Spin Nematic Squeezing and Three-Outcome Bell Correlator

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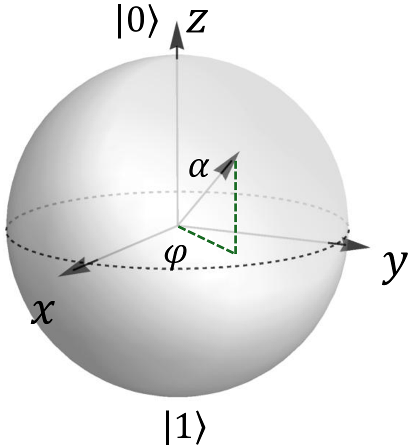
2026/06/17, ICAP29, Wuhan

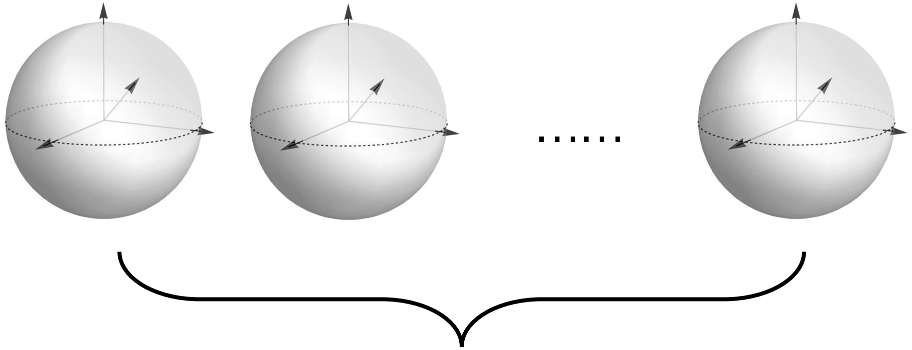
Quantum Spin Squeezing

Qubit: a two-level quantum system



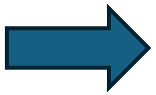
$$|\psi\rangle = \cos\frac{\alpha}{2}|0\rangle + e^{i\varphi}\sin\frac{\alpha}{2}|1\rangle$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

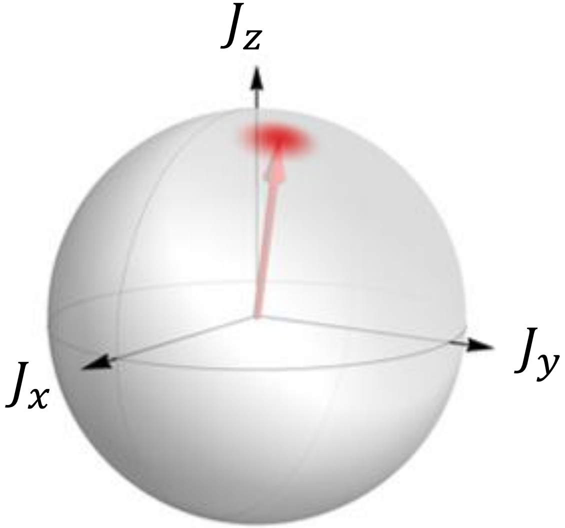


N-qubits

From Qubits to Collective Spin



$$\vec{J} = \sum_{i=1}^{N} \vec{\sigma}_i/2, \qquad J = N/2.$$



Collective Bloch Sphere

Quantum Spin Squeezing

Coherent spin state

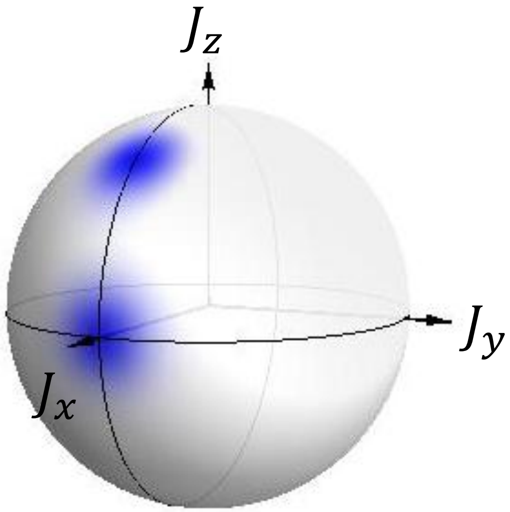
$$\langle J_x \rangle = J = \frac{N}{2}$$

$$\Delta J_y = \Delta J_z = \frac{\sqrt{N}}{2}$$

$$U(\phi) = e^{-i\phi J_y}$$

⇒

$$\langle J_z \rangle_\phi = J \sin\phi$$



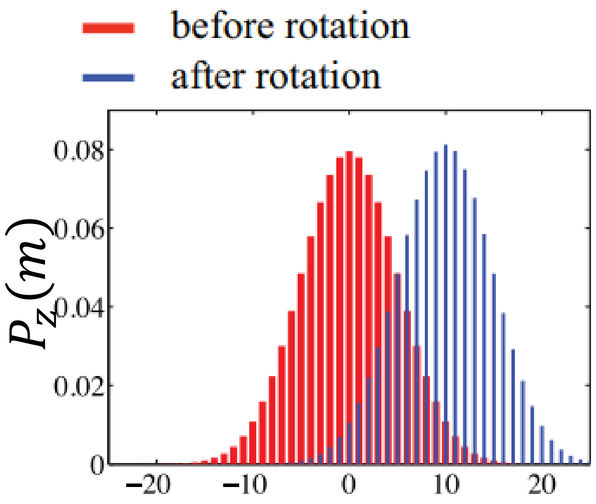
Sensitivity

$$\Delta J_z = \frac{\sqrt{N}}{2}$$

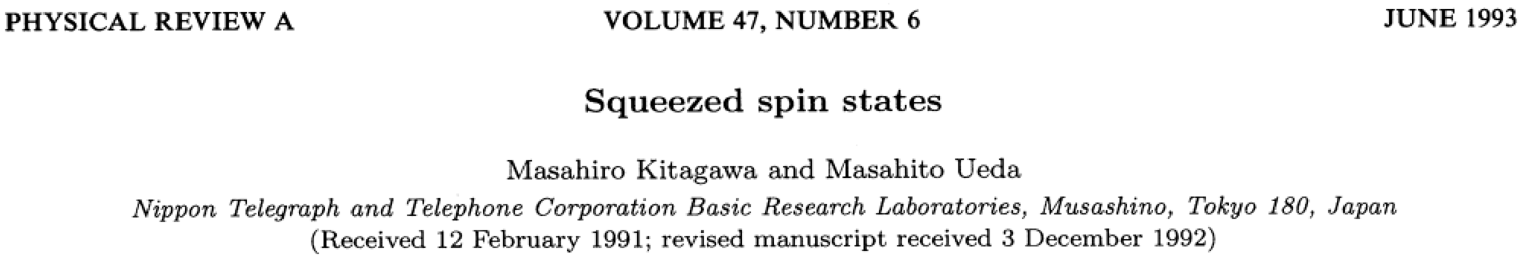
$$|\partial_\phi \langle J_z \rangle| = J = \frac{N}{2}$$

$$\Delta\phi = \frac{\Delta J_z}{|\partial_\phi \langle J_z \rangle|} = \frac{1}{\sqrt{N}}$$

Standard quantum limit (SQL)



Quantum Spin Squeezing



PHYSICAL REVIEW A VOLUME 47, NUMBER 6 JUNE 1993

Squeezed spin states

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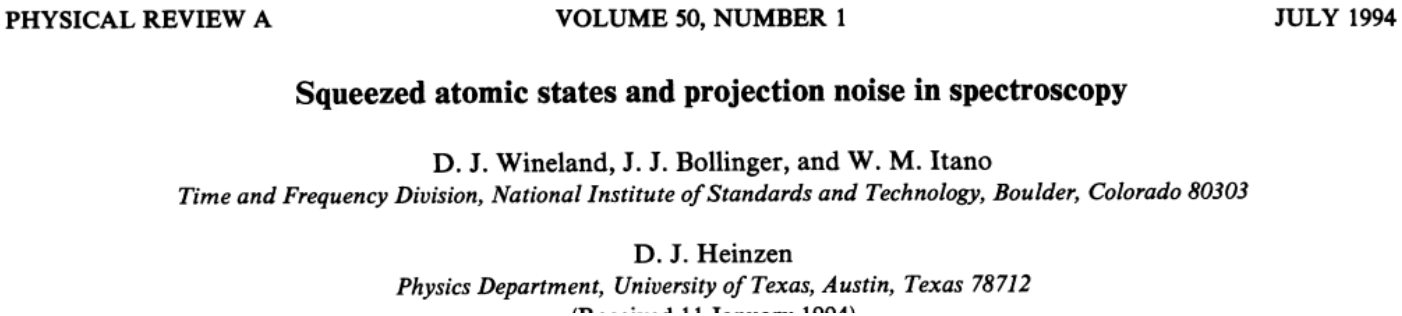
(Received 12 February 1991; revised manuscript received 3 December 1992)



PHYSICAL REVIEW A VOLUME 50, NUMBER 1 JULY 1994

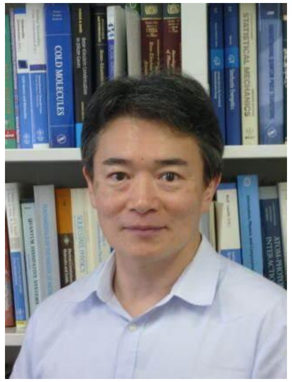
Squeezed atomic states and projection noise in spectroscopy

D. J. Wineland, J. J. Bollinger, and W. M. Itano

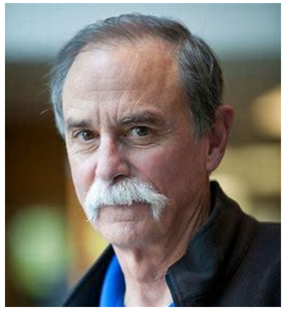
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303

D. J. Heinzen

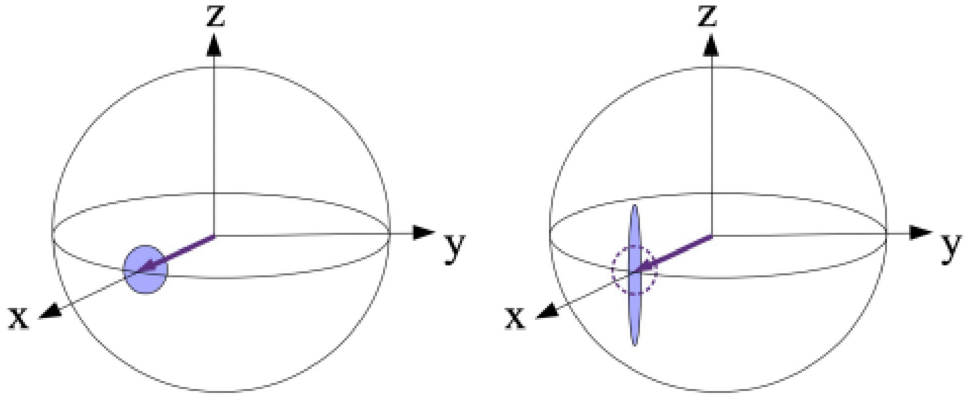
Physics Department, University of Texas, Austin, Texas 78712



Masahito Ueda



D. J. Wineland

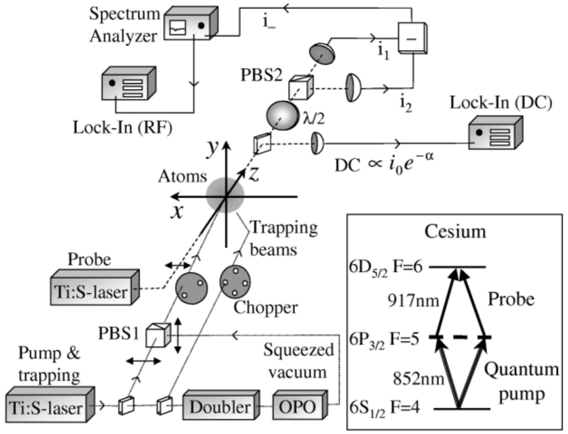


$$\xi_s^2 = \frac{4(\Delta J_\perp)^2}{N} < 1 \qquad \xi_{\mathrm{R}}^2 = \frac{N(\Delta J_\perp)^2_{\min}}{|\langle \mathbf{J} \rangle|^2} < 1$$

Spin Squeezing

Quantum Spin Squeezing

☐ Atom–light Interaction Generated Spin Squeezing



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Eugene Polzik

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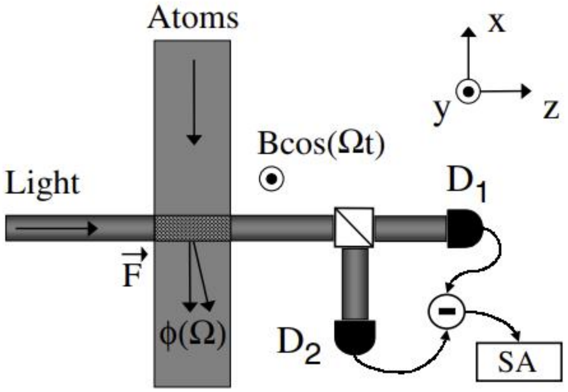
Morgan Mitchell



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Alex Kuzmich

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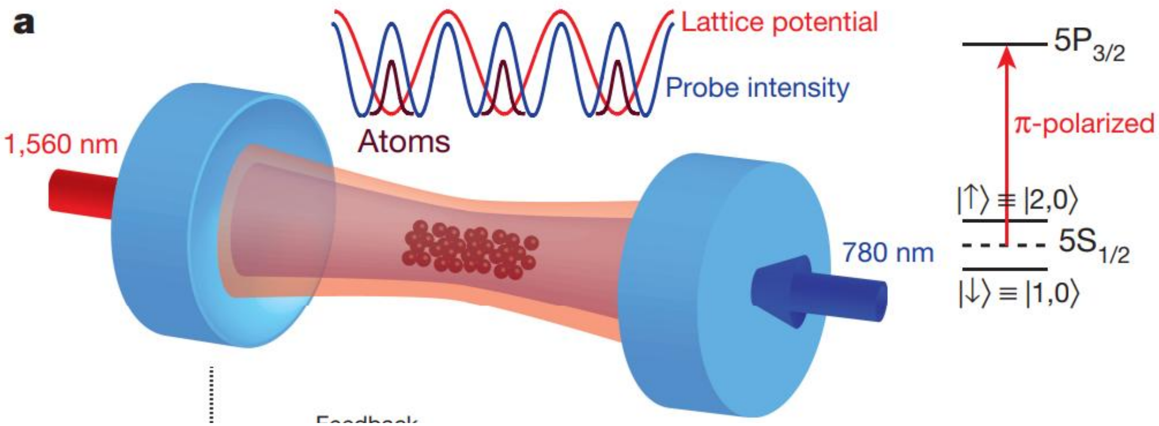
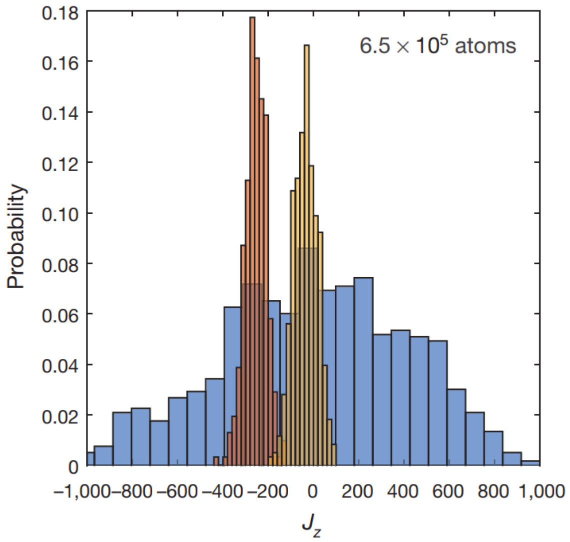
Yanhong Xiao

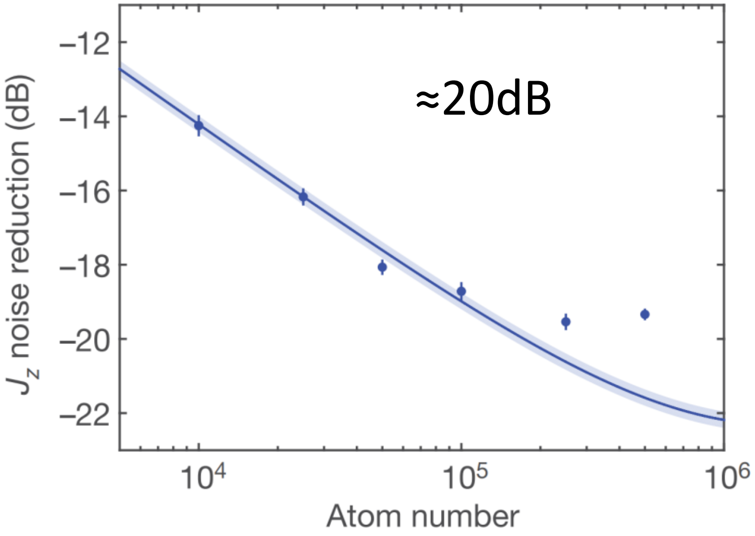
Quantum Spin Squeezing

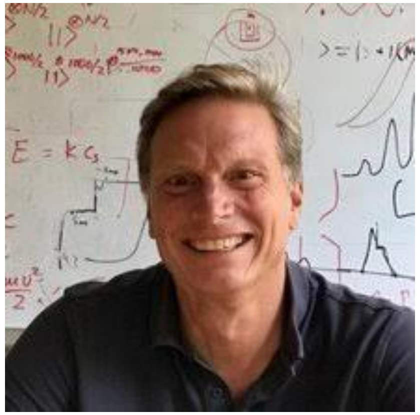
☐ Atom–light Interaction Generated Spin Squeezing

Nature 529, 505 (2016)









Mark Kasevich

Onur Hosten

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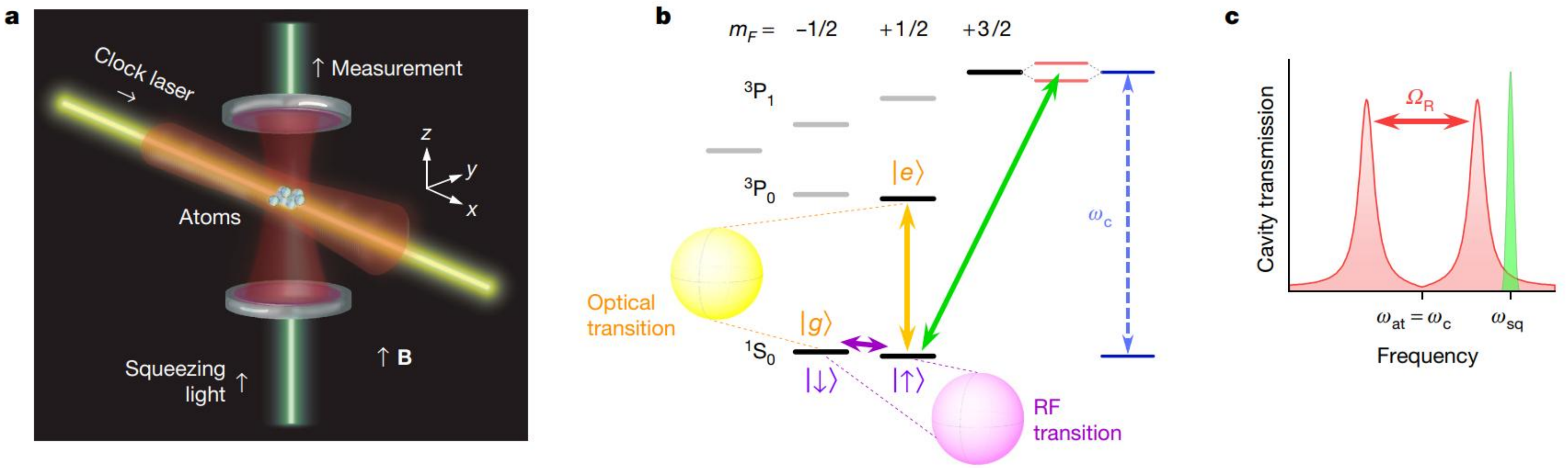
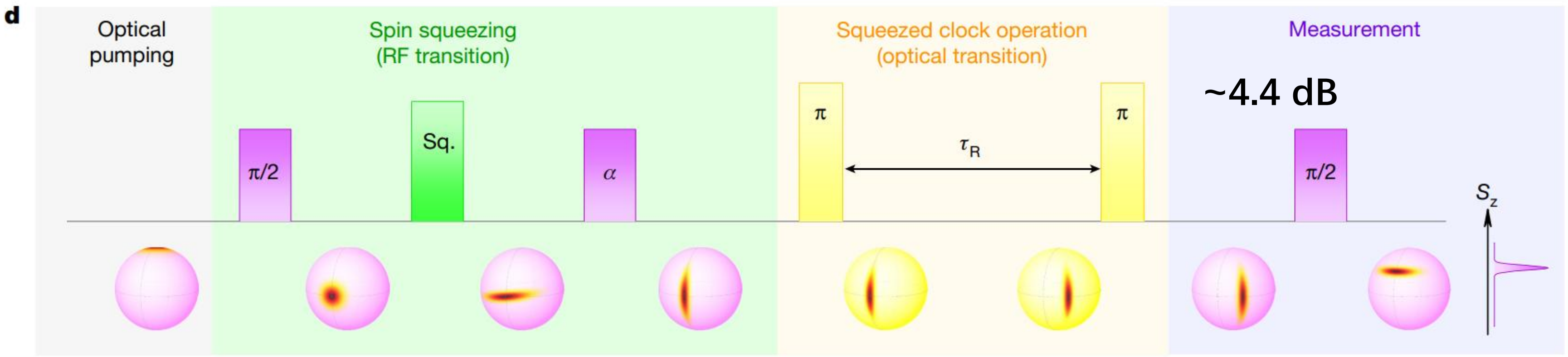
Quantum Spin Squeezing

