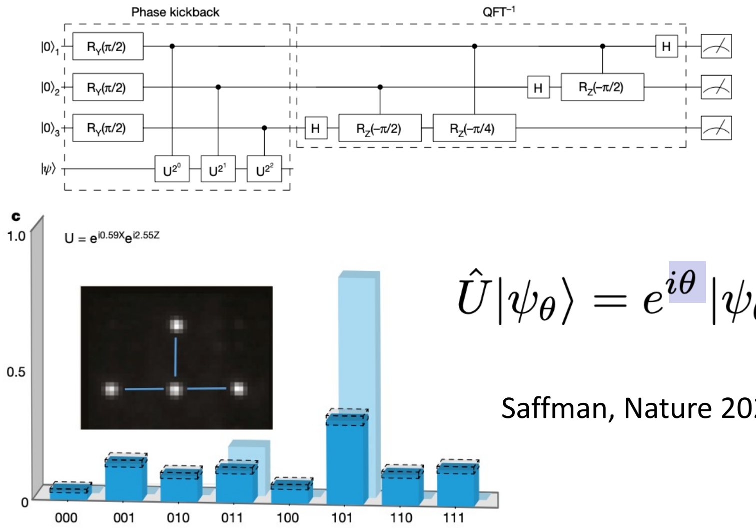
Time optimal CZ gate

Pupillo Quantum 2022

Harvard Nature 2023

Examples of quantum circuits with Rydberg processor

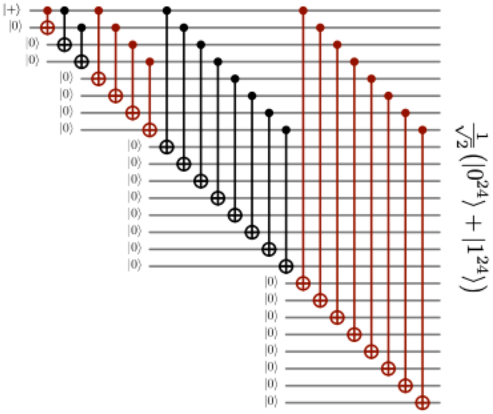
Quantum phase estimation



$$\hat{U}|\psi_\theta\rangle = e^{i\theta}|\psi_\theta\rangle$$

Saffman, Nature 2022

Preparation of cat state



arXiv:2411.11822

Digital simulation of (few?) fermions

arXiv:2501.18554

Digital quantum simulation

Lloyd, Science 1996

$$e^{-iHt} \approx \left(e^{-iH_1 t/n} e^{-iH_2 t/n} \ldots e^{-iH_n t/n}\right)^n$$

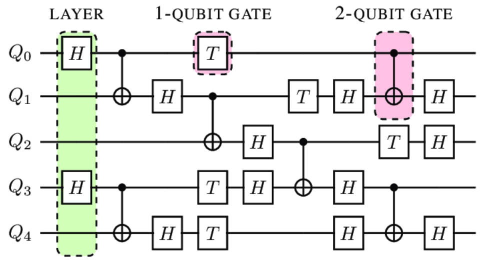
Variational algorithm: find ground state

$$H = \epsilon \hat{J}_z + \frac{V}{2}(\hat{J}_+^2 + \hat{J}_-^2)$$

arXiv:2501.06097

Today: ~ 100 entangling gates (ϵ ~ 10^{-2})

Towards fault tolerant quantum computing



Success: $p \sim e^{-N\epsilon}$

Nb gates

Error/gate

Need Quantum Error Correction

Shor, 1995

Bridge gap: hardware error (ϵ ~ 10^{-3}) / error required for applications ($\epsilon < 10^{-10}$)

QEC: generalize redundancy to quantum ⇒ logical qubit = many physical qubits

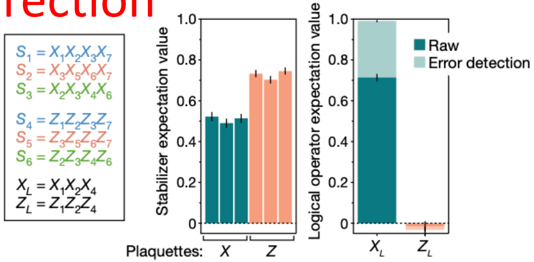
Threshold theorem: $\epsilon_{\mathrm{L}} \sim C \left(\frac{\epsilon}{\epsilon_{\mathrm{th}}}\right)^{\frac{d+1}{2}}$

phys. qubit error

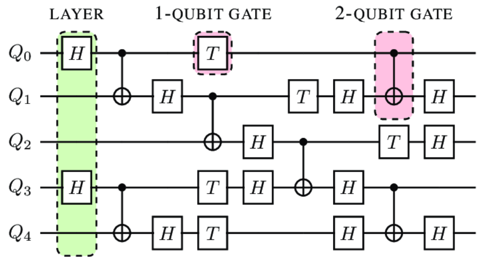
Fowler, PRA 2012

First demonstrations of logical qubits + calculations + correction

Bluvstein *et al.*, Nature 2022
Atom computing, arXiv:2411.11822
Infleqtion, arXiv:2412.07670
Quera, arXiv:2412.15165
Pasqal, arXiv:2605.21276
Atom computing, arXiv:2606.04079



Towards fault tolerant quantum computing



Success: $p \sim e^{-N\epsilon}$

Nb gates

Error/gate

Need Quantum Error Correction

Shor, 1995

Bri ... -10)

QEC ... s

Factorization RSA-2048 (Shor): ~ 10^{12} gates on ~ 200k – 1M qubits …

Quantum chemistry (Fe_7MoS_9Co): 100-1000 logical qubits ~ 10^{14} gates

First demonstrations of logical qubits + calculations + correction

Bluvstein *et al.*, Nature 2022
Atom computing, arXiv:2411.11822
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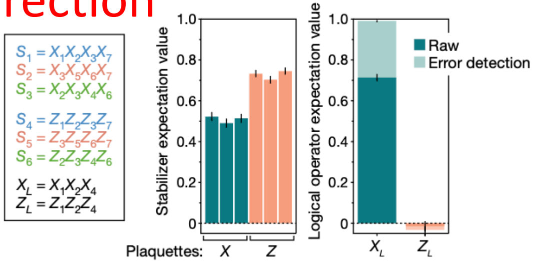

The program

Lecture 1: Arrays of atoms in optical tweezers
Rydberg atoms

Lecture 2: Interactions between Rydberg atoms
Rydberg blockade
Quantum computing with Rydberg atoms

Lecture 3: Quantum simulation: from Rydberg interactions to spin models… and more