Arrays of atoms for Q simulation & computing – Lecture 3

Lecture 1: Arrays of atoms in optical tweezers
Rydberg atoms

Lecture 2: Interactions between Rydberg atoms
Rydberg blockade
Quantum computing with Rydberg atoms

Lecture 3: Quantum simulation: from Rydberg interactions to spin models… and more

Outline – Lecture 3

1. Quantum simulation = many-body physics with synthetic matter

2. From Rydberg interactions to spin models

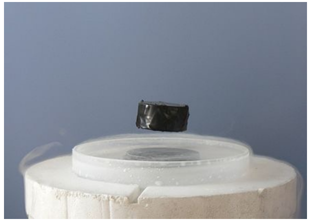
3. Examples of ground-state preparation

4. Examples of out-of-equilibrium dynamics

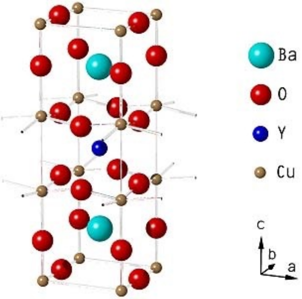
5. Outlook: what we did not discuss… & beyond

The many-body problem: the art of modelling…

wikipedia



YBaCuO



Observe unexpected effects

Ex: high-T_c superconductivity

Microscopic understanding?

Experiment on "real" system

simplify

Too hard to calculate…

Cook up a model

$$H_{\mathrm{model}} = -t \sum_{\langle i,j \rangle, \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \mathrm{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$

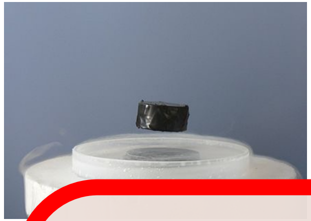
Problem: exponential complexity

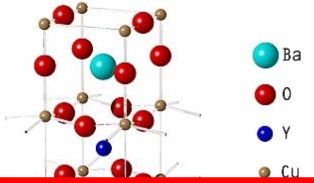
2 d. of freedom (spin…) $\psi_i = \begin{pmatrix} a \\ b \end{pmatrix}$

Many-body wavefunction: $\Psi = \Psi(1, 2, \ldots, N) \Rightarrow \Psi$ requires 2^N numbers

Record *ab-initio* calculation (2025) $N \sim 50 \Rightarrow 2^{50} \sim 10^{15}$ = 1000 Tb RAM !!

The many-body problem: the art of modelling…





Observe unexpected effects

Ex: high-T_c superconductivity

Approximations possible!!

mean-field, perturbation theory, Monte-Carlo,
variational methods: DFT, MPS, Neural Quantum States…

But… can be poorly controlled or not valid
when *interactions dominate*

= **Strongly correlated** systems

E

$\sum_i n_{i\downarrow} n_{i\uparrow}$

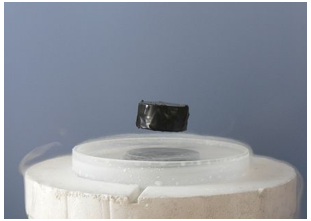
Problem:

Many-

Record *ab-initio* calculation (2025) $N \sim 50 \Rightarrow 2^{50} \sim 10^{15}$ = 1000 Tb RAM !!

One approach: build a synthetic many-body quantum systems

wikipedia



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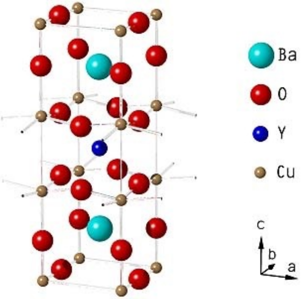

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Lab...

R.P. Feynman

Int. J. Theo. Phys. **21** (1982)

Quantum simulation

Georgescu, Rev. Mod. Phys. (2014)

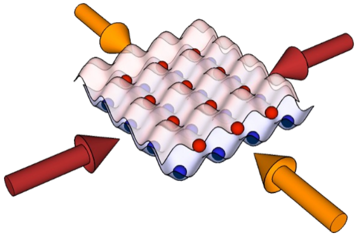
Engineer a system
ruled by $H_{\rm model}$

Measure on system:
Supercond. or not?

Analog versus digital quantum simulation

Analog

The platform implements directly H_{model}



e.g.: Fermi Hubbard, spin models, electrons in B-fields…

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t H_{\text{mod}}(t')dt'\right)|\psi(0)\rangle$$

Non-universal

Georgescu, Rev. Mod. Phys. (2014)

Digital

H_{model} synthesized digitally

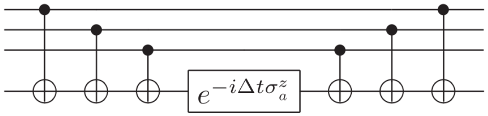
$$H_{\text{mod}} = \sum_{n=1}^{N} H_n$$

e.g. single & 2-qbit operations

$$e^{-iH_{\text{mod}}t} \approx \left(e^{-iH_1 t/n} e^{-iH_2 t/n} \ldots e^{-iH_3 t/n}\right)^n$$