☐ Atom–light Interaction Generated Spin Squeezing



171Yb optical lattice clock



~4.4 dB

Vladan Vuletić

PRL 104, 073602 (2010).
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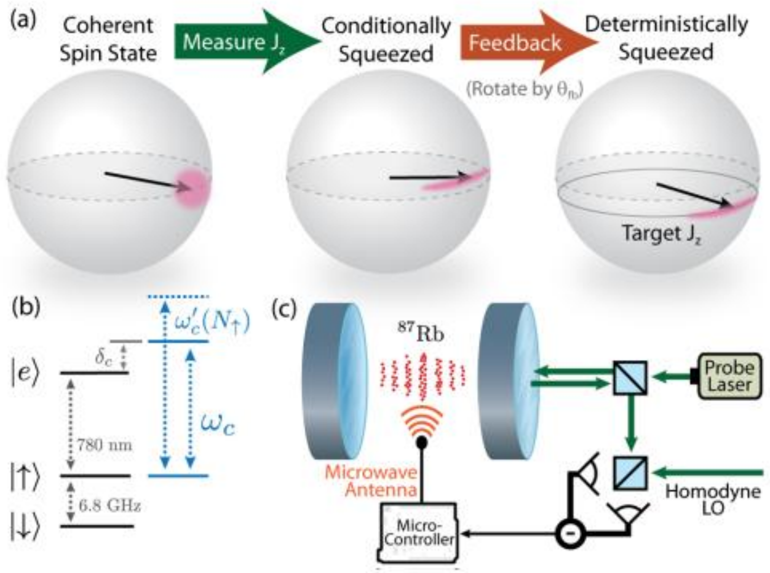
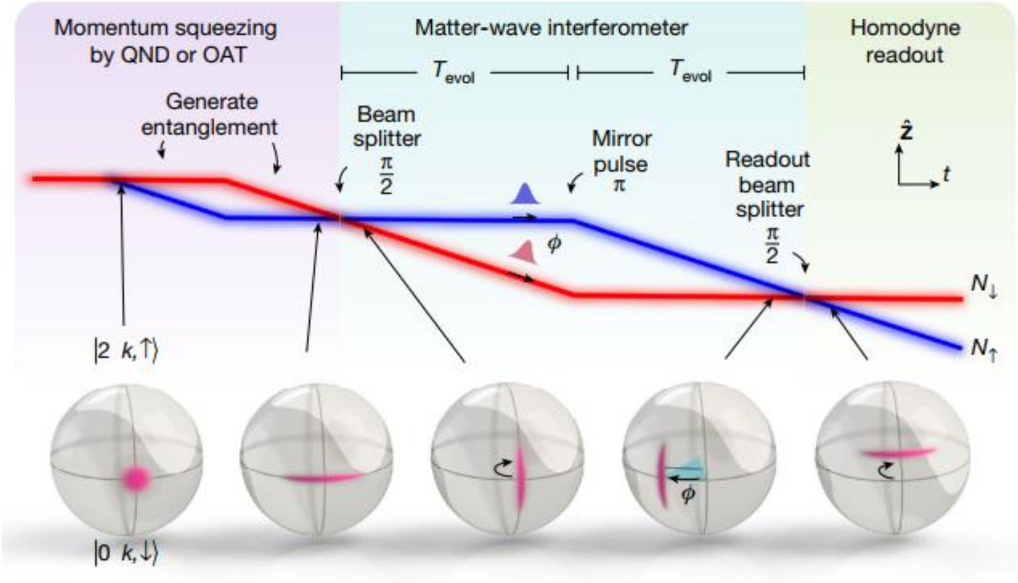
Quantum Spin Squeezing

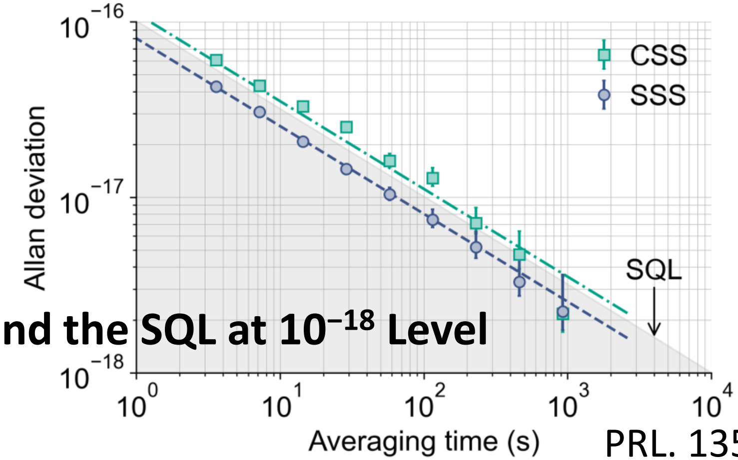
☐ Atom–light Interaction Generated Spin Squeezing



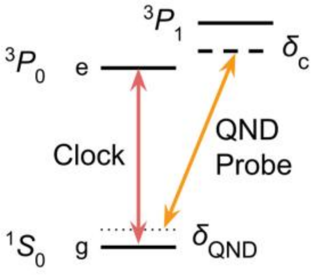
Matter-wave interferometry
~1.7 dB below SQL.



James K. Thompson



beyond the SQL at 10^{-18} Level



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Jun Ye

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Nature 610, 472 (2022).
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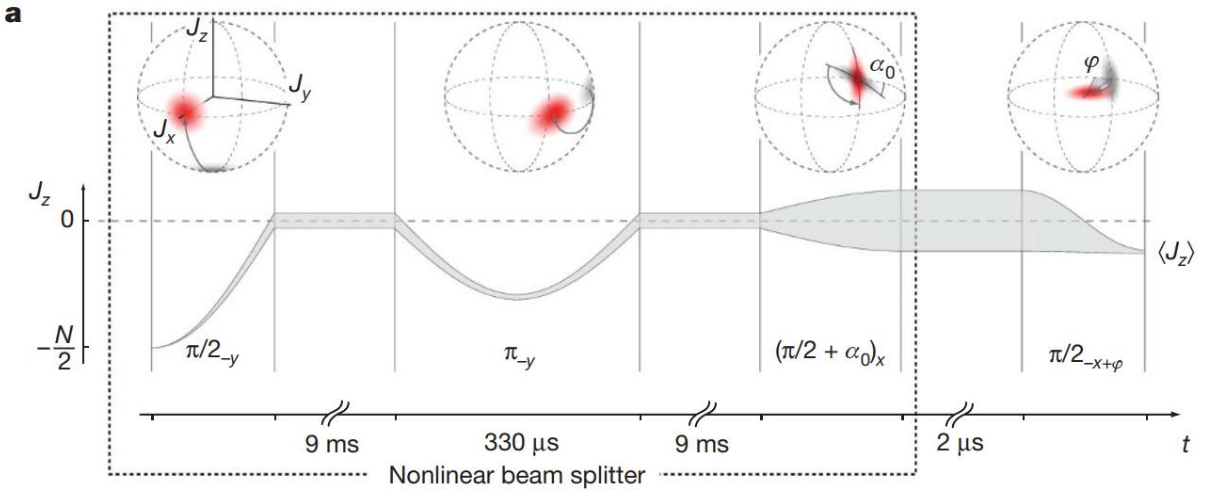
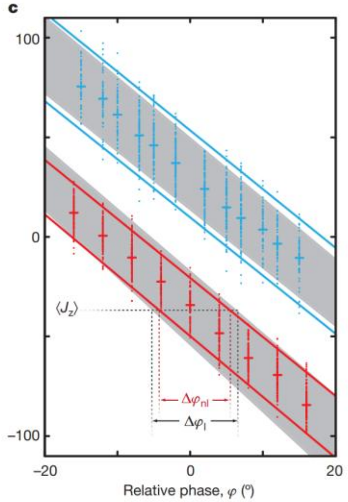
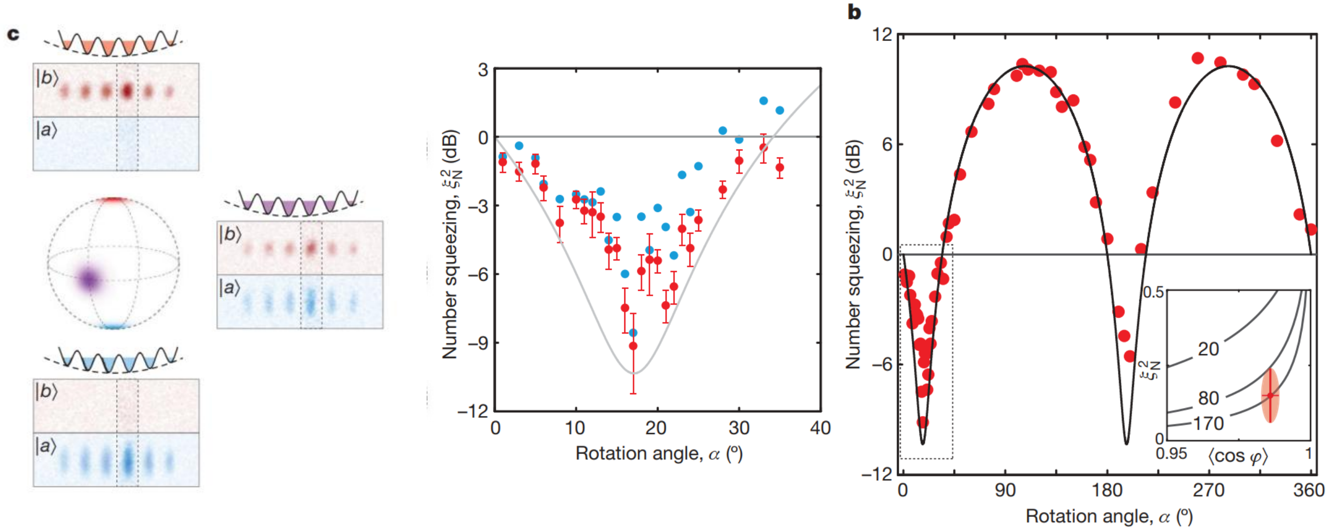
Quantum Spin Squeezing

☐ Collisional Spin Squeezing in BEC





Markus Oberthaler



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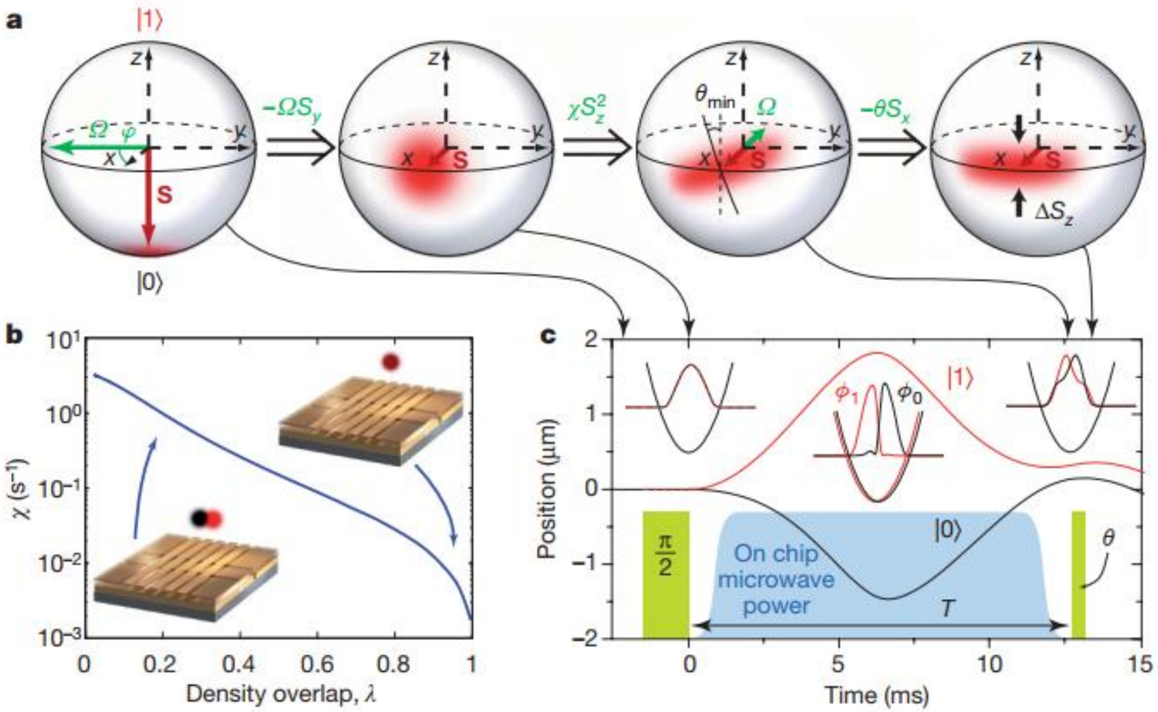
Quantum Spin Squeezing

☐ Collisional Spin Squeezing in BEC



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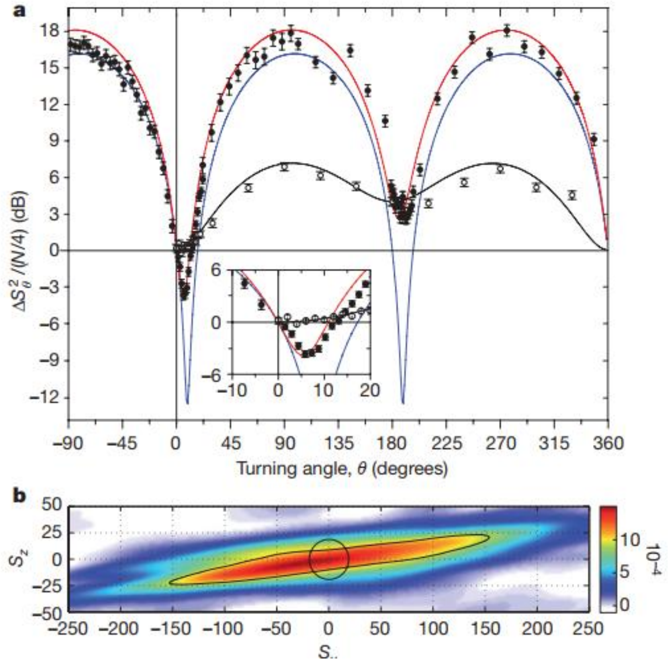

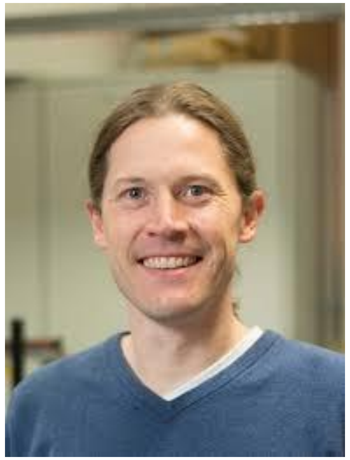

Philipp Treutlein

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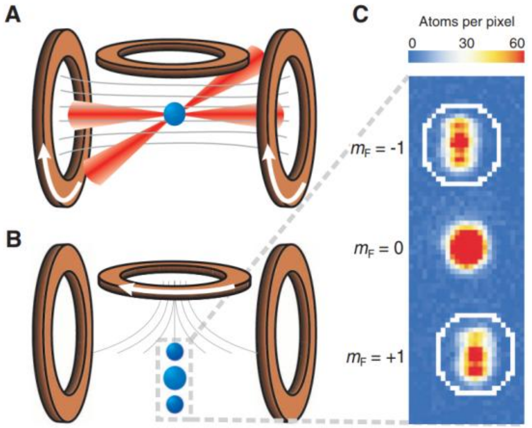
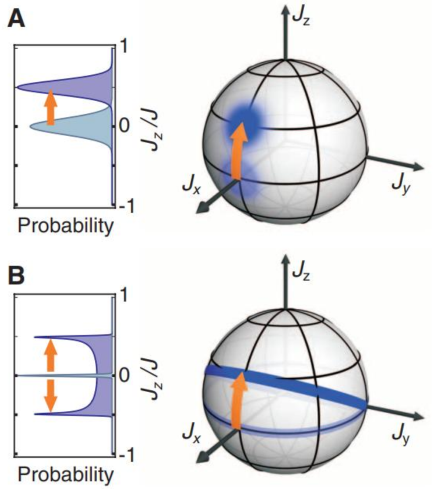
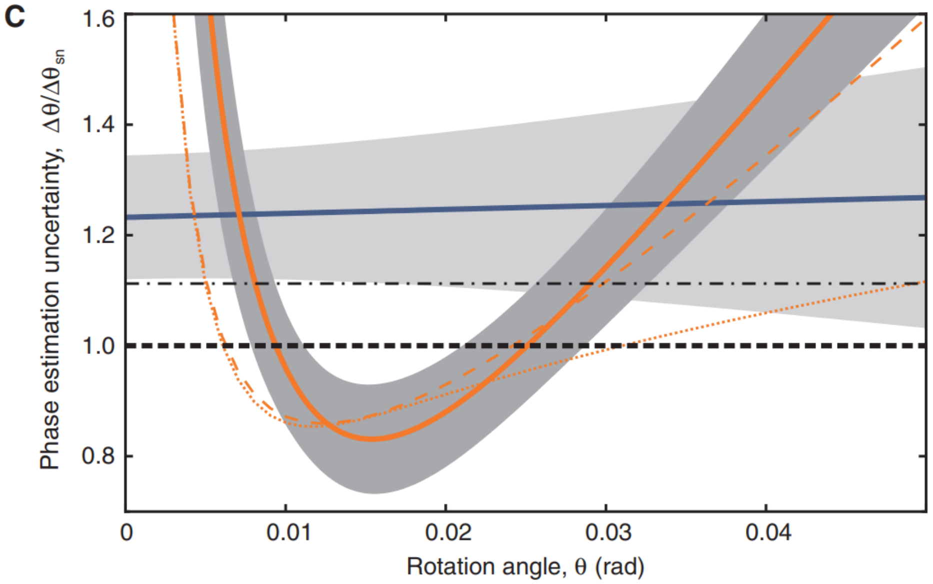
Quantum Spin Squeezing

☐ Collisional Spin Squeezing in BEC







Carsten Klempt

Jan Arlt

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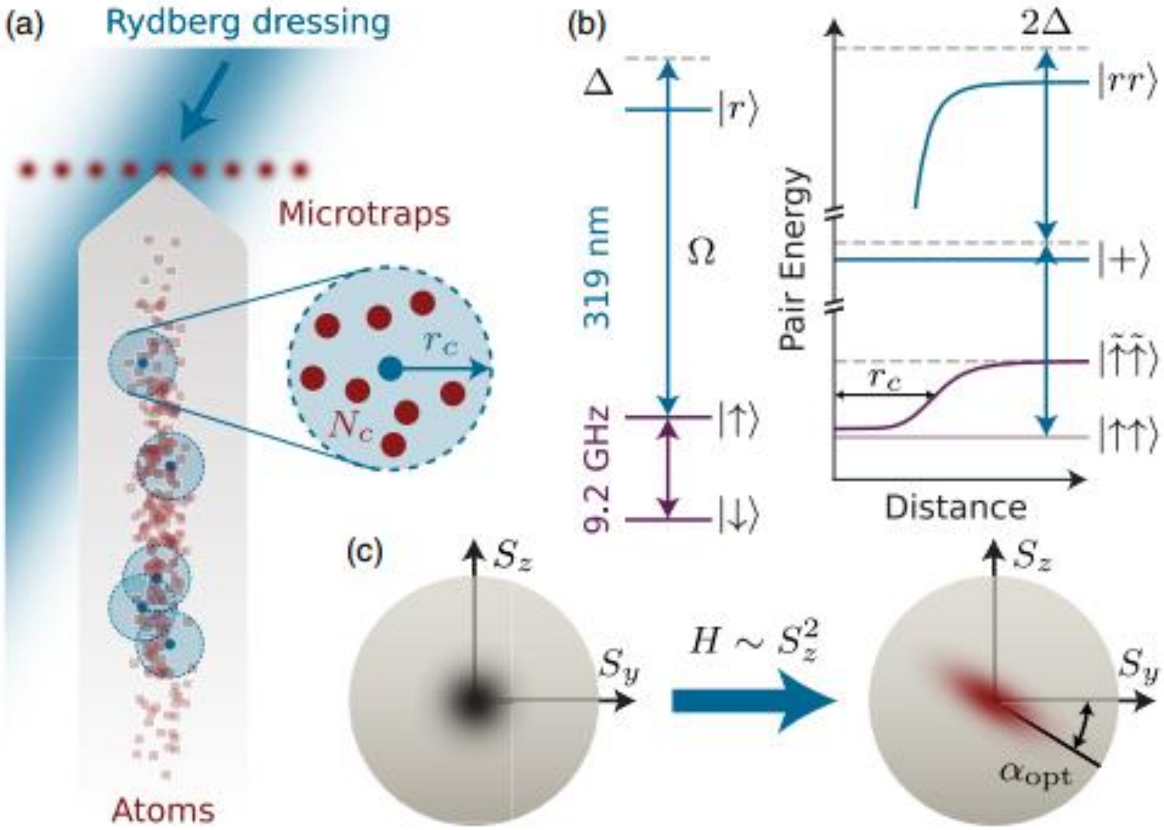
Quantum Spin Squeezing

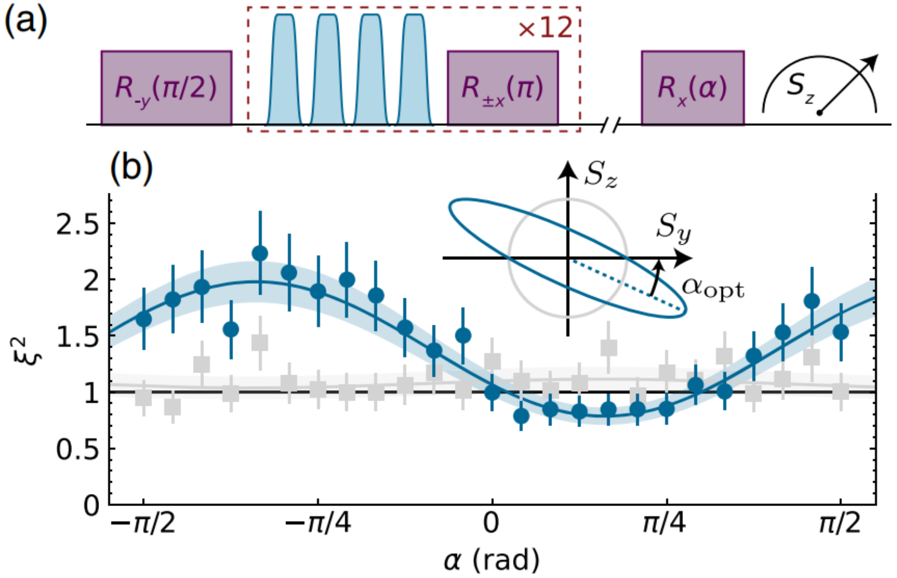
☐ Rydberg-mediated Spin Squeezing

Phys. Rev. Lett. 131, 063401 (2023)



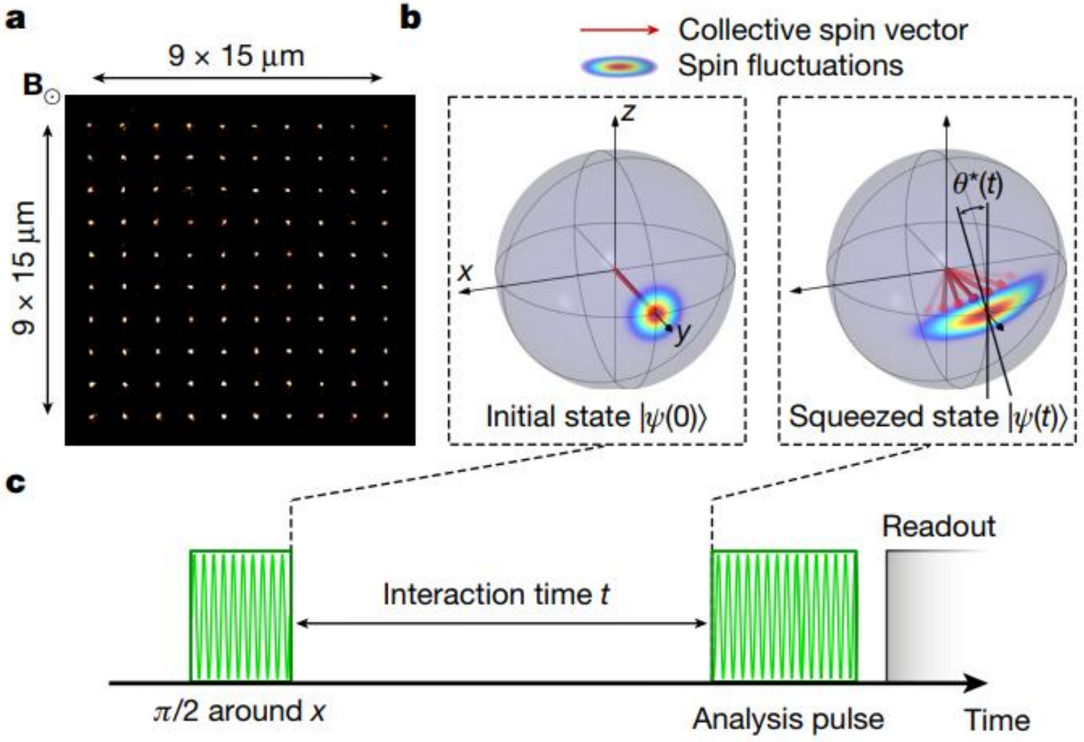
~1.1 dB metrological squeezing

Monika Schleier-Smith

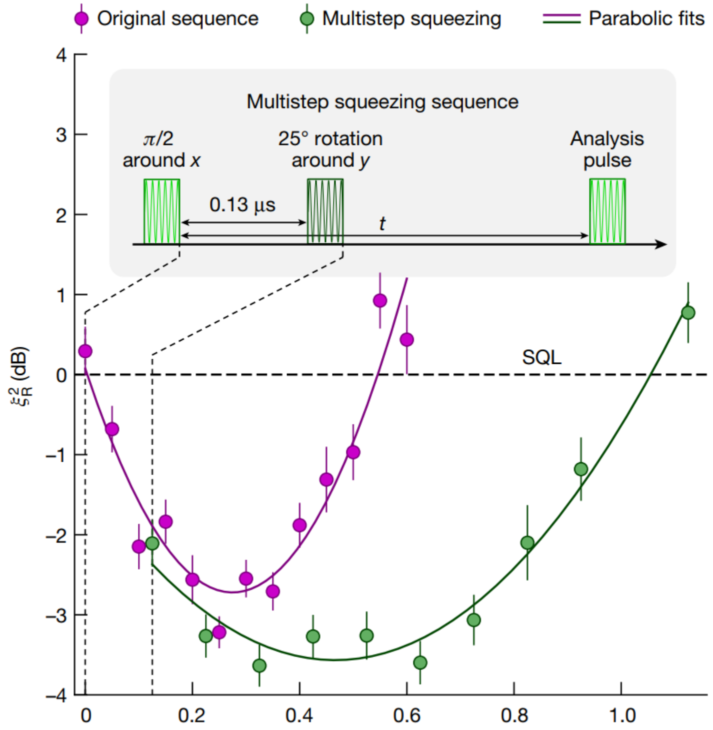


Quantum Spin Squeezing

☐ Rydberg-mediated Spin Squeezing



~3.5 dB observed squeezing and about 5 dB after detection-error correction



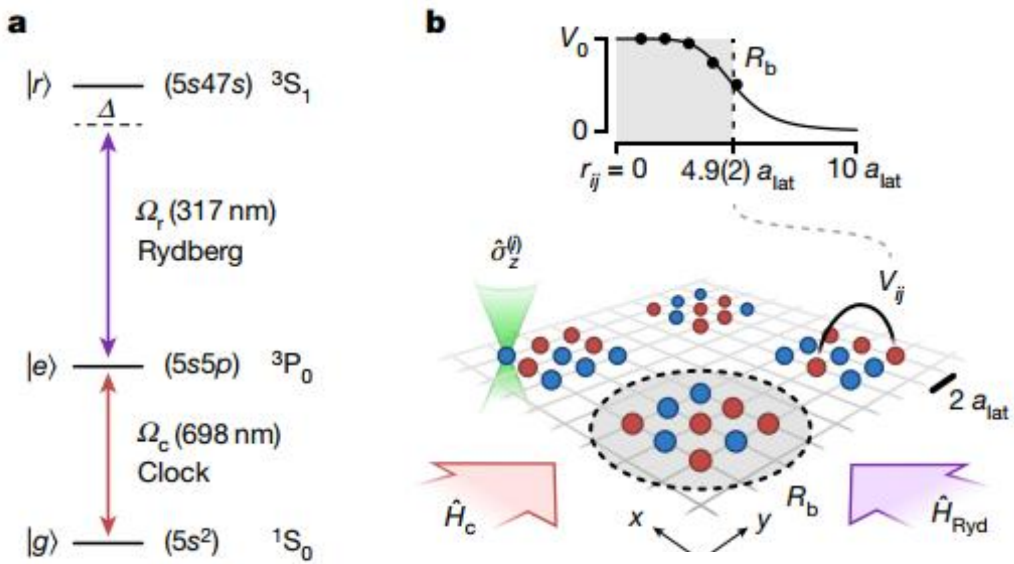
$$H_{XY} = -\frac{J}{2}\sum_{i<j}\frac{a^3}{r_{ij}^3}(\sigma_i^x\sigma_j^x+\sigma_i^y\sigma_j^y),$$

Nature 621, 728 (2023).

Antoine Browaeys

Quantum Spin Squeezing

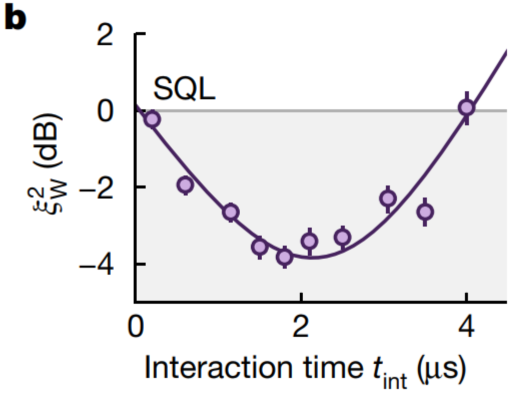
☐ Rydberg-mediated Spin Squeezing



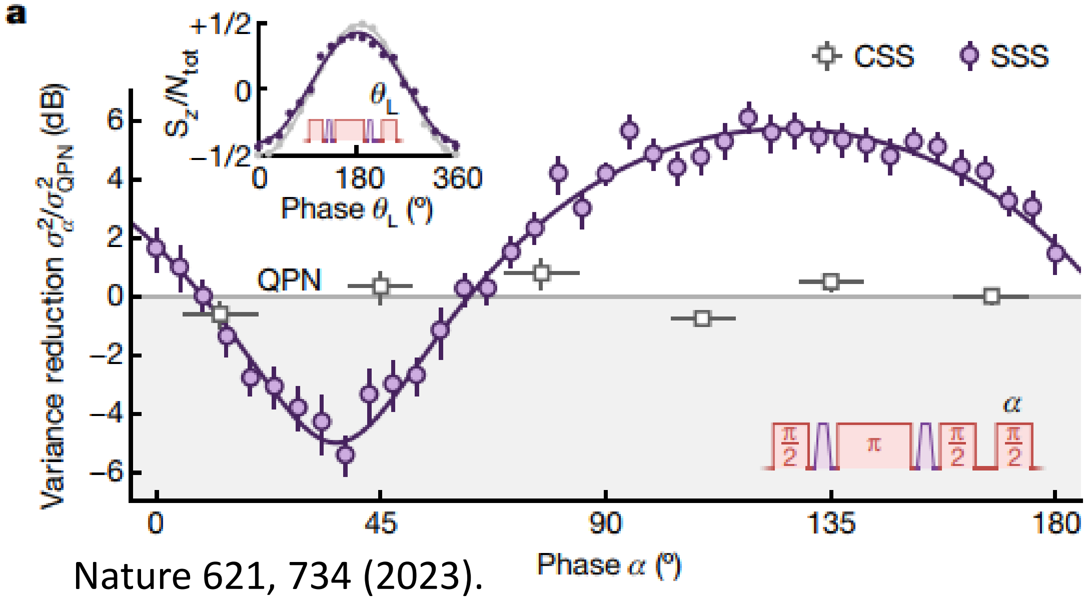
$$\hat{H}_c = \hbar\,\Omega_c\hat{S}_x,$$

$$\hat{H}_{\rm Ryd} = \frac{1}{4}\sum_{i<j} V_{ij}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)} + \frac{1}{2}\sum_{i}\delta_i\hat{\sigma}_z^{(i)},$$

Adam M. Kaufman



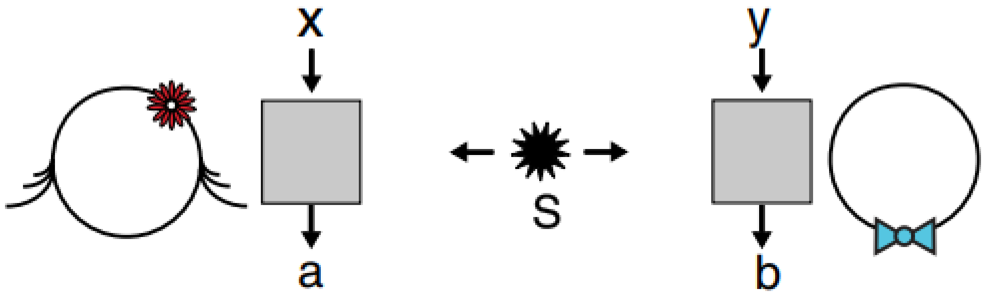
88Sr optical tweezer clock
~4 dB metrological gain

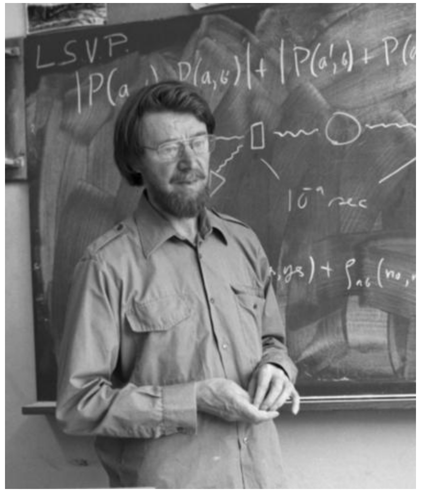


Nature 621, 734 (2023).

Quantum Nonlocality and Bell's Inequalities

In 1935, Einstein, Podolsky and Rosen argued that correlations between distant particles might be explained by **local hidden variables.**





John Stewart Bell
(1928-1990)

Local hidden-variable model λ

$$p(ab|xy) = \int d\lambda\, \rho(\lambda)\, p(a|x,\lambda)p(b|y,\lambda)$$

Correlation function

$$E(x,y) = \sum_{a,b=\pm 1} ab\, p(ab|xy).$$

The local realism implies

$$|E(x,y) - E(x,y')| \leq 1 + E(y,y').$$

Quantum entanglement predicts correlations that violate this bound

Quantum Nonlocality and Bell's Inequalities

Clauser–Horne–Shimony–Holt (CHSH) inequality

A pair of polarization-entangled photons $|\Phi^+\rangle = \frac{|HH\rangle + |VV\rangle}{\sqrt{2}},$

Alice chooses one of two polarization measurement settings

$$x = 0, 1 \quad \Longleftrightarrow \quad \alpha_0, \alpha_1,$$

Bob choose

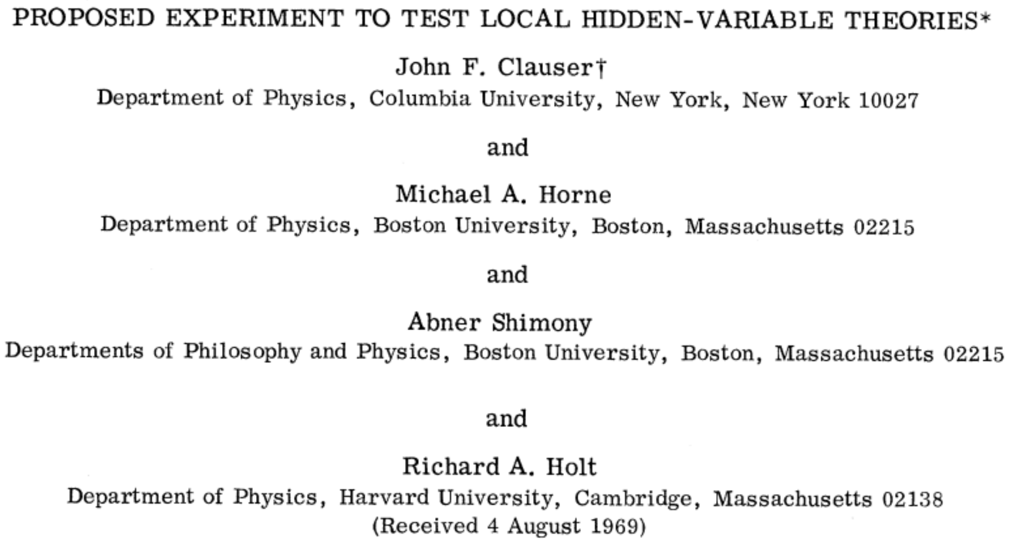
$$y = 0, 1 \quad \Longleftrightarrow \quad \beta_0, \beta_1$$

The CHSH parameter combines four correlations:

$$S = E(0,0) + E(0,1) + E(1,0) - E(1,1).$$

Local hidden-variable theories $|S| \leq 2.$

Quantum mechanics predicts $|S|_{\max} = 2\sqrt{2}.$



PROPOSED EXPERIMENT TO TEST LOCAL HIDDEN-VARIABLE THEORIES*

John F. Clauser†
Department of Physics, Columbia University, New York, New York 10027

and

Michael A. Horne
Department of Physics, Boston University, Boston, Massachusetts 02215

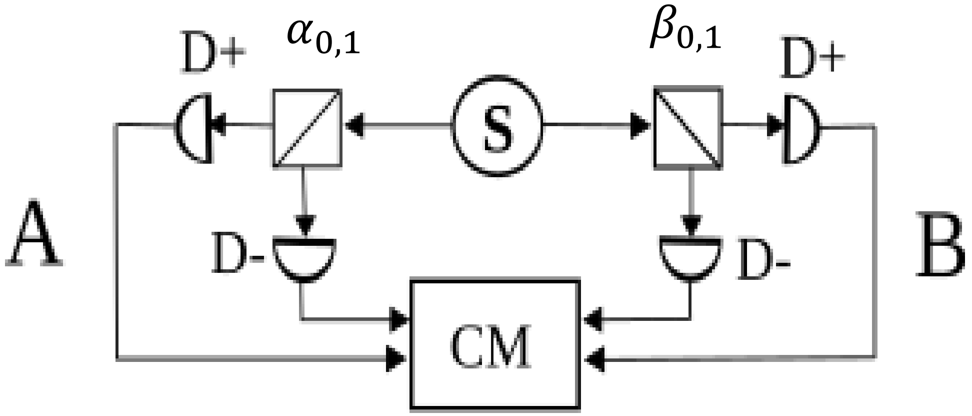
and

Abner Shimony
Departments of Philosophy and Physics, Boston University, Boston, Massachusetts 02215

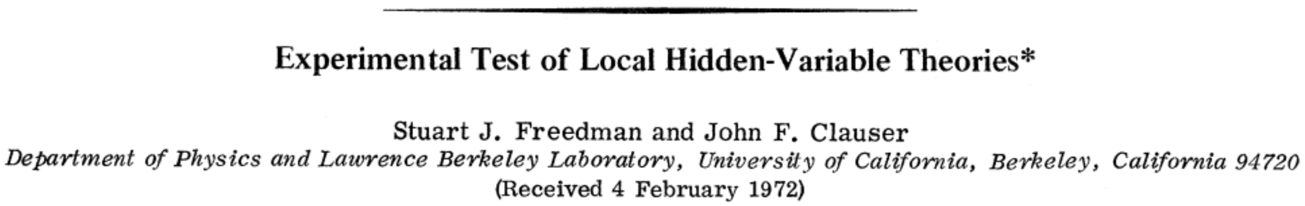
and

Richard A. Holt
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 4 August 1969)

Clauser, Horne, Shimony, and Holt, *Phys. Rev. Lett. 23, 880 (1969).*

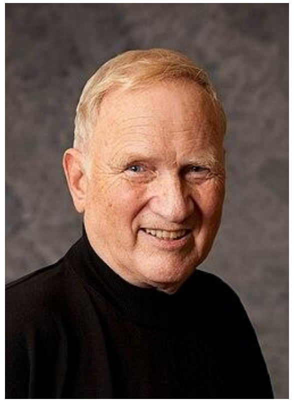


Quantum Nonlocality and Bell's Inequalities



Experimental Test of Local Hidden-Variable Theories*

Stuart J. Freedman and John F. Clauser
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 4 February 1972)



John Clauser

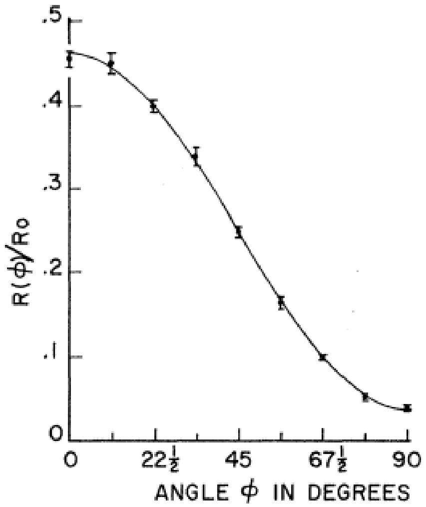
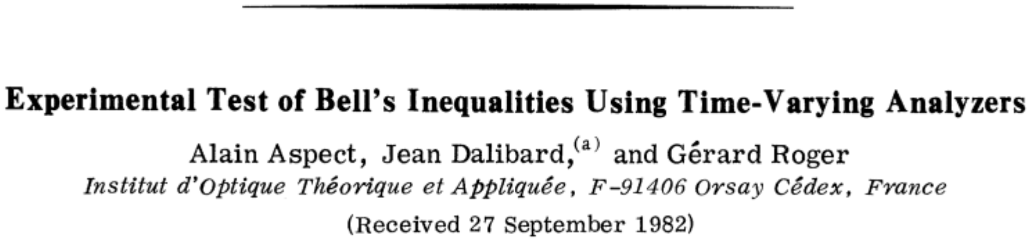
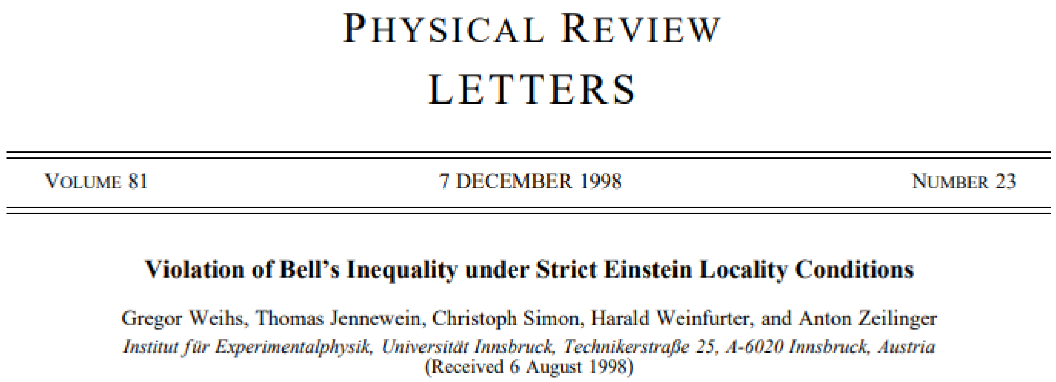


FIG. 3. Coincidence rate with angle φ between the polarizers, divided by the rate with both polarizers removed, plotted versus the angle φ. The solid line is the prediction by quantum mechanics, calculated using the measured efficiencies of the polarizers and solid angles of the experiment.



Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France
(Received 27 September 1982)



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Violation of Bell's Inequality under Strict Einstein Locality Conditions

Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
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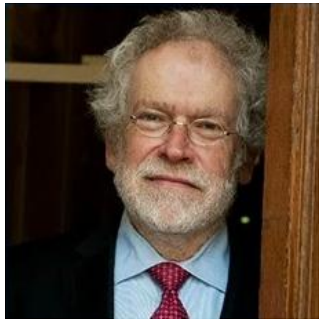
(Received 6 August 1998)

Alain Aspect

Jean Dalibard



Anton Zeilinger

Quantum Nonlocality and Bell’s Inequalities

GHZ Paradox: Bell’s Theorem Without Inequalities

For the three-particle GHZ state,

$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}},$$

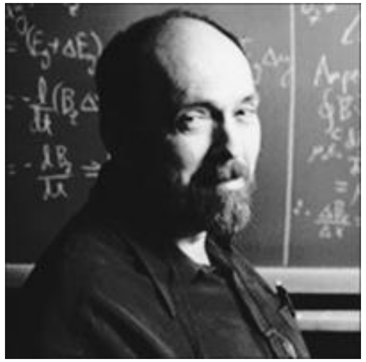
quantum mechanics predicts

$$\langle XXX\rangle = +1, \qquad \langle XYY\rangle = \langle YXY\rangle = \langle YYX\rangle = -1.$$

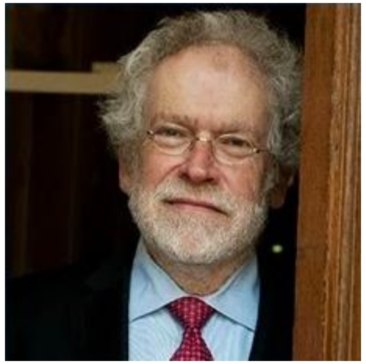
Local realism predicts

$$(XYY)(YXY)(YYX) = XXX = -1$$

Daniel M. Greenberger

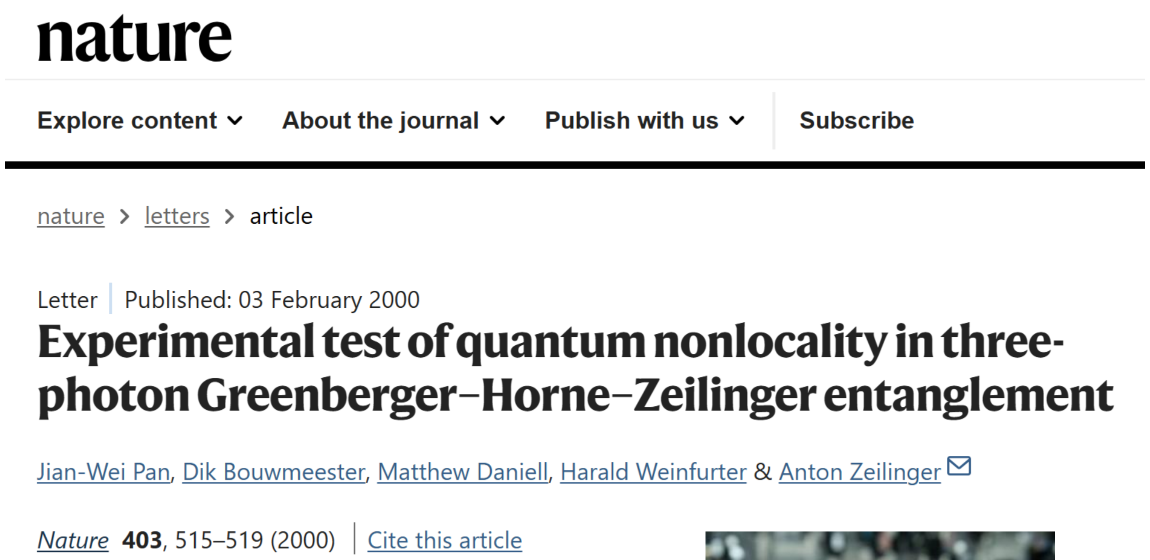


Michael Allan Horne



Anton Zeilinger

D. M. Greenberger, M. A. Horne, and A. Zeilinger, *Going Beyond Bell’s Theorem* (1989).



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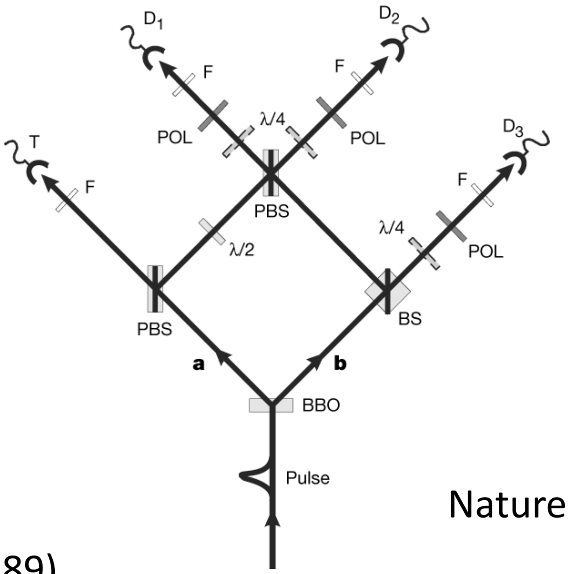
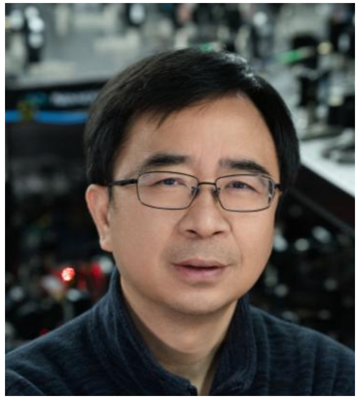
Letter | Published: 03 February 2000

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

Jian-Wei Pan, Dik Bouwmeester, Matthew Daniell, Harald Weinfurter & Anton Zeilinger

Nature **403**, 515–519 (2000) | Cite this article





Jianwei Pan

Nature 403, 515–519 (2000)

NOBELPRISET I FYSIK 2022
THE NOBEL PRIZE IN PHYSICS 2022

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VETENSKAPS-
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Photo: Royal Society

Alain Aspect

Université Paris-Saclay &
École Polytechnique, France

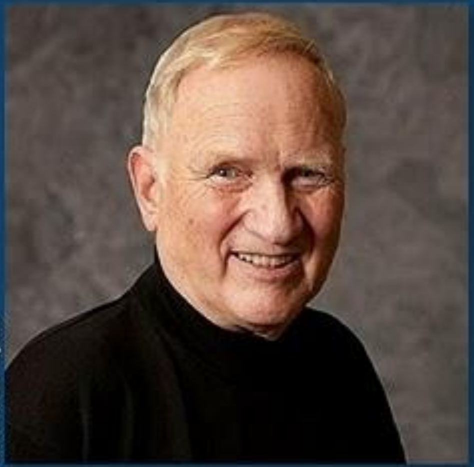


Photo: Peter Lyons

John F. Clauser

J.F. Clauser & Assoc.,
USA


Photo: Sepp Dreissinger

Anton Zeilinger

University of Vienna,
Austria

”för experiment med sammanflätade fotoner som påvisat brott mot Bell-olikheter och banat väg för kvantinformationsvetenskap”

“for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”

#nobelprize

THE NOBEL PRIZE

Bell Correlations with Many-body Quantum States

Why is many-body nonlocality hard?