= “universal” quantum simulation

Ex:



$$H_{\text{mod}} = \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$$

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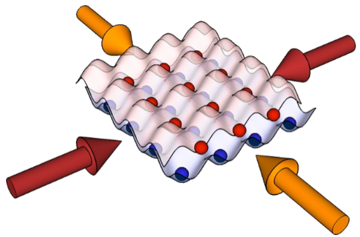

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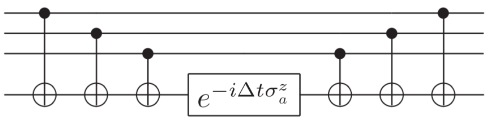

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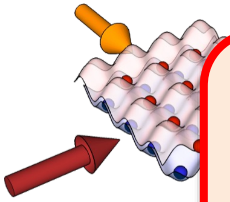

e.g.: Fermi Hubba…
electrons in B-fields…

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H_{model} synthesized digitally

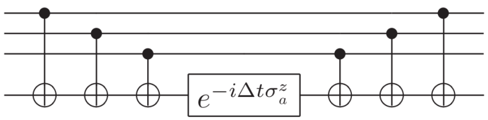
$$H_{\text{mod}} = \sum_{n=1}^{N} H_n$$

2-qbit operations

$$\left(\ldots iH_2 t/n \ldots e^{-iH_3 t/n}\right)^n$$

= "universal" quantum simulation

Ex:



$$H_{\text{mod}} = \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$$

Today: **Hybrid digital / analog**

Same platform = **quantum processors**

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2. From Rydberg interactions to spin models
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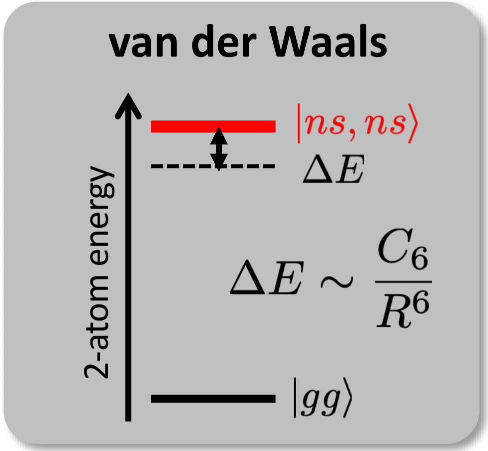
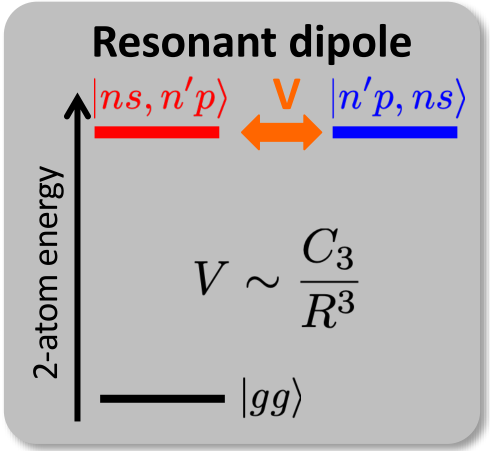
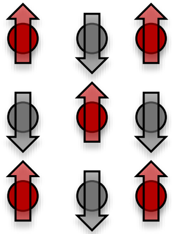
Interactions between Rydberg atoms and spin models

Browaeys & Lahaye, Nat.Phys. (2020)





Spin 1/2



Ising model

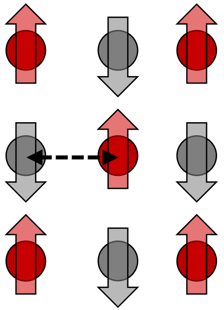
$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j$$

XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$

Spin models: one of the "simplest" many-body systems

Interacting spin ½ particles on a lattice:

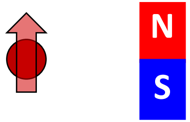


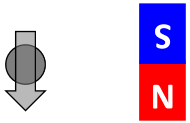
$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