Science 344, 1256 (2014).

Jordi Tura

Maciej Lewenstein

For an N-body Bell experiment, in principle the full correlation vector has dimension $3^N - 1$!

Theory: classical correlation space grows exponentially with N.

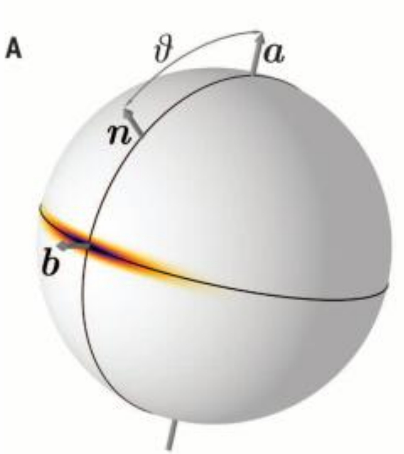
Experiment: high-order N-body correlators are hard to measure.

Can nonlocality be detected using only one- and two-body correlations?

$$S_k = \sum_{i=1}^{N} \langle M_k^{(i)} \rangle, \qquad S_{kl} = \sum_{i \neq j} \langle M_k^{(i)} M_l^{(j)} \rangle, \quad k, l = 0, 1.$$

Permutation symmetry reduces the inequality to

$$-2S_0 + \frac{1}{2} S_{00} - S_{01} + \frac{1}{2} S_{11} + 2N \geq 0$$



- $\mathbf{n}$: mean-spin direction
- $\mathbf{a}$: squeezed direction
- $\hat{S}_{\mathbf{n}}$: collective polarization
- $\hat{S}_{\mathbf{a}}^2$: collective spin noise

$$\boxed{\hat{W} = -\left| \frac{\hat{S}_n}{N/2} \right| + (a \cdot n)^2 \frac{\hat{S}_a^2}{N/4} + 1 - (a \cdot n)^2}$$

Bell Correlations with Many-body Quantum States

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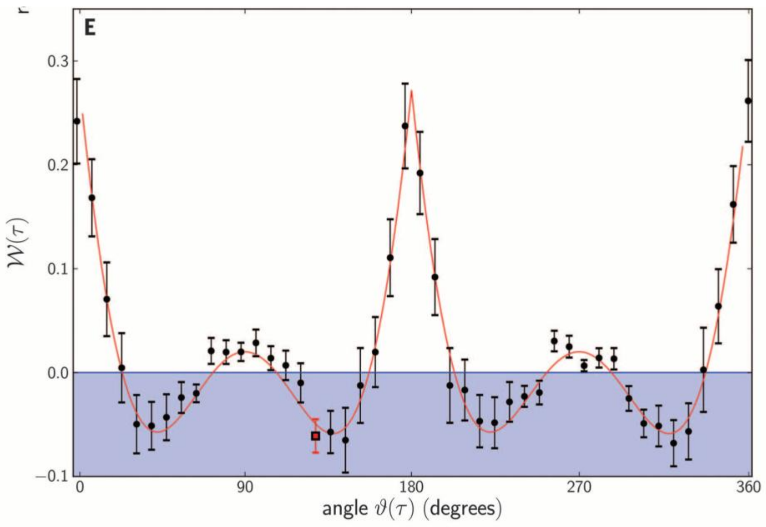
HOME > SCIENCE > VOL. 352, NO. 6284 > BELL CORRELATIONS IN A BOSE-EINSTEIN CONDENSATE

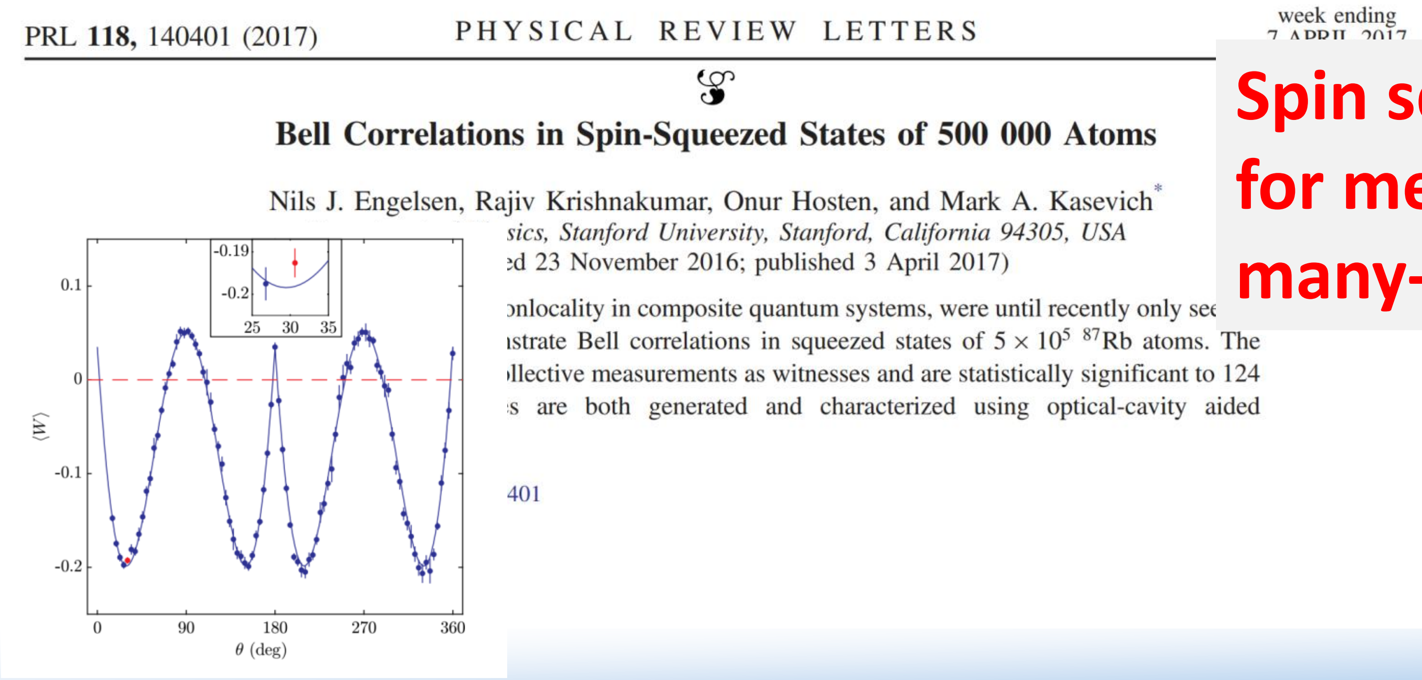
REPORT

Bell correlations in a Bose-Einstein condensate

ROMAN SCHMIED, JEAN-DANIEL BANCAL, BAPTISTE ALLARD, MATTEO FADEL, VALERIO SCARANI, PHILIPP TREUTLEIN, AND NICOLAS SANGOUARD Authors Info & Affiliations

SCIENCE • 22 Apr 2016 • Vol 352, Issue 6284 • pp. 441-444 • DOI: 10.1126/science.aad8665





PRL **118,** 140401 (2017) PHYSICAL REVIEW LETTERS week ending 7 APRIL 2017

Bell Correlations in Spin-Squeezed States of 500 000 Atoms

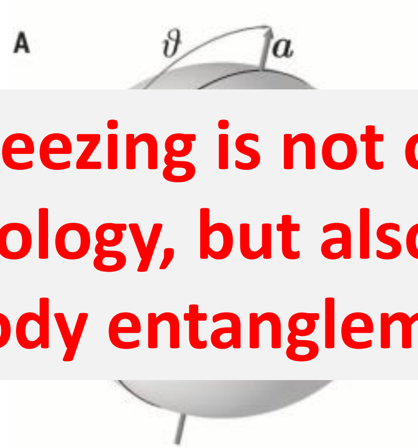
Nils J. Engelsen, Rajiv Krishnakumar, Onur Hosten, and Mark A. Kasevich*

sics, *Stanford University, Stanford, California 94305, USA*

ed 23 November 2016; published 3 April 2017)

onlocality in composite quantum systems, were until recently only see
strate Bell correlations in squeezed states of 5×10^5 ^{87}Rb atoms. The
llective measurements as witnesses and are statistically significant to 124
s are both generated and characterized using optical-cavity aided

401



Spin squeezing is not only a resource for metrology, but also a witness of many-body entanglement!

$$\hat{W} = -\left|\frac{\hat{S}_{\boldsymbol{n}}}{N/2}\right| + (\boldsymbol{a}\cdot\boldsymbol{n})^2 \frac{\hat{S}_{\boldsymbol{a}}^2}{N/4} + 1 - (\boldsymbol{a}\cdot\boldsymbol{n})^2$$

Spin-1 Bose Einstein Condensate

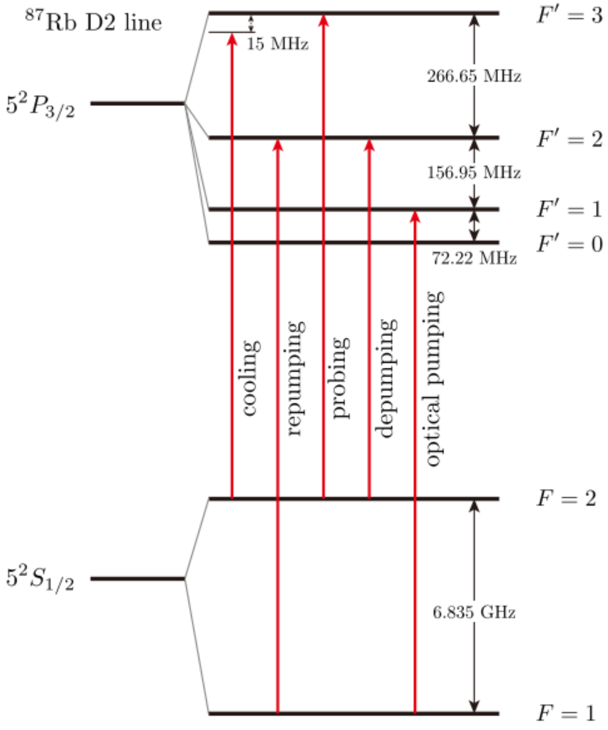
MIT, ENS, JILA, MPQ, Berkeley,
Hannover, NIST, Gatech,…

T.-L Ho, Phys. Rev. Lett. **81**, 742 (1998).
T. Ohmi and K. Machida, J. Phys. Soc. Jpn **67**, 1822 (1998).
C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. **81**, 5257 (1998).

$$H = \frac{c_2}{2N}[2(\hat{a}_1^+\hat{a}_{-1}^+\hat{a}_0\hat{a}_0 + h.c.) + (2N_0 - 1)(N - N_0)] - qN_0$$

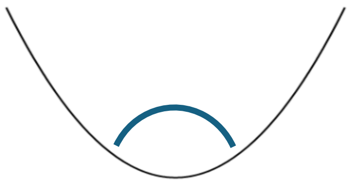
Spin-mixing dynamics

Effective quadratic Zeeman shift

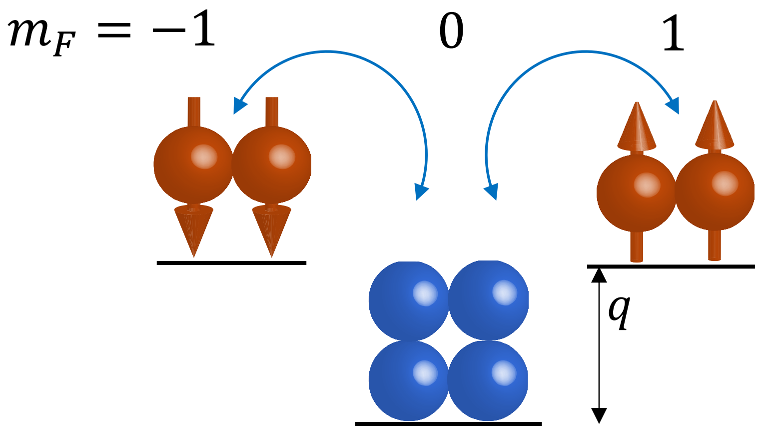


- External degree

Single-mode approximation



- Internal degree (F=1)



Spin-1 Bose Einstein Condensate

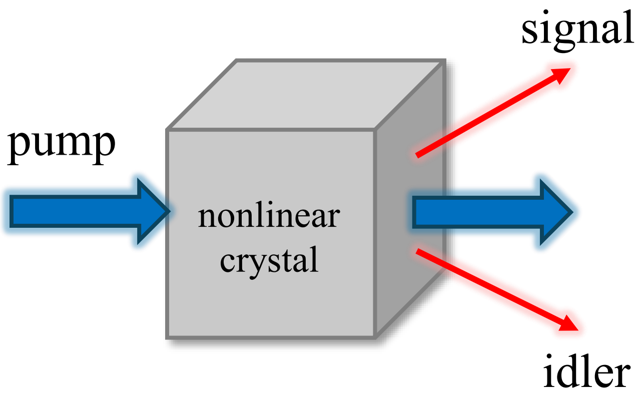
L.-M. Duan, A. Sørensen, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **85**, 3991 (2000).
H. Pu and P. Meystre, *Phys. Rev. Lett.* **85**, 3987 (2000).

$$H = \frac{c_2}{2N}\left[2(\boldsymbol{\hat{a}_1^\dagger \hat{a}_{-1}^\dagger \hat{a}_0 \hat{a}_0 + h.c.}) + (2N_0 - 1)(N - N_0)\right] - qN_0$$

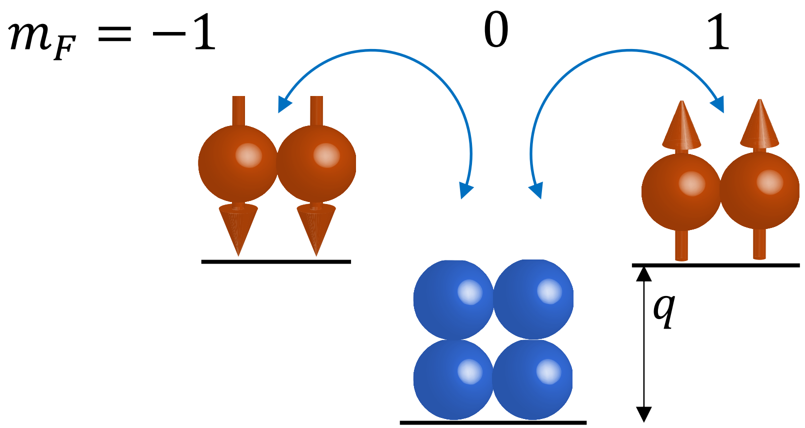
$$q = |c_2| \qquad N_0 \approx N \quad \Rightarrow \quad \hat{a}_0^\dagger \to \sqrt{N} e^{i\theta_0}$$

$$H_{\text{OPA}} = i\hbar\chi^{(2)}\left(\alpha_{\text{pump}}^* \hat{a}\hat{b} - \alpha_{\text{pump}}\hat{a}^\dagger\hat{b}^\dagger\right)$$

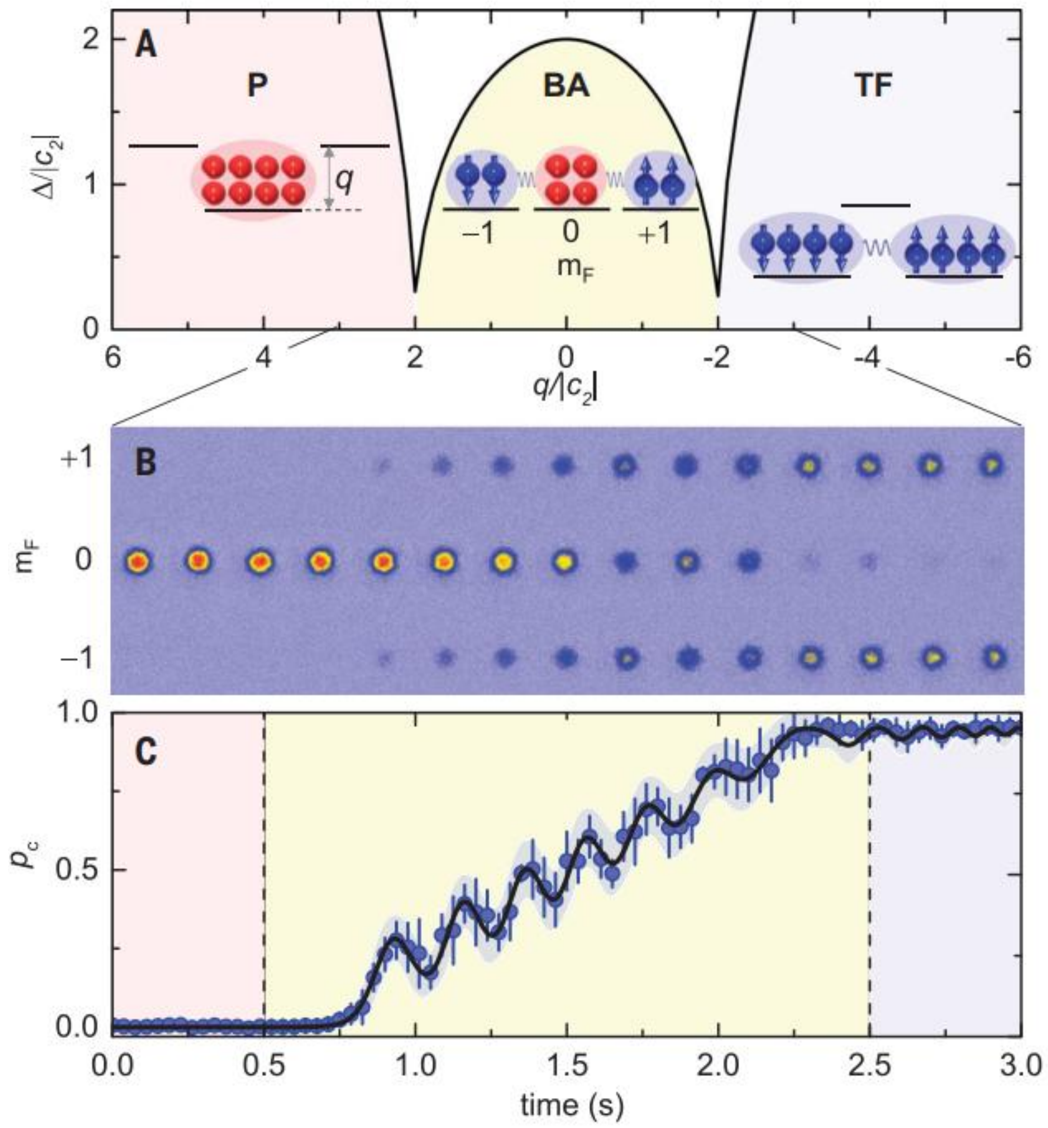
$$\hat{H} \simeq c_2\left(e^{2i\theta_0}\hat{a}_1\hat{a}_{-1} + e^{-2i\theta_0}\hat{a}_1^\dagger\hat{a}_{-1}^\dagger\right)$$



Optical parametric amplification (OPA)

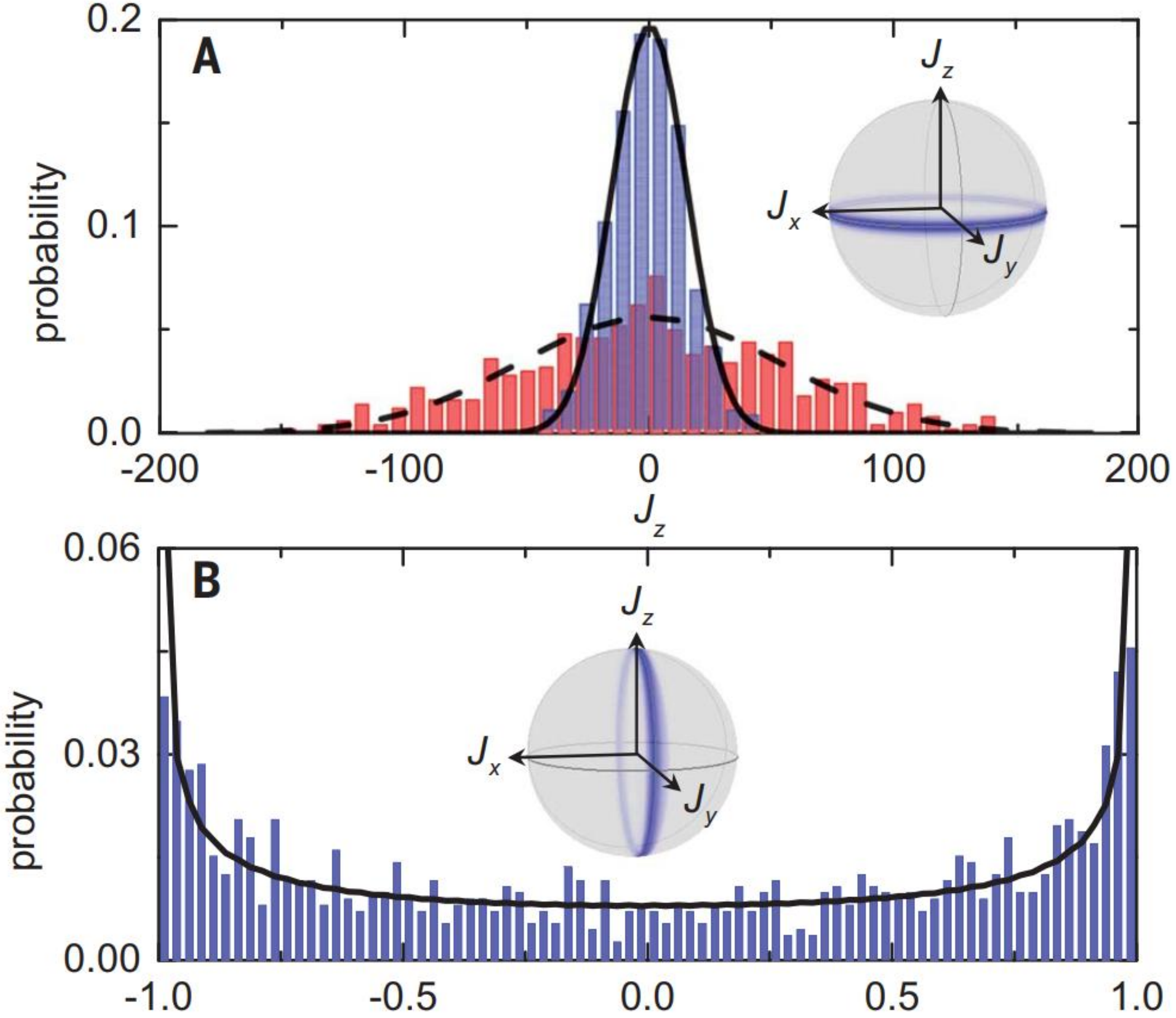


Spin-1 Bose Einstein Condensate

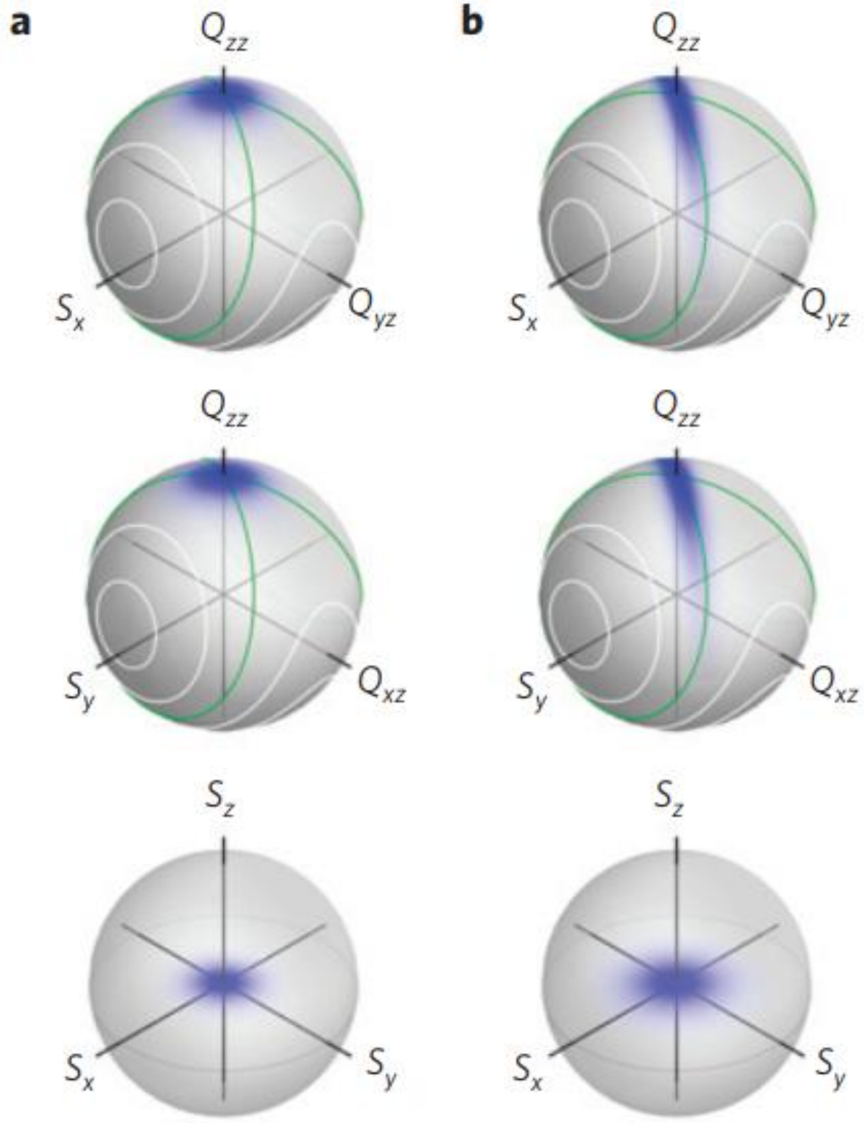


Twin-Fock state ~11,000 rubidium-87 atoms

Science **355**, 620 (2017).



Spin-1 Bose Einstein Condensate



Spin (dipole) operators

$$\boxed{L_x = \frac{1}{\sqrt{2}}\left(a_1^\dagger a_0 + a_0^\dagger a_1 + a_{-1}^\dagger a_0 + a_0^\dagger a_{-1}\right),}$$

$$L_y = \frac{i}{\sqrt{2}}\left(-a_1^\dagger a_0 - a_0^\dagger a_{-1} + a_0^\dagger a_1 + a_{-1}^\dagger a_0\right),$$

$$L_z = a_1^\dagger a_1 - a_{-1}^\dagger a_{-1}.$$

Spin-1 Nematic (Quadrupole) Operators

$$\boxed{Q_{yz} = \frac{i}{\sqrt{2}}\left(-a_1^\dagger a_0 + a_0^\dagger a_1 - a_{-1}^\dagger a_0 + a_0^\dagger a_{-1}\right),}$$

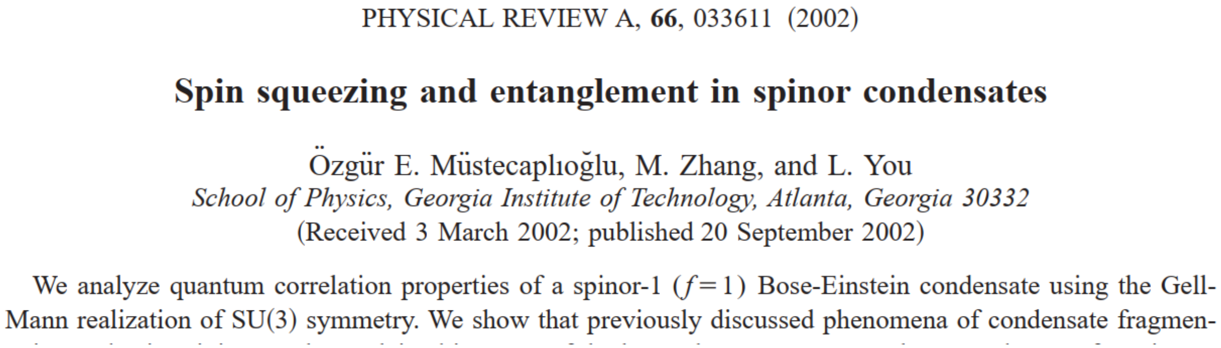
$$Q_{xz} = \frac{1}{\sqrt{2}}\left(a_1^\dagger a_0 + a_0^\dagger a_1 - a_{-1}^\dagger a_0 - a_0^\dagger a_{-1}\right),$$

$$Q_{xx} = \frac{2}{3}a_0^\dagger a_0 - \frac{1}{3}a_1^\dagger a_{-1} - \frac{1}{3}a_{-1}^\dagger a_{-1} + a_{-1}^\dagger a_1 + a_1^\dagger a_{-1},$$

$$Q_{yy} = \frac{2}{3}a_0^\dagger a_0 - \frac{1}{3}a_1^\dagger a_{-1} - \frac{1}{3}a_{-1}^\dagger a_{-1} - \hat{a}_{-1}^\dagger a_1 - a_1^\dagger a_{-1},$$

$$\boxed{Q_{zz} = \frac{2}{3}a_1^\dagger a_1 - \frac{4}{3}a_0^\dagger a_0 + \frac{2}{3}a_{-1}^\dagger a_{-1},}$$

Spin-1 Bose Einstein Condensate



PHYSICAL REVIEW A, **66**, 033611 (2002)

Spin squeezing and entanglement in spinor condensates

Özgür E. Müstecaplıoğlu, M. Zhang, and L. You

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

(Received 3 March 2002; published 20 September 2002)

We analyze quantum correlation properties of a spinor-1 ($f=1$) Bose-Einstein condensate using the Gell-Mann realization of SU(3) symmetry. We show that previously discussed phenomena of condensate fragmen-

Özgür E. Müstecaplıoğlu

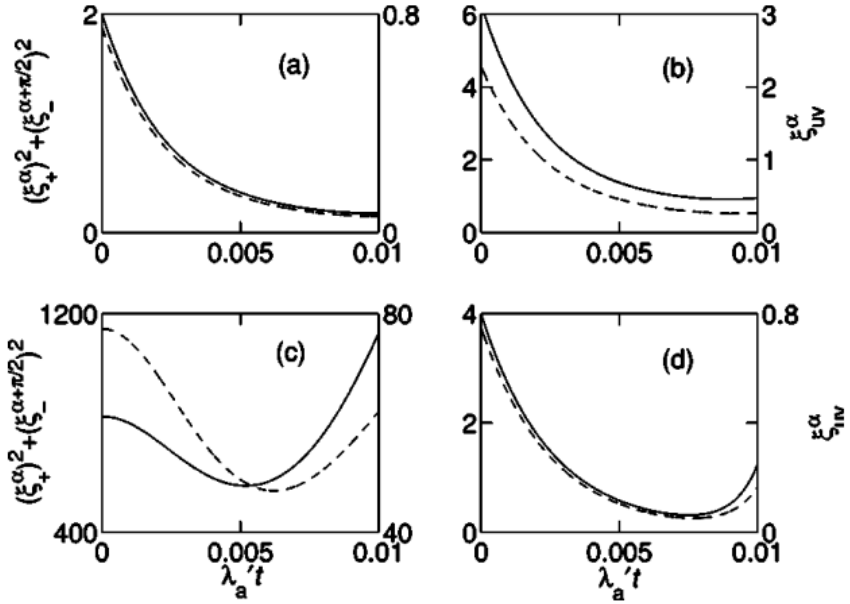


FIG. 5. Time-dependent U-V squeezing parameter (dashed curve) and two-mode entanglement criterion (solid curve) for N

U-V pseudospins $U: |-1\rangle \leftrightarrow |0\rangle, \qquad V: |+1\rangle \leftrightarrow |0\rangle$

$$U_+ = a_-^\dagger a_0, \qquad V_+ = a_+^\dagger a_0$$

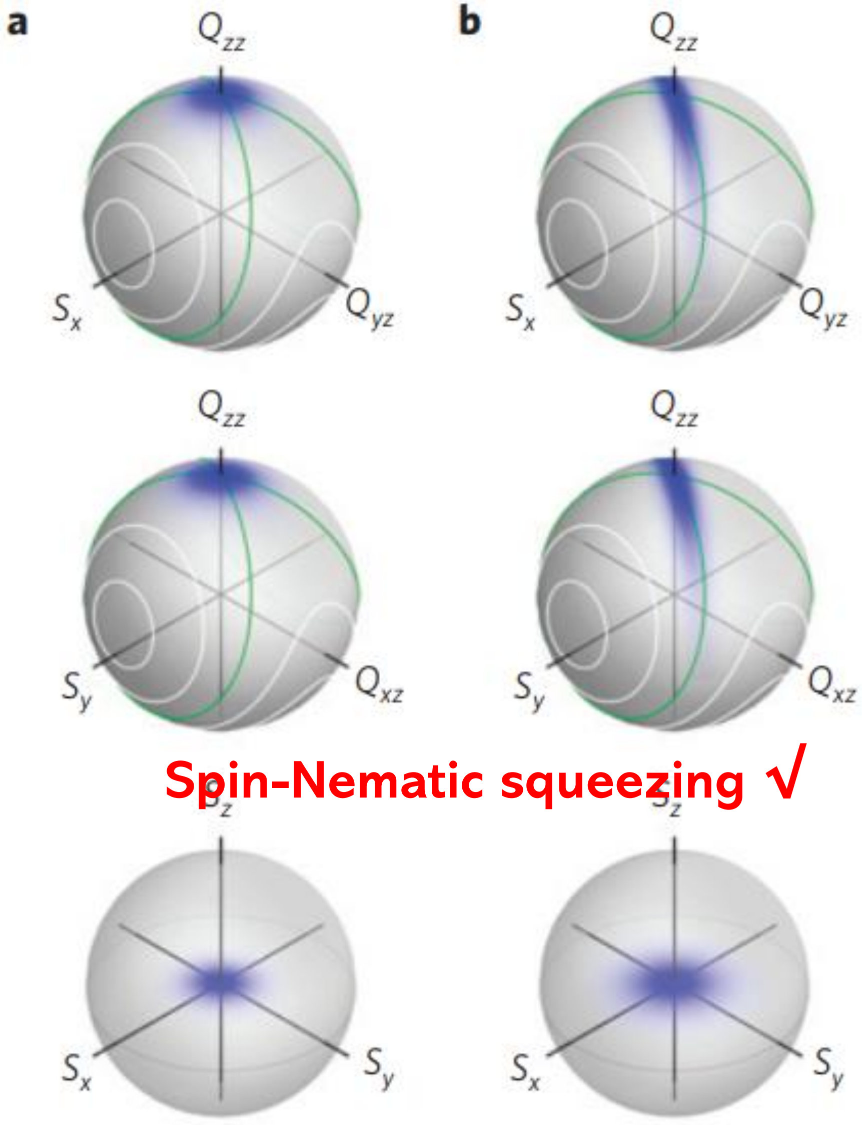
$$U_- = a_0^\dagger a_-, \qquad V_- = a_0^\dagger a_+$$

U and V pseudospins combine into spin dipoles and nematic quadrupoles

$$F_x \propto U_x + V_x, \quad Q_{xz} \propto V_x - U_x$$

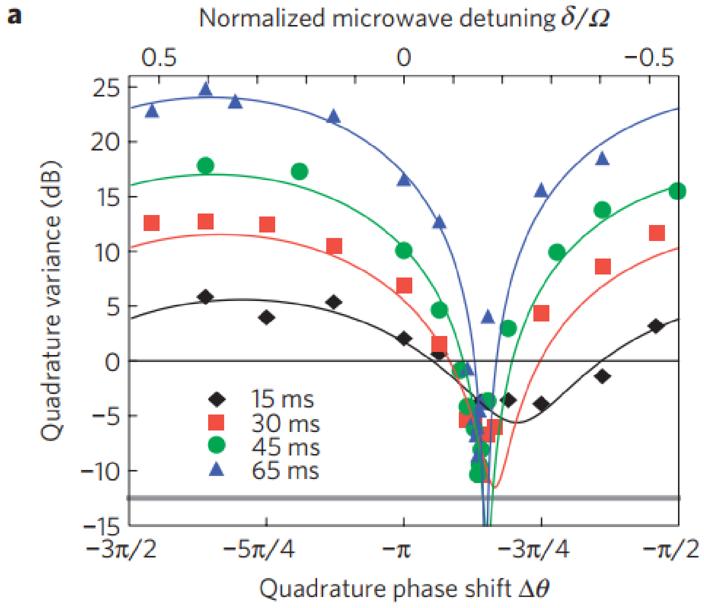
$$F_y \propto V_y - U_y, \quad Q_{yz} \propto U_y + V_y$$

Spin-1 Bose Einstein Condensate



Spin-Nematic squeezing √

Spin squeezing ×



Michael Chapman

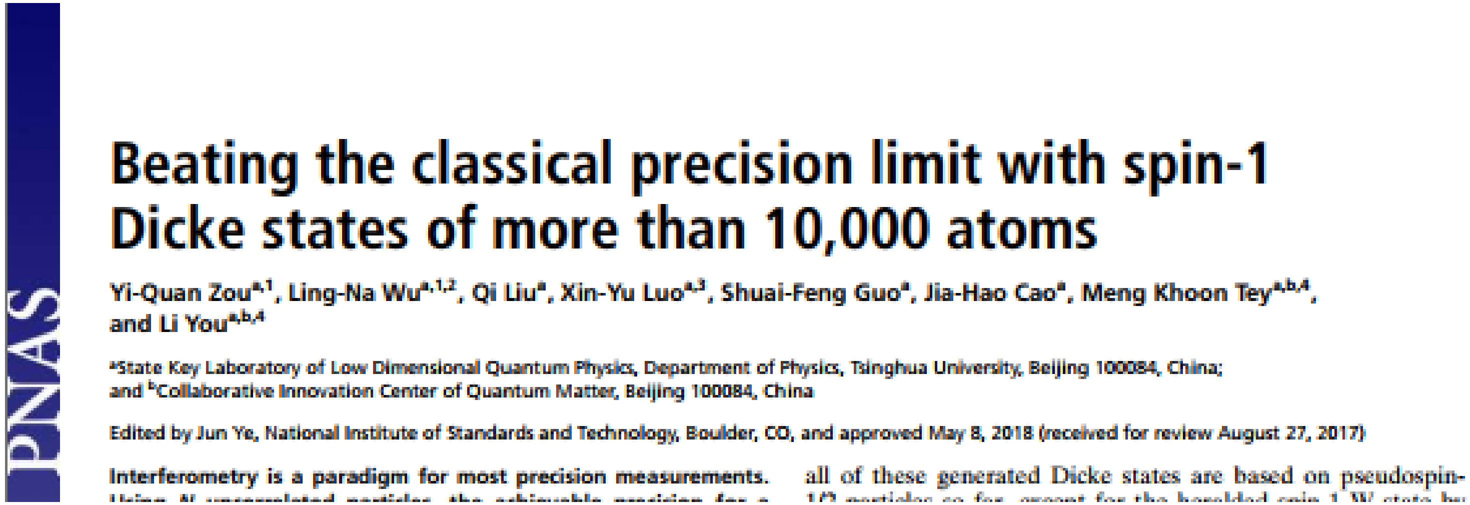
nature physics

LETTERS

Spin-nematic squeezed vacuum in a quantum gas

C. D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans and M. S. Chapman*

Spin-1 Bose Einstein Condensate



PNAS

Beating the classical precision limit with spin-1 Dicke states of more than 10,000 atoms

Yi-Quan Zou[a,1], Ling-Na Wu[a,1,2], Qi Liu[a], Xin-Yu Luo[a,3], Shuai-Feng Guo[a], Jia-Hao Cao[a], Meng Khoon Tey[a,b,4], and Li You[a,b,4]

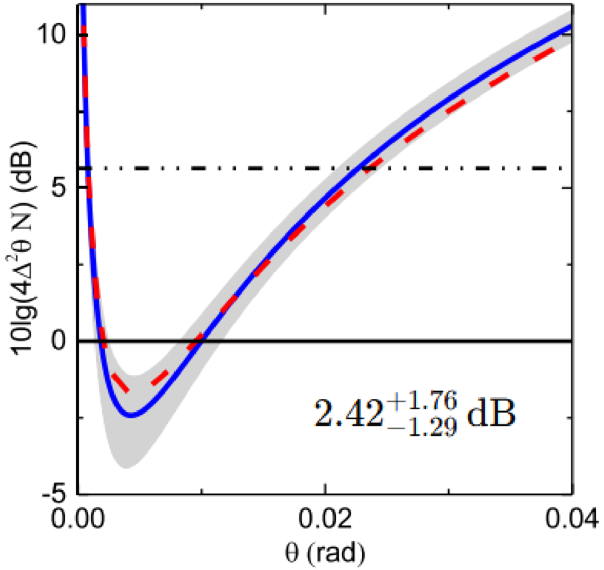
[a]State Key Laboratory of Low Dimensional Quantum Physics, Department of Physics, Tsinghua University, Beijing 100084, China; and [b]Collaborative Innovation Center of Quantum Matter, Beijing 100084, China

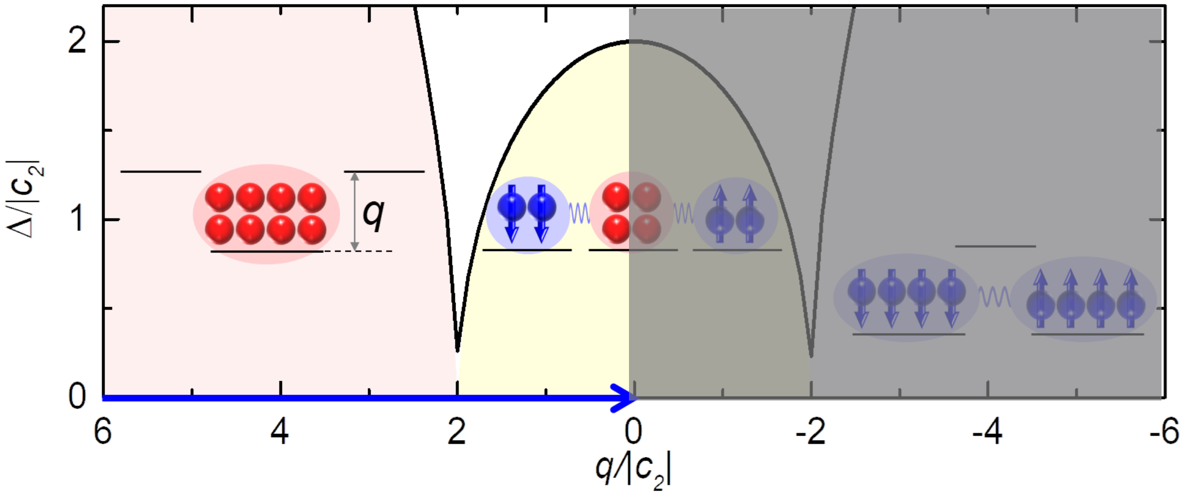
Edited by Jun Ye, National Institute of Standards and Technology, Boulder, CO, and approved May 8, 2018 (received for review August 27, 2017)

Interferometry is a paradigm for most precision measurements.

all of these generated Dicke states are based on pseudospin-

beyond the three-mode SQL

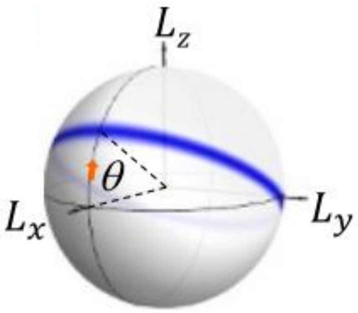




$$L_x = \frac{1}{\sqrt{2}}\left(a_{+1}^\dagger a_0 + a_0^\dagger a_{+1} + a_0^\dagger a_{-1} + a_{-1}^\dagger a_0\right)$$

$$L_y = \frac{1}{\sqrt{2}i}\left(a_{+1}^\dagger a_0 - a_0^\dagger a_{+1} + a_0^\dagger a_{-1} - a_{-1}^\dagger a_0\right)$$