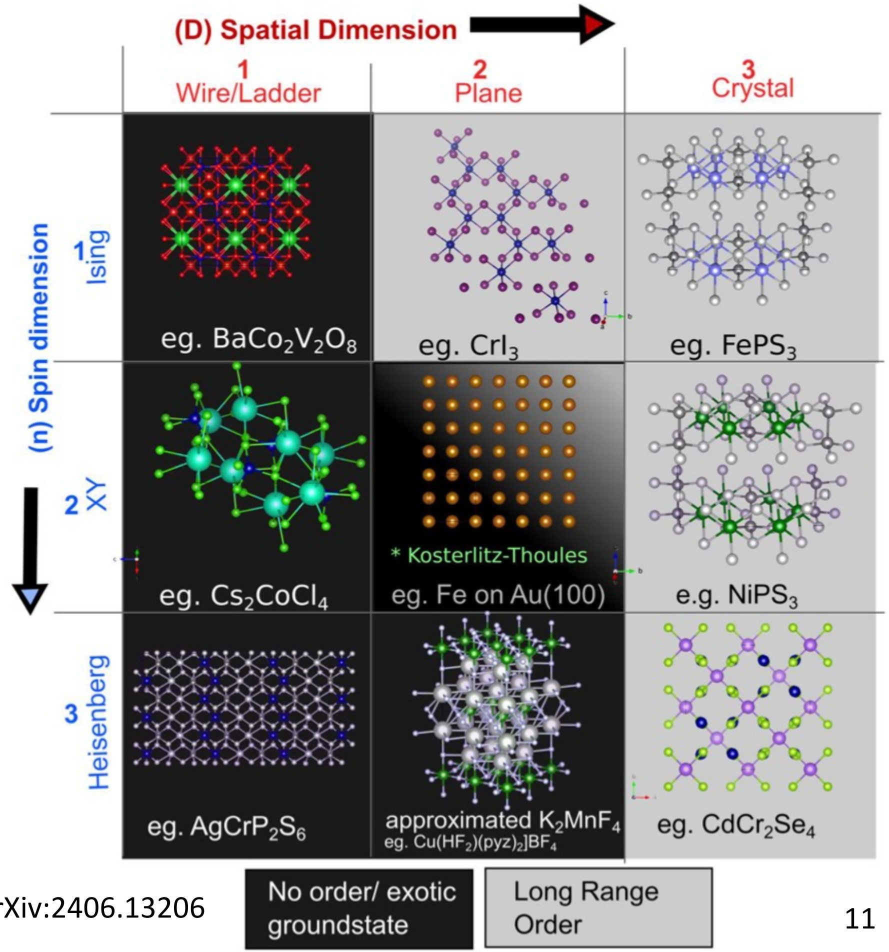
Heisenberg

Ising $$\hat{H}_{\text{Ising}} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

XY model $$\hat{H}_{\text{XY}} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^{+} \hat{\sigma}_j^{-} + \hat{\sigma}_i^{-} \hat{\sigma}_j^{+} \right)$$







arXiv:2406.13206

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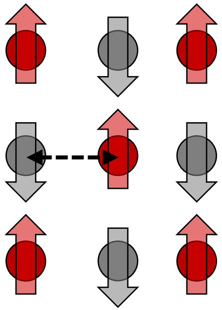

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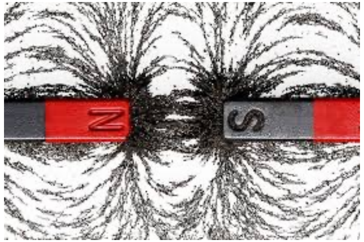
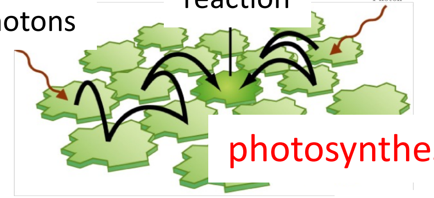
Magnetism



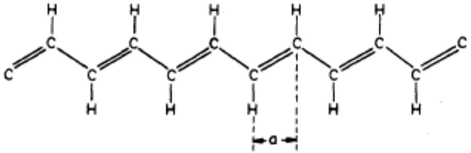
Transport of excitations



photosynthesis

Light scattering

excitons



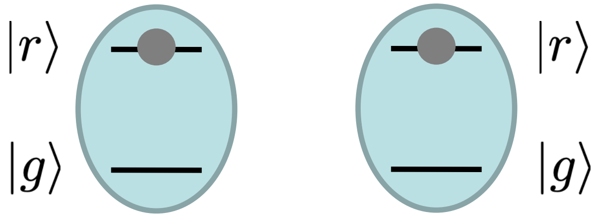
Spin models = generic systems to study many-body questions:

Quantum phase transition, out-of equilibrium, topology, entanglement…

From van der Waals interaction to the Ising spin model

Browaeys & Lahaye, Nat.Phys. (2020)

van der Waals: $\Delta E \sim \dfrac{C_6}{R^6}$



$C_6 \propto n^{11}$ $\Rightarrow$ switchable interaction

Ground state: $n = 5$

Rydberg: $n = 50$

$\times 10^{11}$

Ising - like!!

$$\hat{H}_{\text{int}} = \frac{C_6}{R^6}\hat{n}_1\hat{n}_2 \sim J\,\hat{\sigma}_1^z\hat{\sigma}_2^z$$