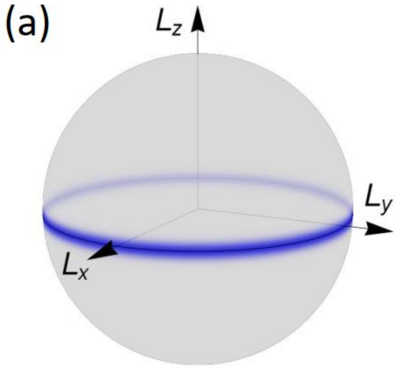
$$L_z = a_{+1}^\dagger a_{+1} - a_{-1}^\dagger a_{-1}$$



Z. Zhang and L.-M. Duan, *Phys. Rev. Lett.* **111**, 180401 (2013).

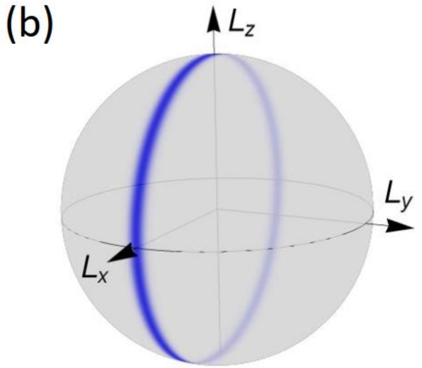
Spin-1 Bose Einstein Condensate

(a)



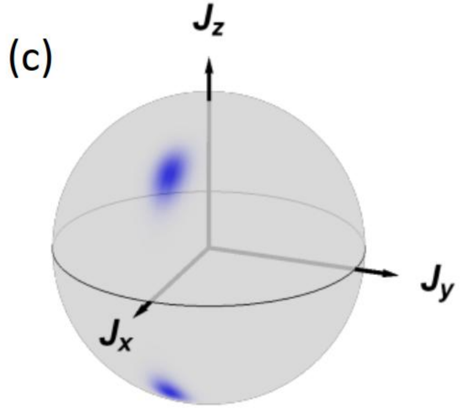
$$\propto \int_0^{2\pi} d\varphi \left(\frac{1}{2} e^{-i\varphi} |1\rangle + \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{2} e^{i\varphi} |-1\rangle \right)^{\otimes N}$$

(b)



$$|\psi\rangle \propto \int_0^{2\pi} d\theta \left(\frac{1+\sin\theta}{2} |1\rangle + \frac{1}{\sqrt{2}} \cos\theta |0\rangle + \frac{1-\sin\theta}{2} |-1\rangle \right)^{\otimes N}$$

(c)



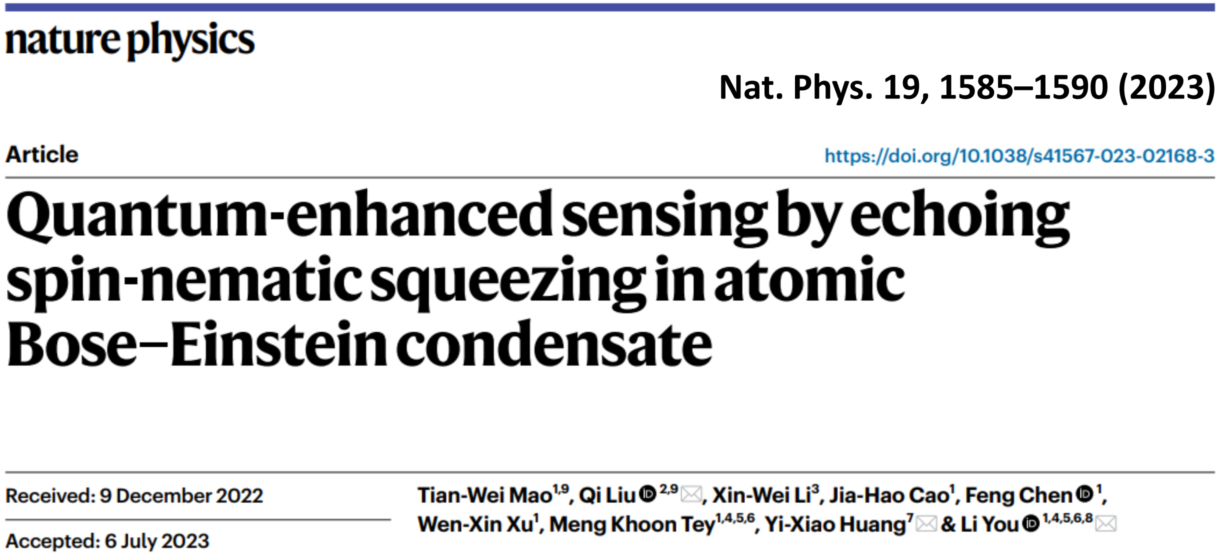
$$N_0 = \frac{N}{2} \cos^2\theta$$

⇨

$$\propto \left(|\theta_0, N_s\rangle + |\pi + \theta_0, N_s\rangle \right)$$

$$\theta_0 = \arccos(\sqrt{2N_0/N})$$

Quantum Metrology by Echoing the Spin-nematic Squeezing



nature physics

Nat. Phys. 19, 1585–1590 (2023)

Article

https://doi.org/10.1038/s41567-023-02168-3

Quantum-enhanced sensing by echoing spin-nematic squeezing in atomic Bose–Einstein condensate

Received: 9 December 2022

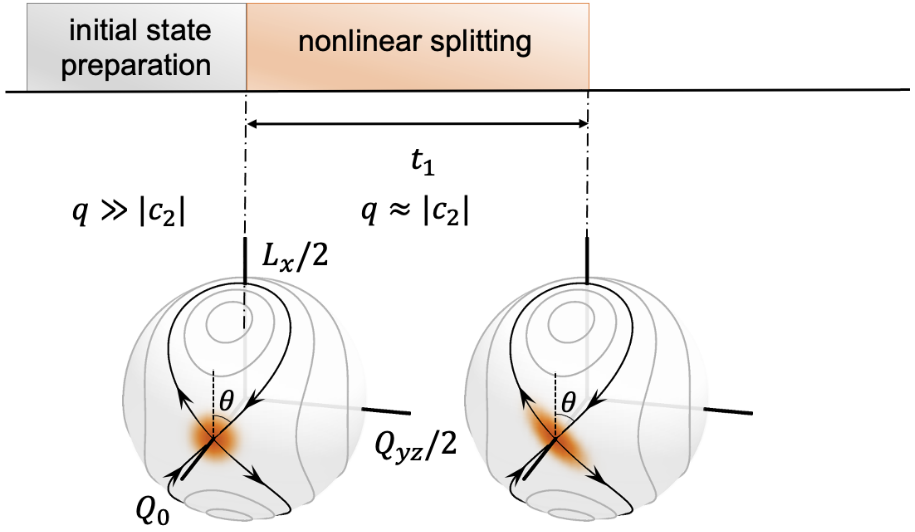
Accepted: 6 July 2023

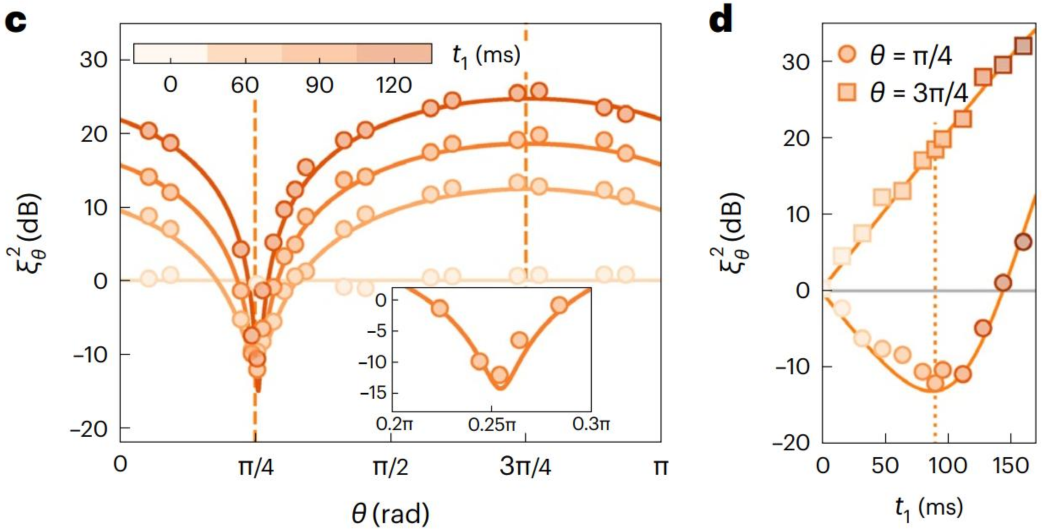
Tian-Wei Mao[1,9], Qi Liu[2,9], Xin-Wei Li[3], Jia-Hao Cao[1], Feng Chen[1], Wen-Xin Xu[1], Meng Khoon Tey[1,4,5,6], Yi-Xiao Huang[7] & Li You[1,4,5,6,8]

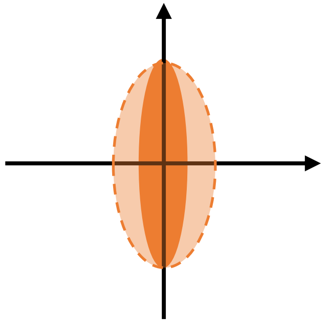
Mao Tianwei

Liu Qi

optimal squeezing detected ~ 12dB

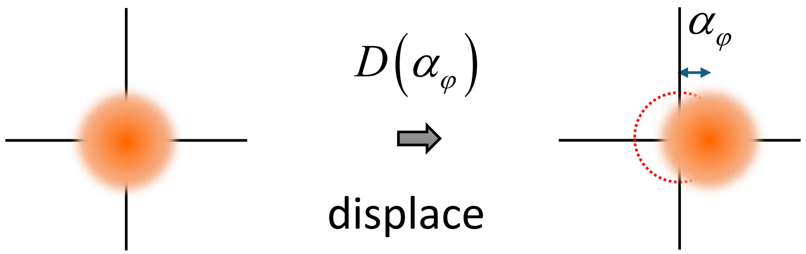






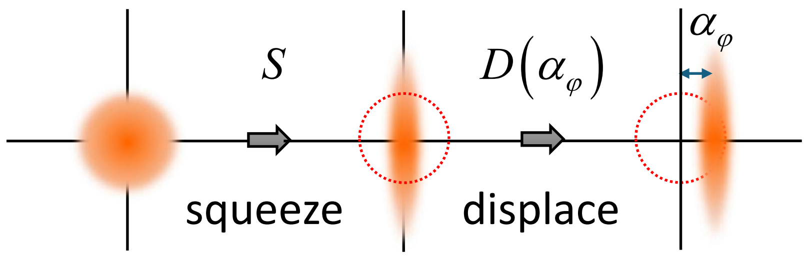
Quantum Metrology by Echoing the Spin-nematic Squeezing

E. Davis, G. Bentsen, and M. Schleier-Smith, Phys. Rev. Lett.116, 053601 (2016)
F. Fröwis, P. Sekatski, and W. Dür, Phys. Rev. Lett.116, 090801 (2016)



$$\Delta\varphi \geq \Delta\varphi_{\mathrm{SQL}} = \frac{1}{\sqrt{N}}$$

$$\Delta\varphi = \frac{\Delta M}{|d\langle M\rangle/d\varphi|}$$

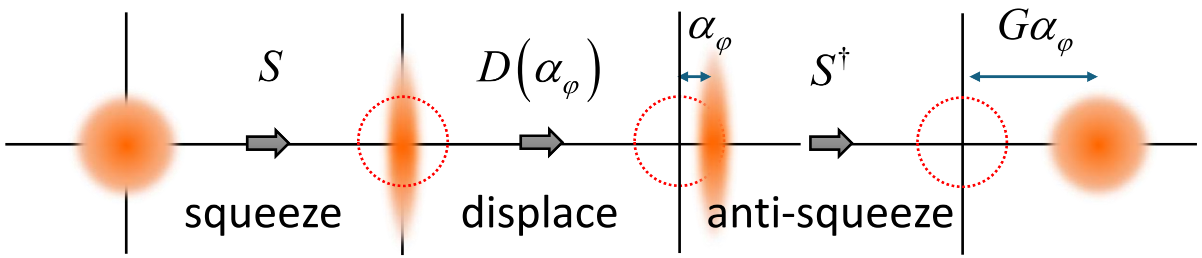


$$\Delta\varphi < \Delta\varphi_{\mathrm{SQL}}$$

$$\Delta\varphi = \frac{\xi\Delta M}{|d\langle M\rangle/d\varphi|}$$

$$\Delta\varphi = \frac{\xi\Delta M + \sigma_{\mathrm{det}}}{|d\langle M\rangle/d\varphi|}$$

Sensitive to detection noise



$$\Delta\varphi = \frac{\Delta M + \sigma_{\mathrm{det}}}{G|d\langle M\rangle/d\varphi|}$$

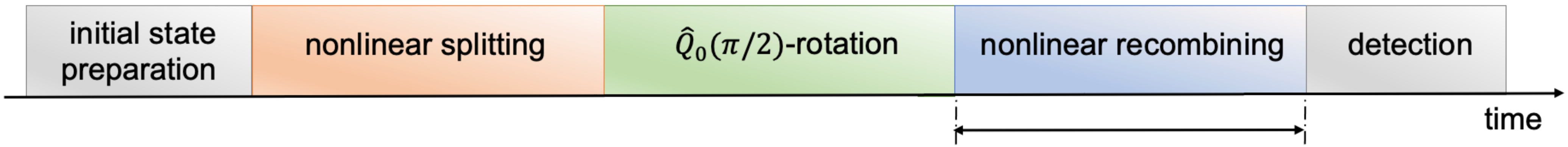
$$\Delta\varphi = \frac{\Delta M/G + \sigma_{\mathrm{det}}/G}{|d\langle M\rangle/d\varphi|}$$

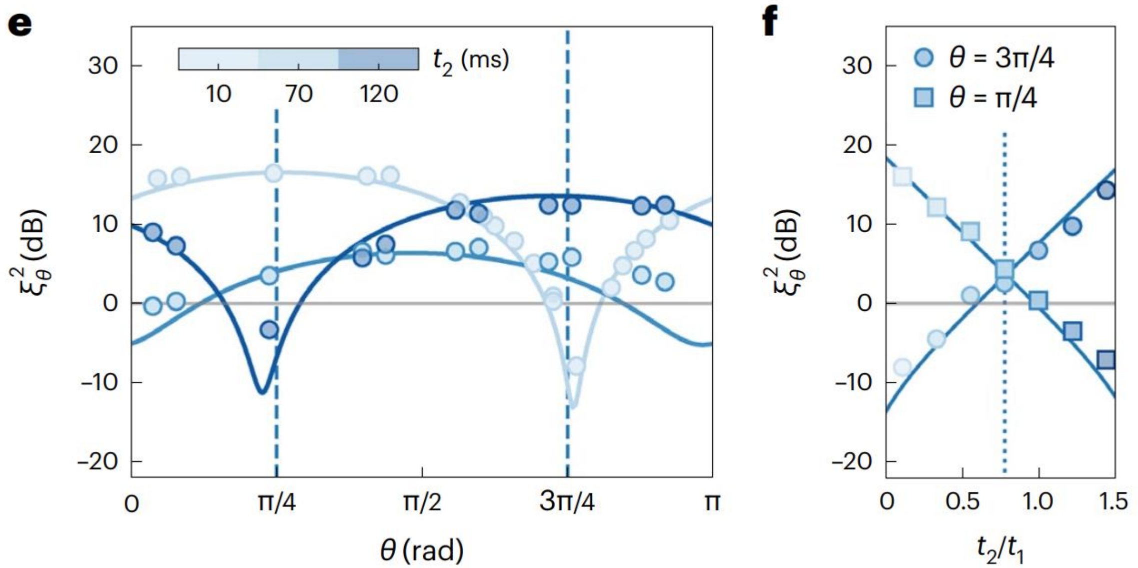
Robust to detection noise

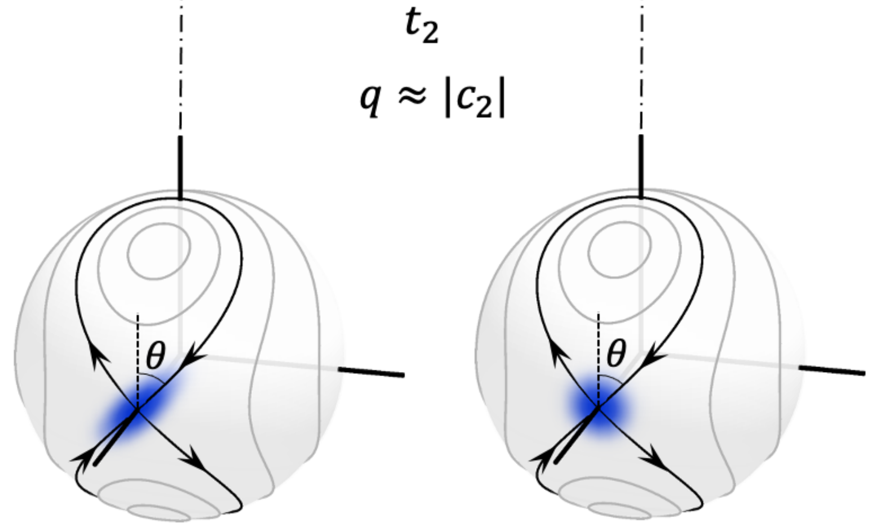
S. C. Burd, R. Srinivas, J. J. Bollinger, et al.,Science 364, 1163(2019)

Quantum Metrology by Echoing the Spin-nematic Squeezing

$N = 26500, B = 0.537\ \mathrm{G}, |c_2| = 2\pi \times 3.9\ \mathrm{Hz}$



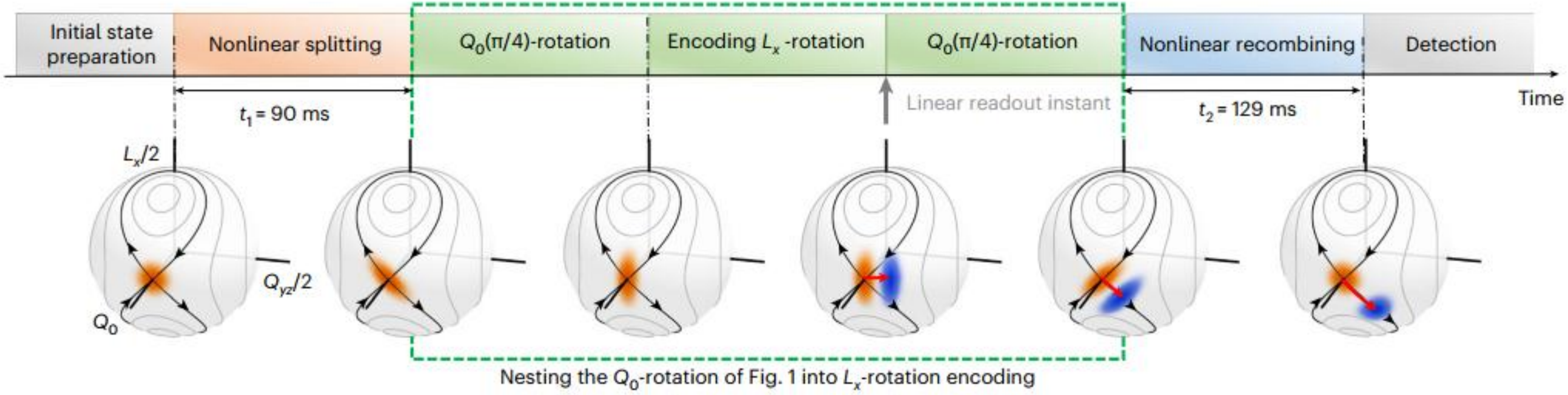


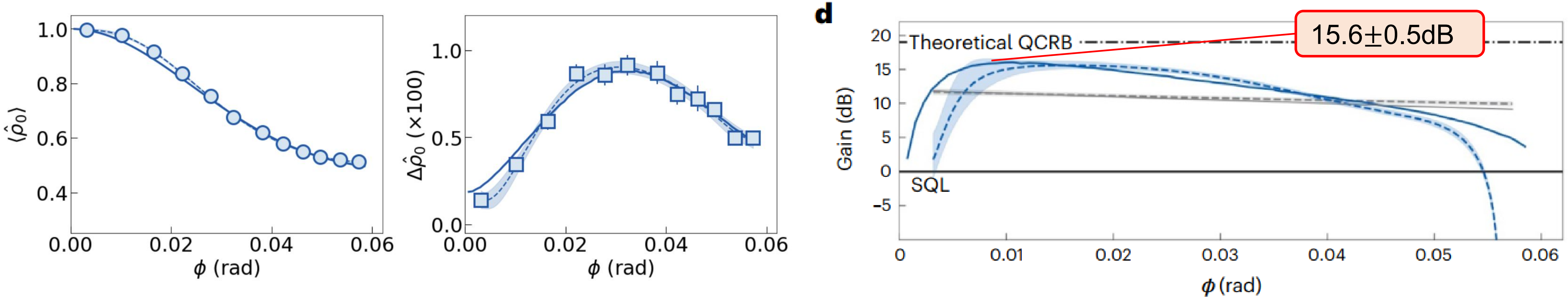


The effectively time-reversed evolution best disentangles the probe state at

$$t_2 = 0.78 t_1 = 70\ \mathrm{ms}$$

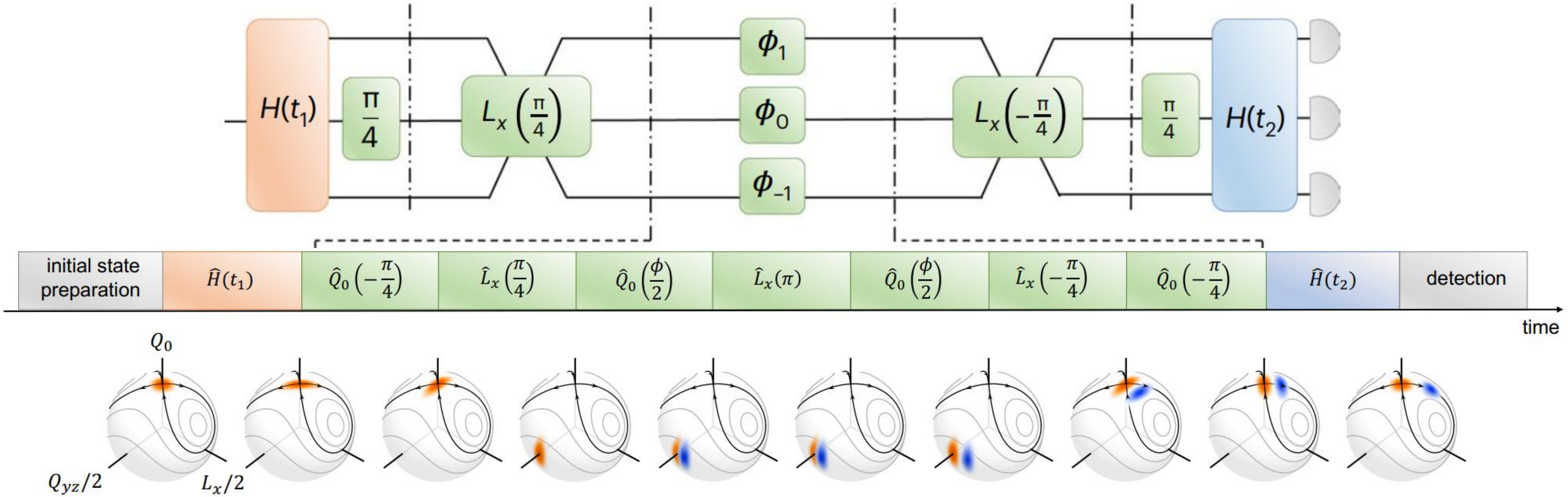
Quantum Metrology by Echoing the Spin-nematic Squeezing