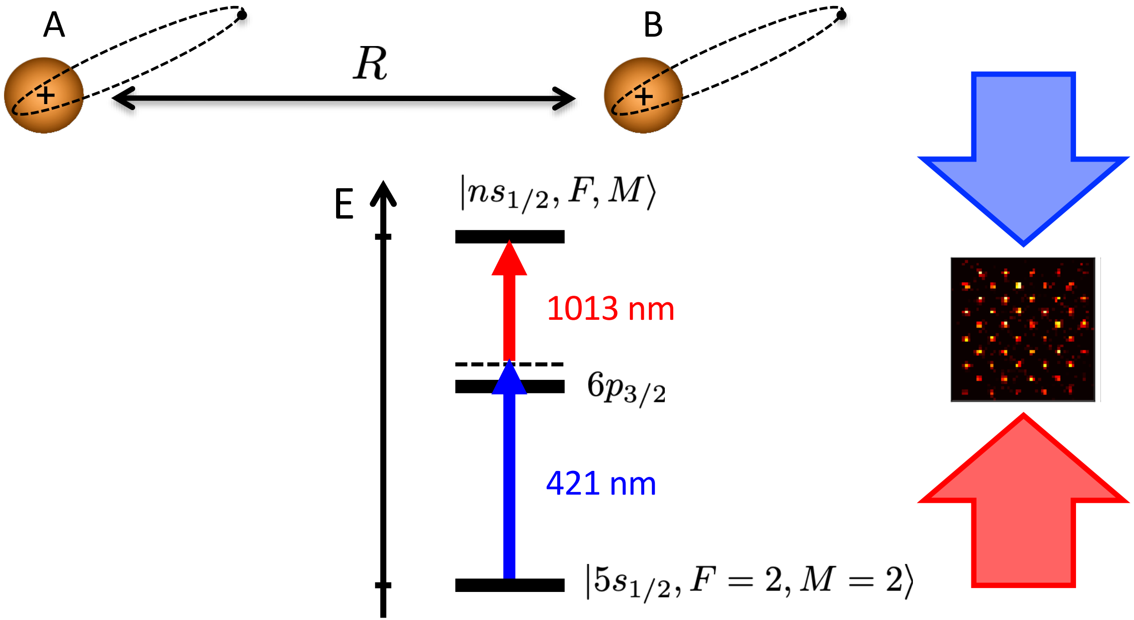
"Equivalent" to interaction between spins

$U_{\text{int}} = J$ $\qquad$ $U_{\text{int}} = -J$

Rydberg $\quad n_{1,2} = 1$

Ground state $\quad n_{1,2} = 0$

From van der Waals interaction to the Ising spin model



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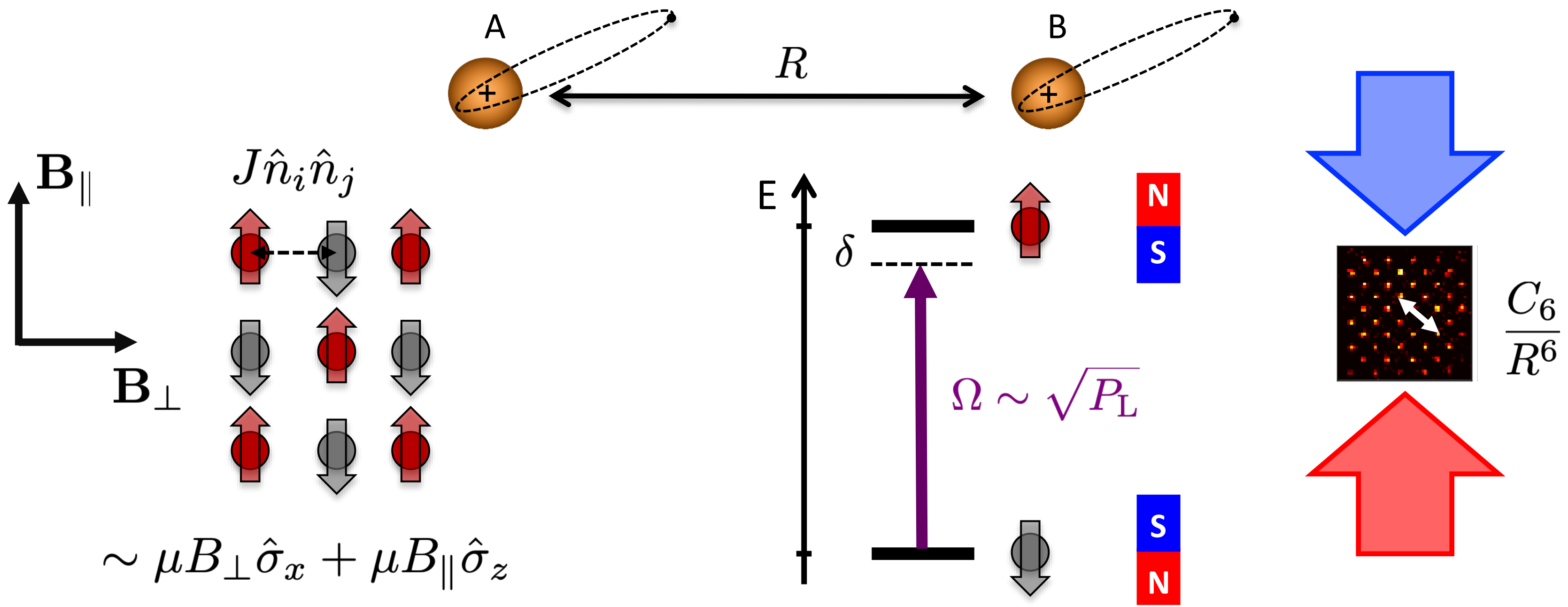
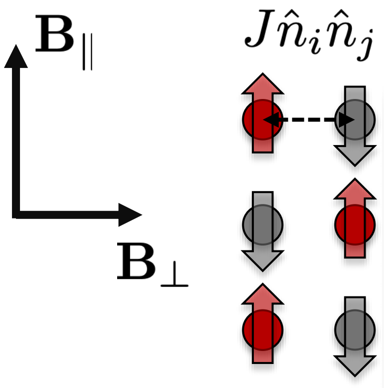