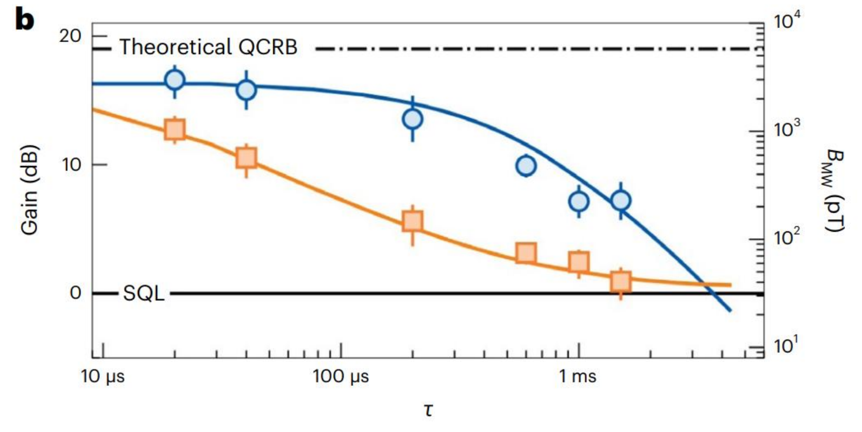


15.6±0.5dB

Quantum Metrology by Echoing the Spin-nematic Squeezing



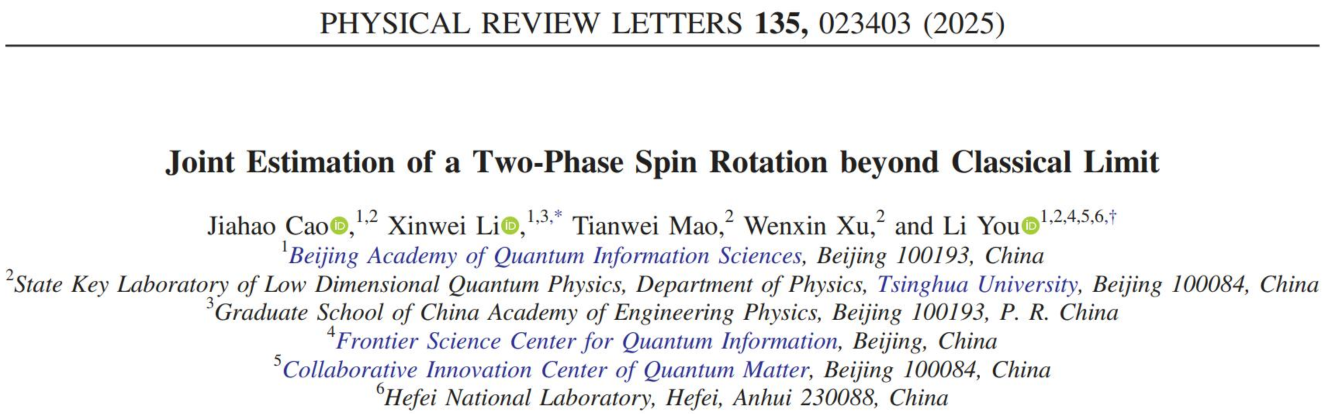
Ramsey interferometry nested with echoing sequence



optimal gain: 16.5 dB

single-shot sensitivity for microwave field sensing: 40.6 pT

Joint Estimation of a Two-phase Spin Rotation



PHYSICAL REVIEW LETTERS **135,** 023403 (2025)

Joint Estimation of a Two-Phase Spin Rotation beyond Classical Limit

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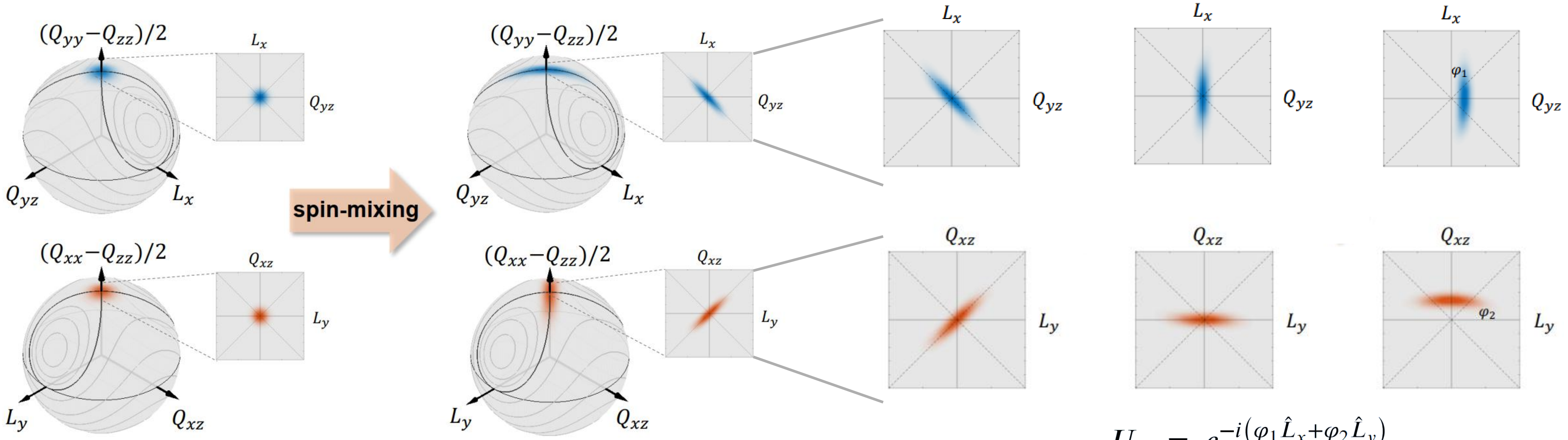
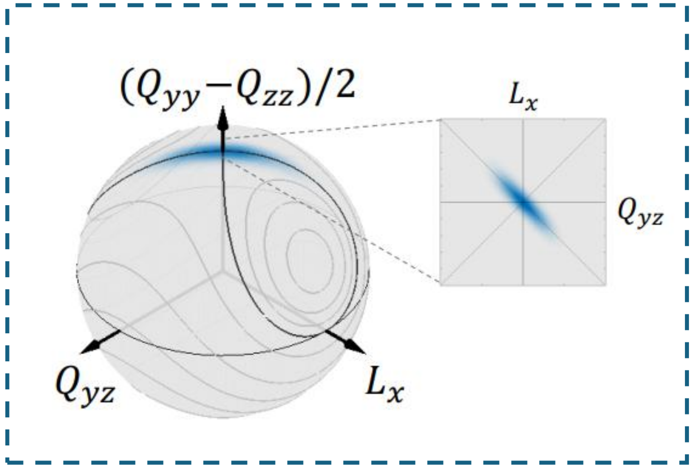
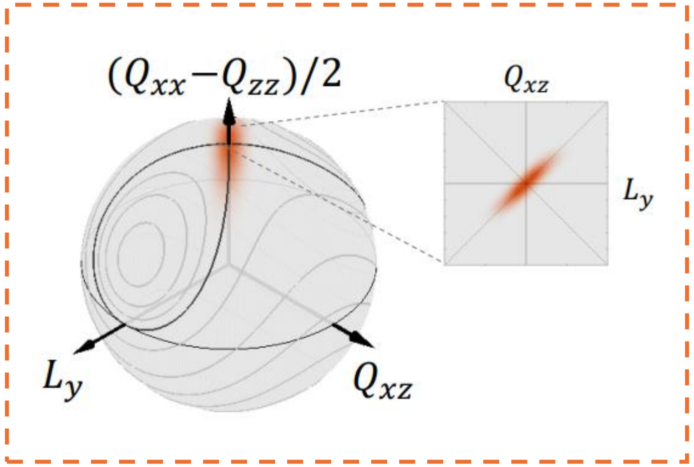
Li Xinwei

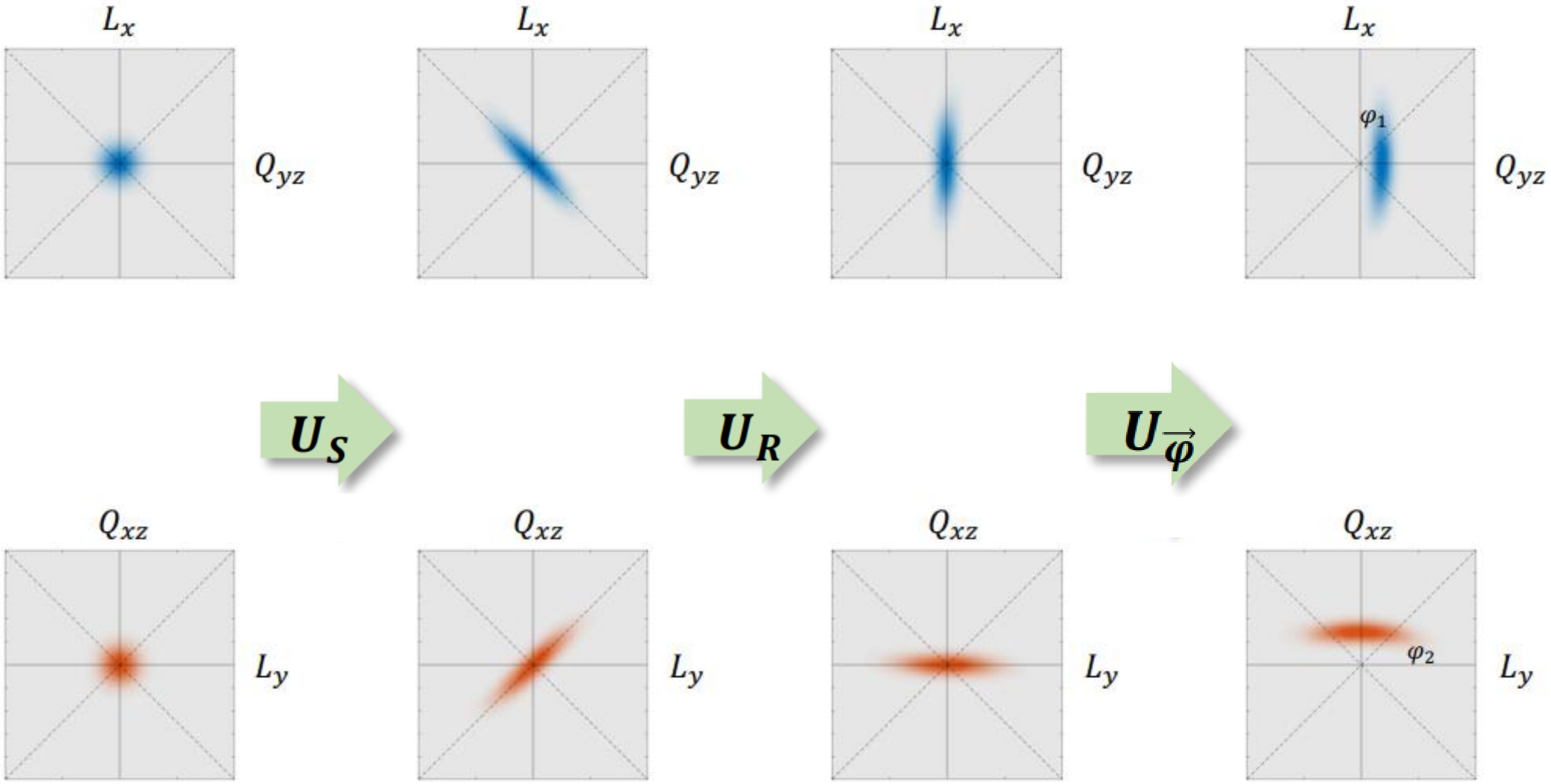


$$U_{\vec{\varphi}} = e^{-i\left(\varphi_1 \hat{L}_x + \varphi_2 \hat{L}_y\right)}$$

Joint Estimation of a Two-phase Spin Rotation

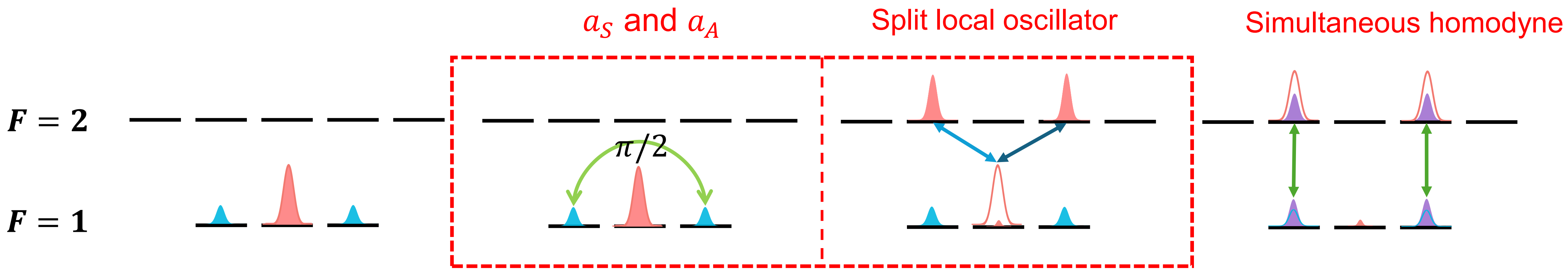






- Sensitive to the two-phase rotation $U_{\vec{\varphi}} \;=\; e^{-i\left(\varphi_1 \hat{L}_x+\varphi_2 \hat{L}_y\right)}$
- To measure Q_{yz} and Q_{xz} simultaneously

Joint Estimation of a Two-phase Spin Rotation

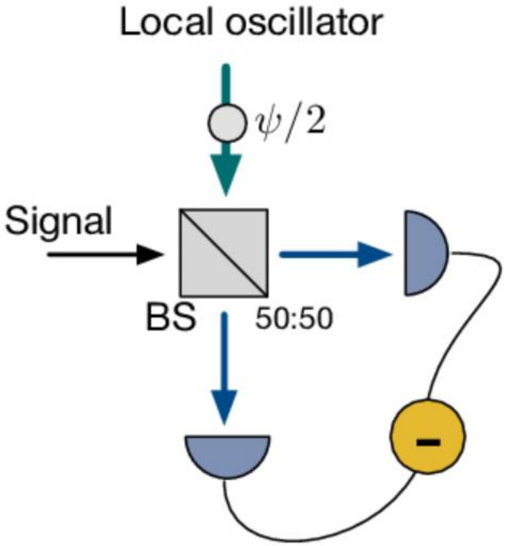


$$\langle N^{out}_{2,-1} - N^{out}_{1,-1}\rangle = \frac{i\varepsilon_1}{\sqrt{2}}\langle -a^\dagger_{1,+1}a_{1,0} - a^\dagger_{1,-1}a_{1,0} + a^\dagger_{1,0}a_{1,+1} + a^\dagger_{1,0}a_{1,-1}\rangle = \varepsilon_1\langle Q_{yz}\rangle$$

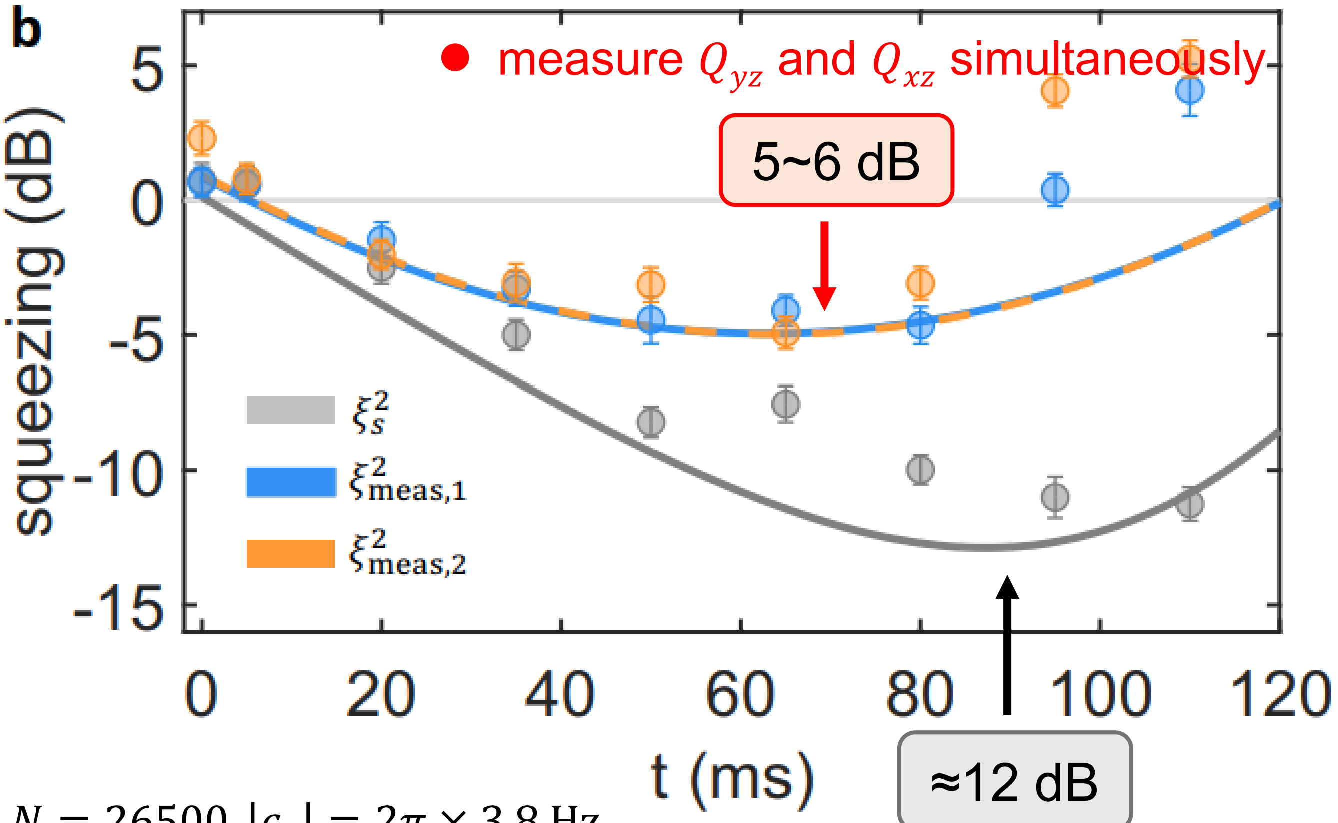
$$\langle N^{out}_{2,+1} - N^{out}_{1,+1}\rangle = \frac{\varepsilon_2}{\sqrt{2}}(a^\dagger_{1,+1}a_{1,0} - a^\dagger_{1,-1}a_{1,0} + a^\dagger_{1,0}a_{1,+1} - a^\dagger_{1,0}a_{1,-1}) = \varepsilon_2\langle Q_{xz}\rangle$$

$$\varepsilon_1 \approx \varepsilon_2 \approx 1/\sqrt{2}$$

$$\frac{1}{\sqrt{2}}a_{1,+1} + \frac{1}{\sqrt{2}}a_{1,-1} = a_S \qquad \frac{1}{\sqrt{2}}a_{1,+1} - \frac{1}{\sqrt{2}}a_{1,-1} = a_A$$



Joint Estimation of a Two-phase Spin Rotation



● measure Q_{yz} and Q_{xz} simultaneously

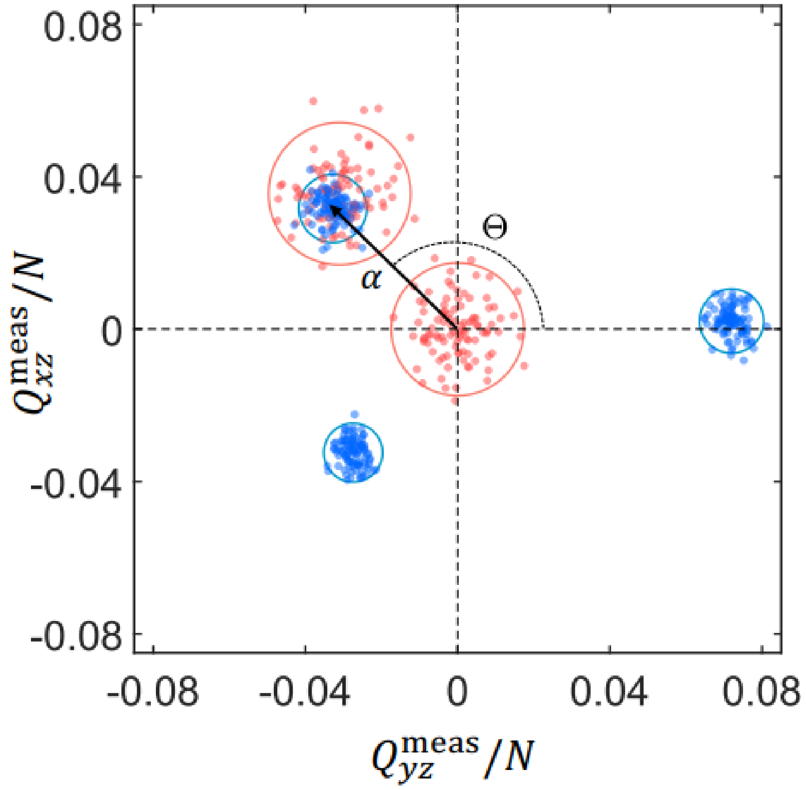
5~6 dB

≈12 dB

$N = 26500, |c_2| = 2\pi \times 3.8\ \mathrm{Hz}$

$$\xi^2 = 20\log_{10}\left(\Delta Q_{yz(xz)}/\sqrt{N}\right)$$

● only measure Q_{yz} or Q_{xz}



Blue dots: spin-nematic squeezed state

Red dots: polar state $|0N0\rangle$

Three-outcome Bell inequalities

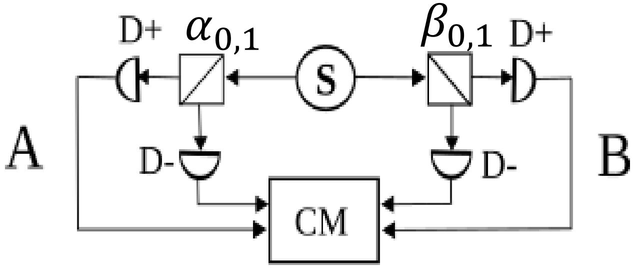
$$\hat{m}_0 = \cos\theta\,\hat{q}_{yz} + \sin\theta\,\hat{q}_{xy}$$

$$\hat{m}_1 = \cos\theta\,\hat{q}_{yz} - \sin\theta\,\hat{q}_{xy}$$

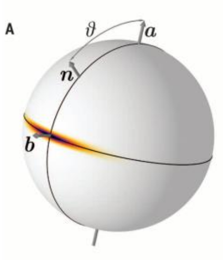
A general Bell scenario is defined by: (N, m, d)

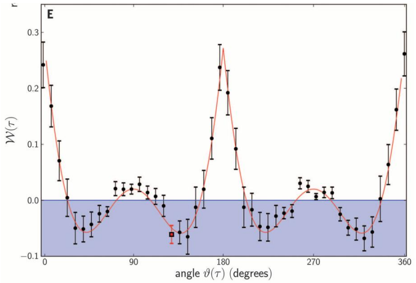
N	**number of parties**
m	**measurement choices per party**
d	**possible outcomes per measurement**

- CHSH-type $(\mathbf{2}, \mathbf{2}, \mathbf{2})$

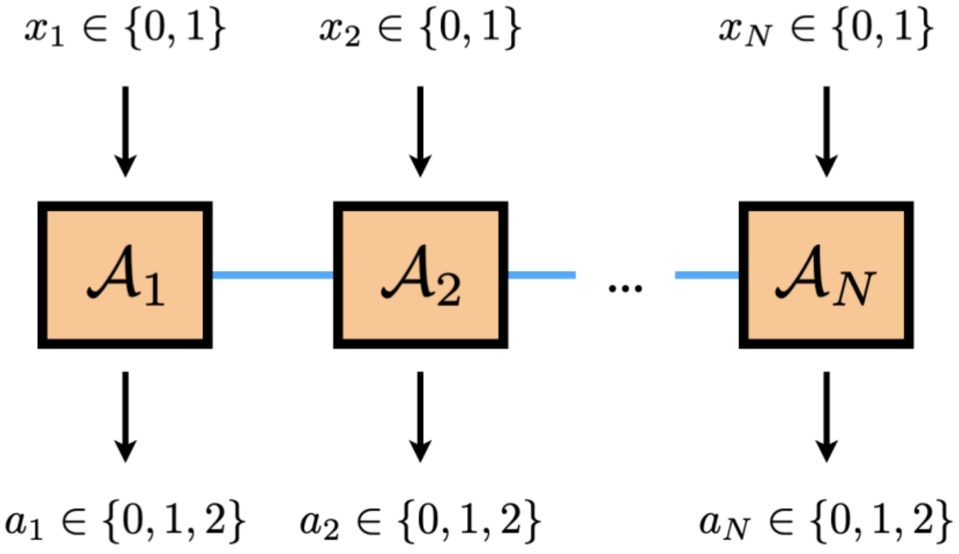


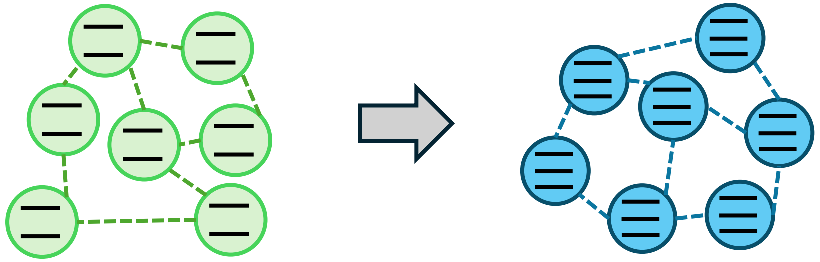
- Spin squeezed state $(\boldsymbol{N}, \mathbf{2}, \mathbf{2})$





Spin-1 atoms naturally realize a three-outcome system $(\boldsymbol{N}, \mathbf{2}, \mathbf{3})$





Three-outcome Bell inequalities

Full many-body distribution $p(a_1, \ldots, a_N | x_1, \ldots, x_N)$

⇩ Keep only one- and two-body probabilities

$$P_{a|x}, \qquad P_{ab|xy}$$

⇩ permutation invariance

$$P_{a|x} = \sum_i p(a_i|x_i) \qquad P_{ab|xy} = \sum_{i \neq j} p(a_i b_j | x_i y_j)$$

⇩ Spin-exchange symmetry

$$\boxed{B = \widetilde{P}_\alpha + \widetilde{P}_{\alpha\alpha} - 2\widetilde{P}_{\alpha\beta} \geq 0.}$$

Bell inequality

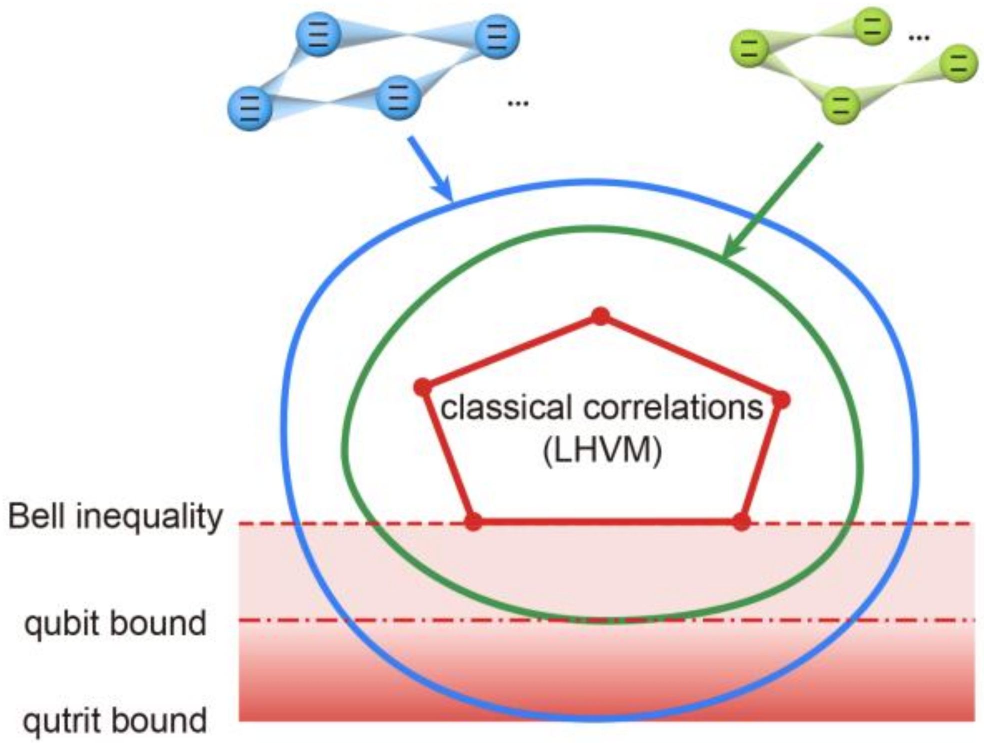
$$\widetilde{P}_\alpha = P_{-1|0} + P_{-1|1} + P_{+1|0} + P_{+1|1},$$

$$\widetilde{P}_{\alpha\alpha} = P_{-1,-1|00} + P_{-1,-1|11} + P_{+1,+1|00} + P_{+1,+1|11},$$

$$\widetilde{P}_{\alpha\beta} = P_{-1,+1|01} + P_{+1,-1|01}.$$

Guillem Müller-Rigat

Matteo Fadel



Three-outcome Bell inequalities

$$B = \widetilde{P}_{\alpha} + \widetilde{P}_{\alpha\alpha} - 2\widetilde{P}_{\alpha\beta} \geq 0.$$

$$\Updownarrow$$

$$\boxed{W = \frac{3}{N}\Big(\underbrace{\langle \hat{Q}_{yz}^2 \rangle + \langle \hat{Q}_{xz}^2 \rangle}_{\text{nematic fluctuations}}\Big) + \underbrace{\frac{5}{2N}\langle \hat{Q}_{zz} \rangle}_{\text{depletion from mF=0}} + \frac{10}{3} \geq 2}$$

Bell witness that converts the probabilities into measurable observables.

$W \geq 2$: **LHVM bound**

$1 \leq W < 2$: **Bell correlated / Qubits bound**

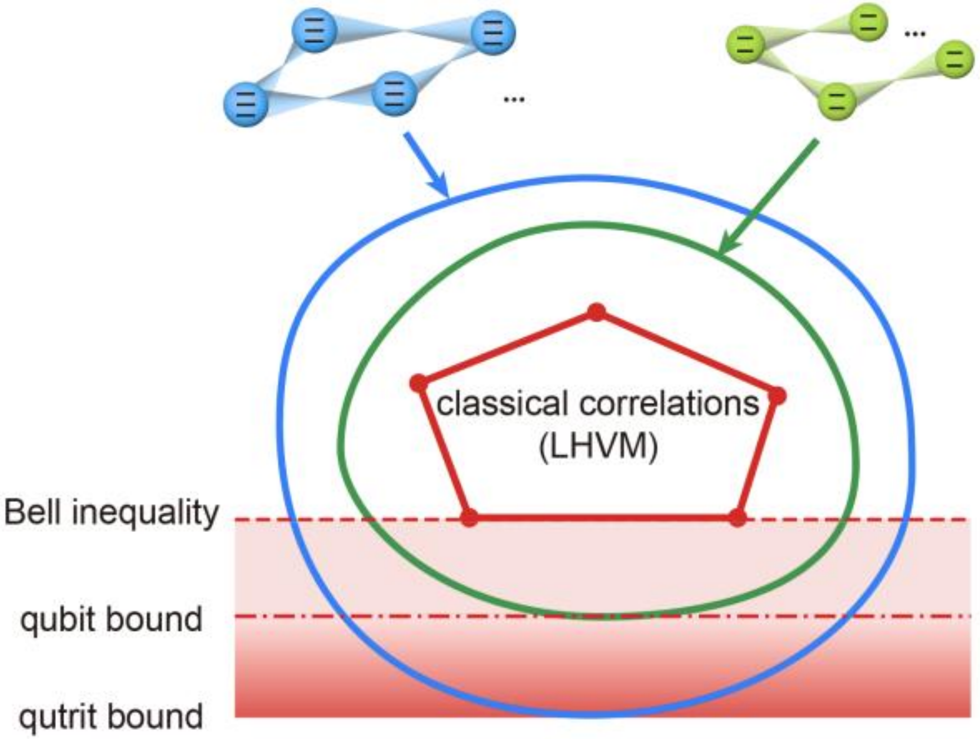
$W < 1$: **Qutrit Bell correlations**

$W < 1$ can not be explained by N qubits correlation!

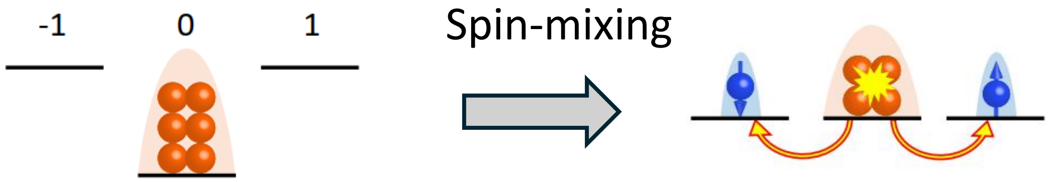
$$W < 1 - \frac{k-1}{N} \quad \Rightarrow \quad \text{at least } k \text{ qutrits}$$

Guillem Müller-Rigat

Matteo Fadel

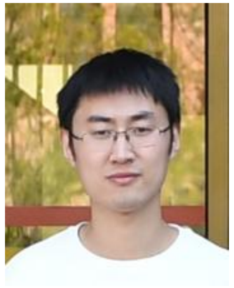


Three-outcome Bell inequalities

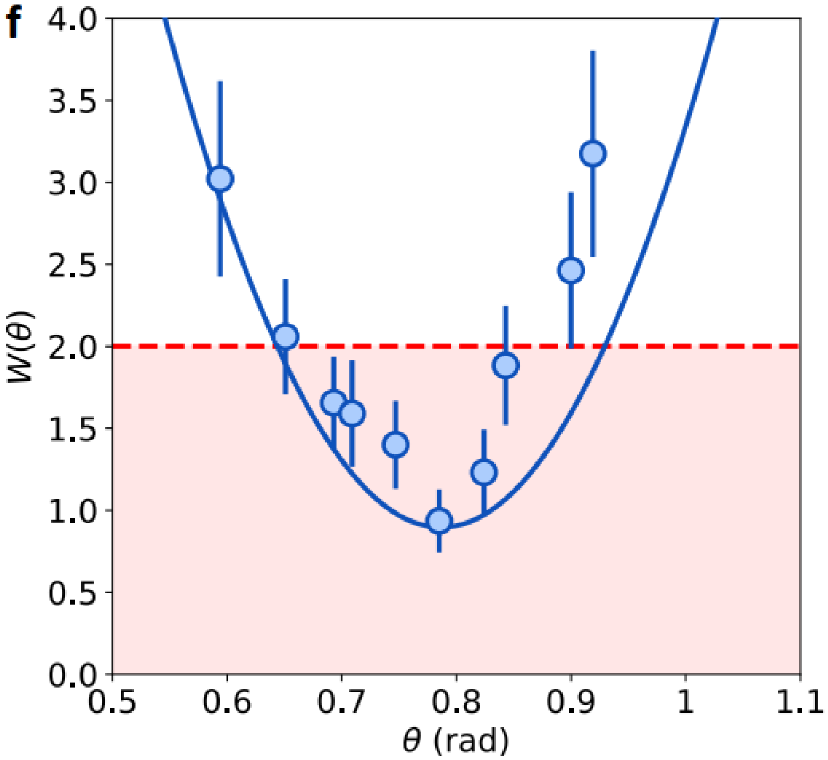


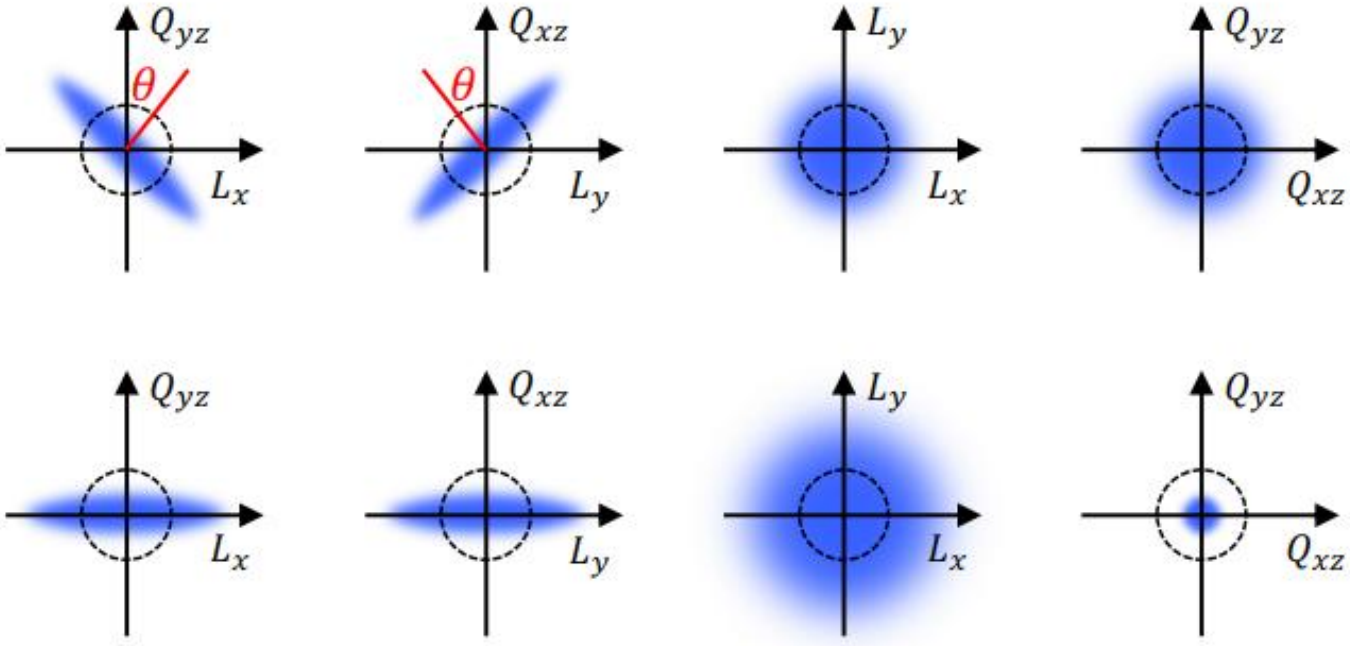
Wenxin Xu

Junshuang Hu

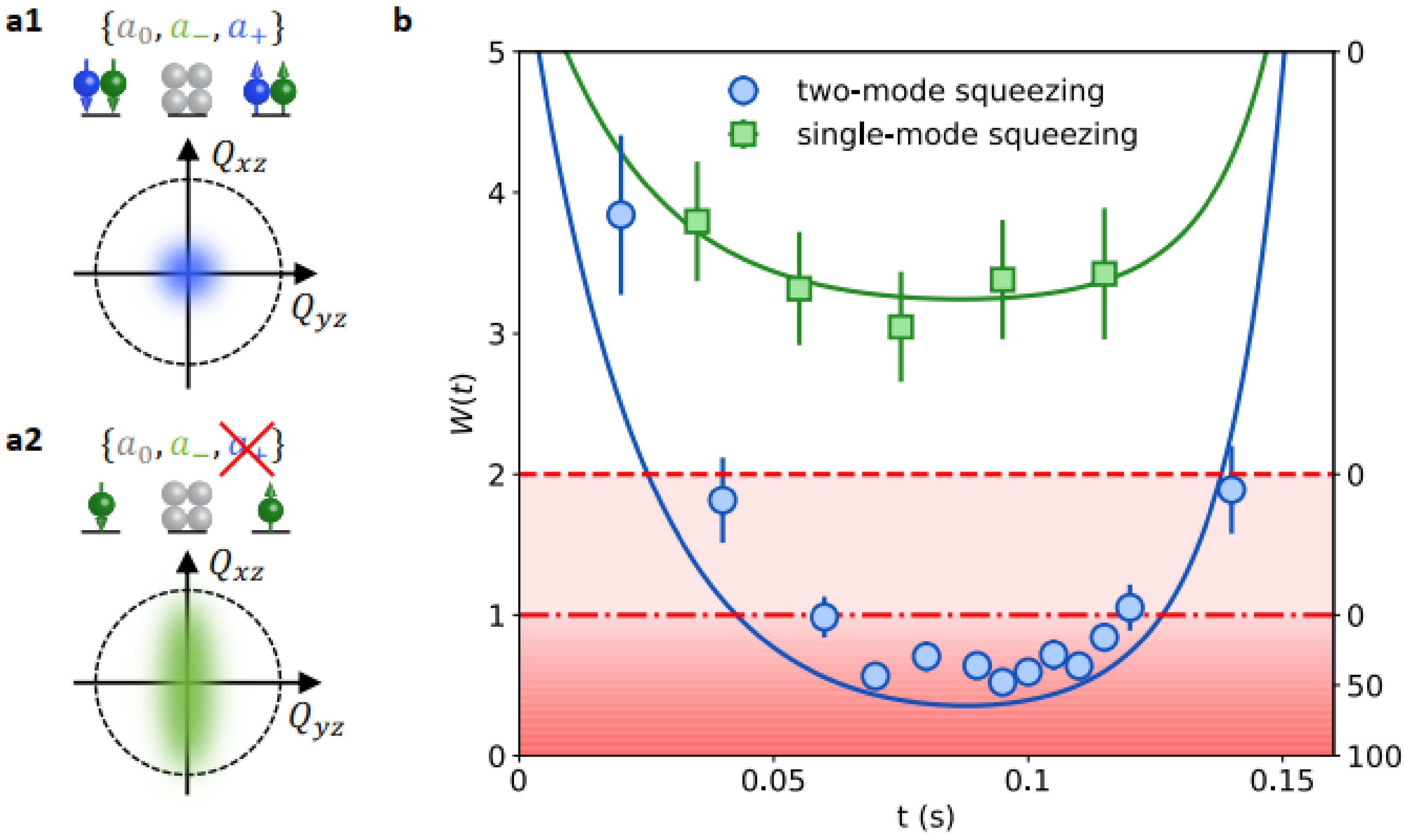


Qi Liu





Three-outcome Bell inequalities



$$N = 31{,}000 \pm 380$$

$$\xi^2 = -8.2 \pm 0.8 \text{ dB}$$

$$W_{\min} = 0.52(7) < 1$$

At least 48% of atoms require a Qutrit description!

Observing Bell Inequality Violation Beyond the Qubit Bound in a Spinor Bose–Einstein Condensate

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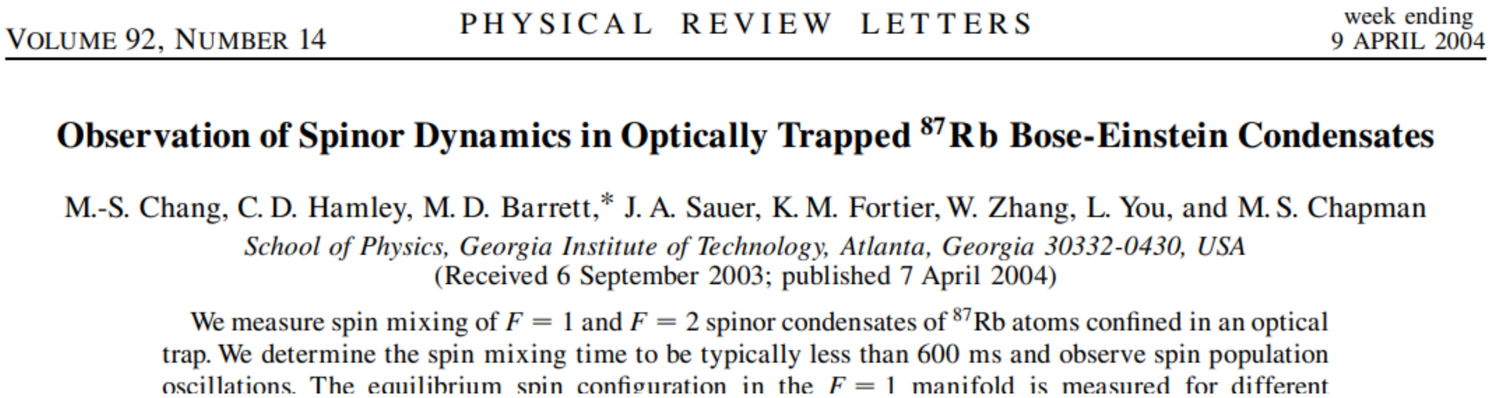
(Dated: March 12, 2026)

Bell nonlocality—quantum correlations that cannot be explained by any local hidden-variable theory—lies at the heart of both fundamental tests of quantum mechanics and applications of quantum information science. Yet, demonstrations of Bell nonlocality in many-body systems have largely relied on effective two-level (qubit) descriptions, leaving the device-independent certification of higher-dimensional nonlocality an open challenge. Here, we observe Bell correlations in a spin-1 ^{87}Rb Bose-Einstein condensate by measuring a permutationally invariant Bell witness accessible with collective spin and nematic observables. Using spin mixing dynamics in an ensemble of $N \simeq 3.1 \times 10^4$ atoms, we generate spin-nematic squeezing of -11.9 ± 0.7 dB and obtain a violation that exceeds the maximal bound attainable with any collection of qubits, thereby providing device-independent evidence of genuine multipartite qutrit nonlocality. Our results establish spinor condensates as a platform for high-dimensional Bell tests in the many-body regime and open new avenues for certifying dimensionality with coarse-grained measurements.

Spin-1 Bose Einstein Condensate



VOLUME 92, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 9 APRIL 2004

Observation of Spinor Dynamics in Optically Trapped ^{87}Rb Bose-Einstein Condensates

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(Received 6 September 2003; published 7 April 2004)

We measure spin mixing of $F = 1$ and $F = 2$ spinor condensates of ^{87}Rb atoms confined in an optical trap. We determine the spin mixing time to be typically less than 600 ms and observe spin population oscillations. The equilibrium spin configuration in the $F = 1$ manifold is measured for different