Quantum Ising model:

$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta\sum_i \hat{\sigma}_z^i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

Laser: $B_\perp$ $B_\parallel$ spin-spin interactions

From van der Waals interaction to the Ising spin model

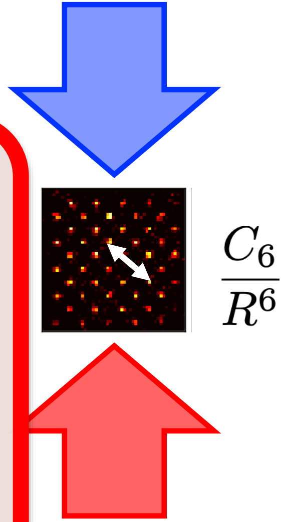


$\sim \mu B_{\perp}\hat{\sigma}_x +$

Quantum simulation:

Emulate a system by another one

Similar equations lead to same solutions!!



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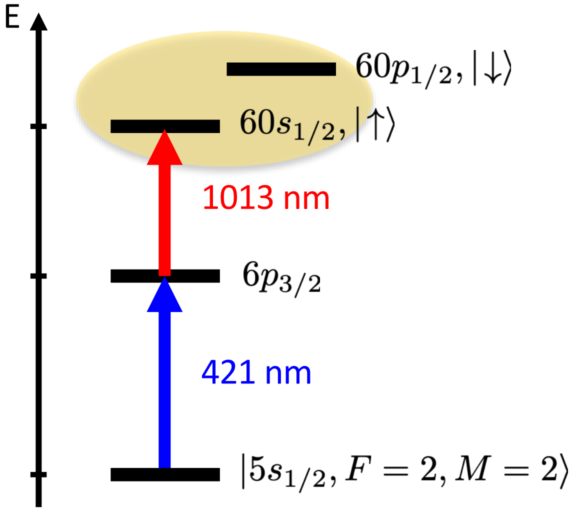
Laser: $B_{\perp}$ $B_{\parallel}$ spin-spin interactions

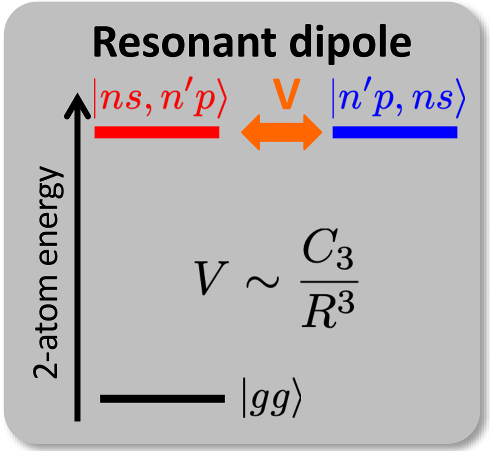
Controlled parameters:

From negligible to dominant interactions

Resonant interaction between Rydbergs and XY spin model

Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)





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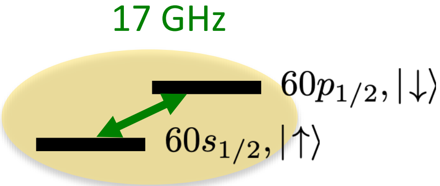
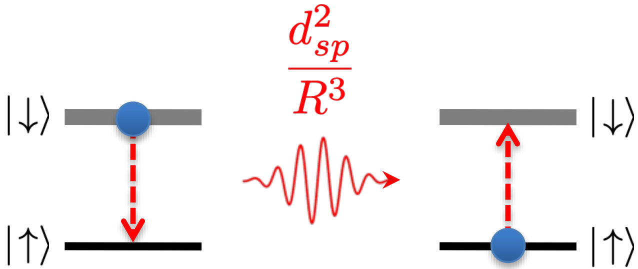
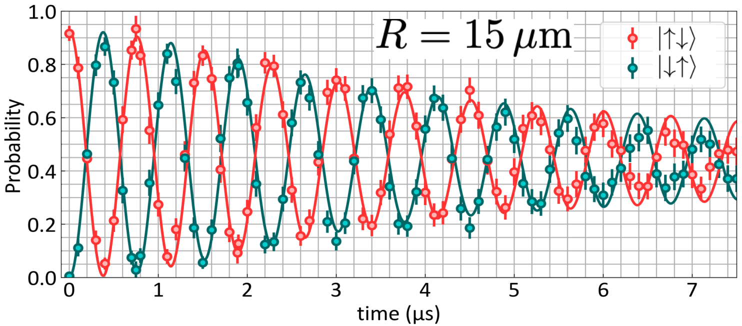




Non radiative “exchange” of excitation



$$\hat{H}_{\text{XY}} = \frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+)$$

$$= 2\frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y)$$

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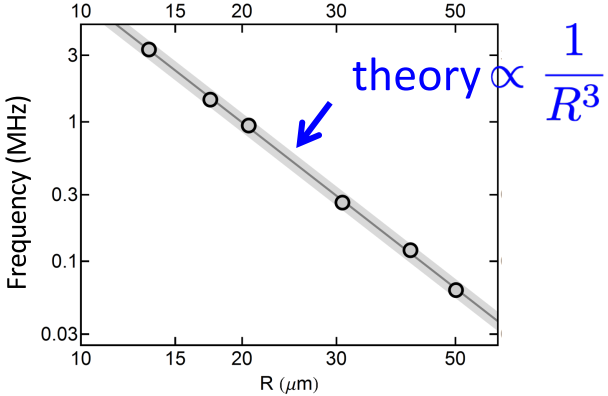

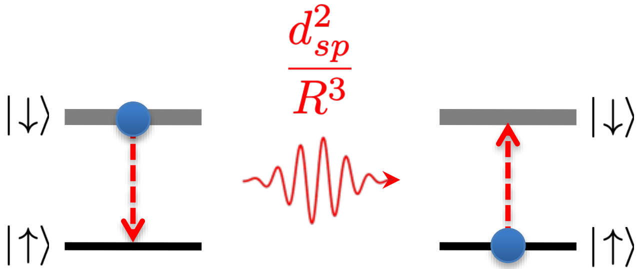

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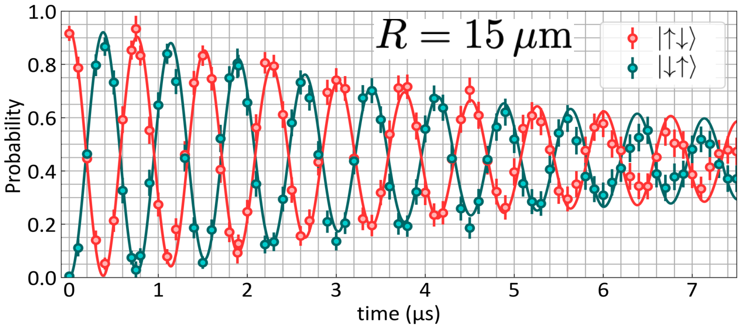
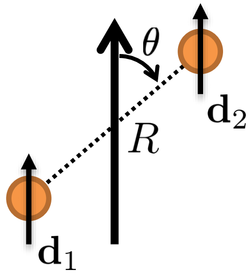
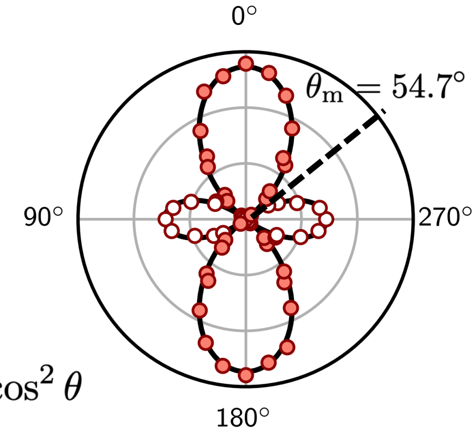


Quantization axis (B)





$$C_3(\theta) \propto 1 - 3\cos^2\theta$$

$$\hat{H}_{XY} = \frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+)$$

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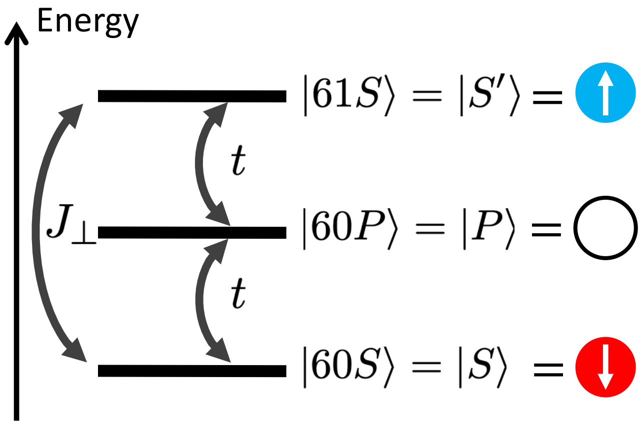
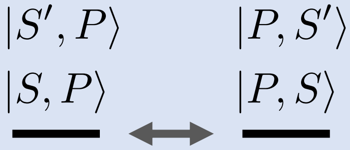
Beyond spin ½: $t - J - V$ model

Idea: Homeier *et al.*, PRL 2024

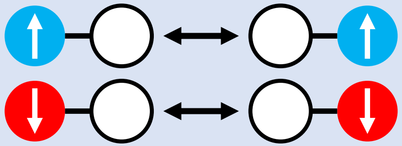


Qiao *et al.* Nature (2025)

direct dipolar exchange

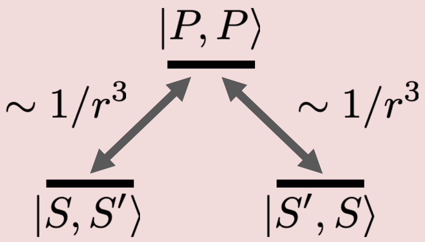


$t \propto 1/r^3$

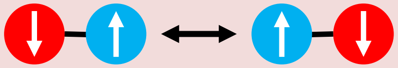


Dipolar tunneling

Indirect dipolar exchange



$J_\perp, J_z \propto 1/r^6$

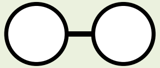


spin exchange

van der Waals shift

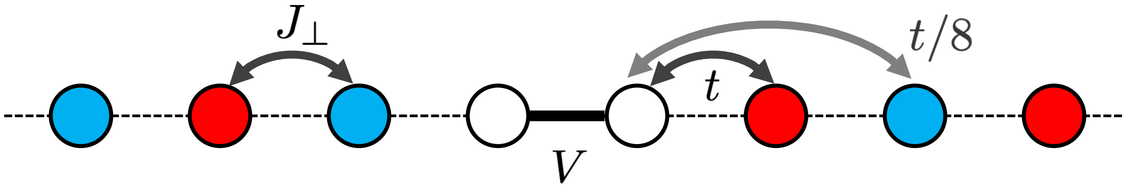
$|S,S\rangle, |P,P\rangle$...

$V \propto 1/r^6$



hole-hole repulsion

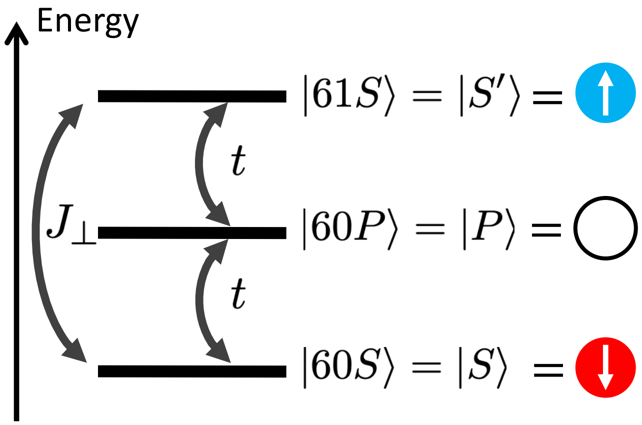
Bosonic t-J-V model:



Long-range hopping...

Beyond spin ½: $t - J - V$ model

Idea: Homeier *et al.*, PRL 2024



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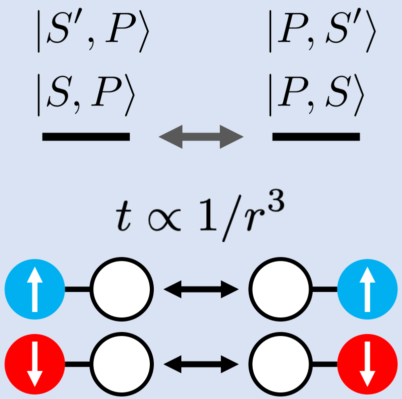
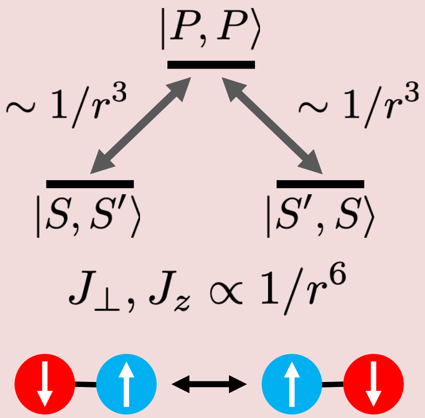


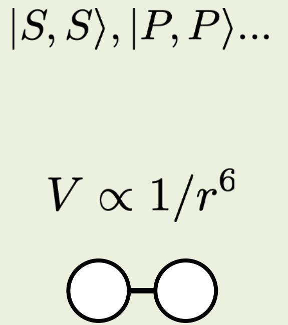
Dipolar tunneling

Indirect dipolar exchange



spin exchange

van der Waals shift



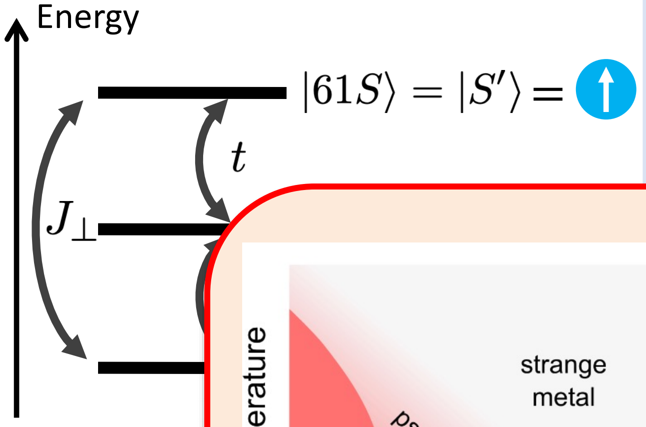
hole-hole repulsion

Bosonic t-J-V model:

$$\hat{H}_{tJV} = -\sum_{i\neq j}\sum_{\sigma=\downarrow,\uparrow}\frac{t_\sigma}{r_{ij}^3}\left(\hat{a}^\dagger_{i,\sigma}\hat{a}^\dagger_{j,h}\hat{a}_{i,h}\hat{a}_{j,\sigma}\right) + \sum_{i\neq j}\frac{1}{r_{ij}^6}\left[J^z\hat{S}_i^z\hat{S}_j^z + \frac{J_\perp}{2}\left(\hat{S}_i^+\hat{S}_j^-\right)\right] + \sum_{i\neq j}\frac{V}{r_{ij}^6}\hat{n}_i^h\hat{n}_j^h$$

Beyond spin ½: $t - J - V$ model

Idea: Homeier *et al.*, PRL 2024



Qiao…

direct dipolar exchange

$|S', P\rangle$ $|P, S'\rangle$

Indirect dipolar exchange

$|P, P\rangle$

van der Waals shift

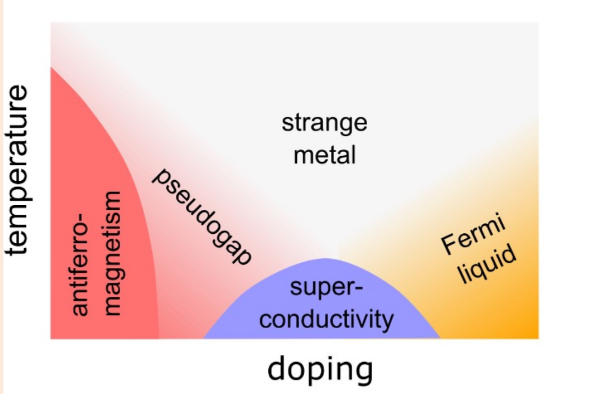
$|S, S\rangle, |P, P\rangle \ldots$

$\propto 1/r^6$

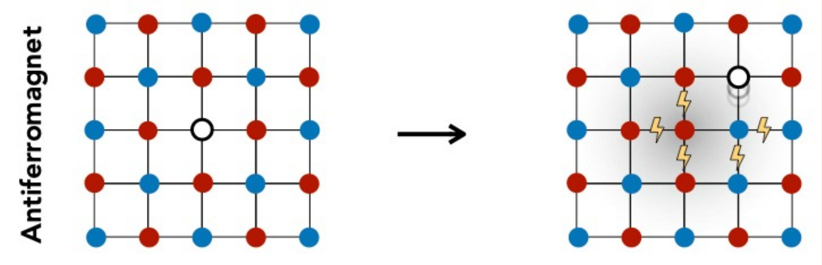
…le-hole …pulsion

Bosonic…

$\hat{H}_{tJV}$ … $\hat{n}_i^h \hat{n}_j^h$



Cuprate away from zero doping…



Competition hole propagation / magnetic ordering

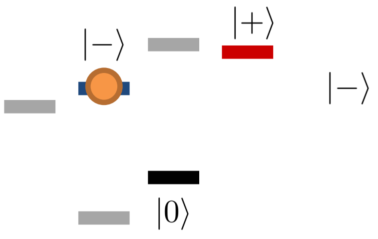
Conclusion: many variants of spin Hamiltonians

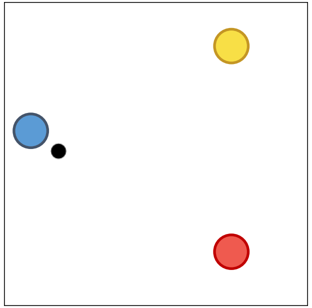
Quantum Ising
s = 1/2

Hardcore
boson

Bosons/ Fermions
Softcore
potential

Spin-orbit coupling





Lienhard, PRX 2021

XY, s = 1/2
$\frac{1}{R^3}, \frac{1}{R^6}$

XYZ
Heisenberg
s = 1/2
Floquet

t- J model

In various *addressable* geometries: 1D (OBC, PBC), 2D : square, triangle, Kagome…

Warning: mapping is only approximate (on top of uncontrolled parameters)…:

XY has small Ising; neglect quadrupolar interactions; not exactly 2 levels…

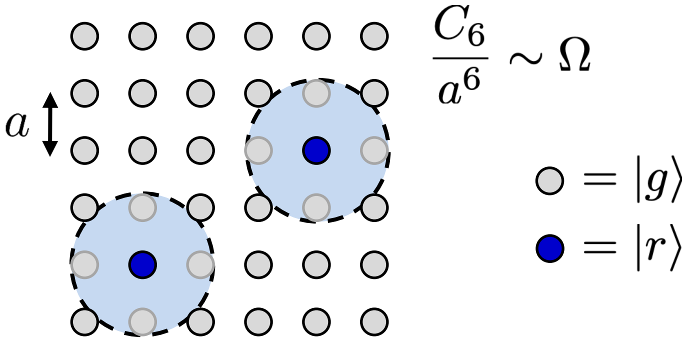
Hard to assess the impact…!!

Outline – Lecture 3

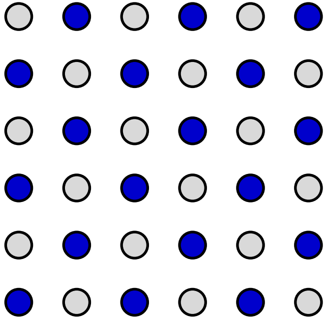
1. Quantum simulation = many-body physics with synthetic matter
2. From Rydberg interactions to spin models
3. Examples of ground-state preparation
4. Examples of out-of-equilibrium dynamics
5. Outlook: what we did not discuss… & beyond

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction



Anti-ferromagnetic ground state



2D Ising anti-ferromagnet on a square

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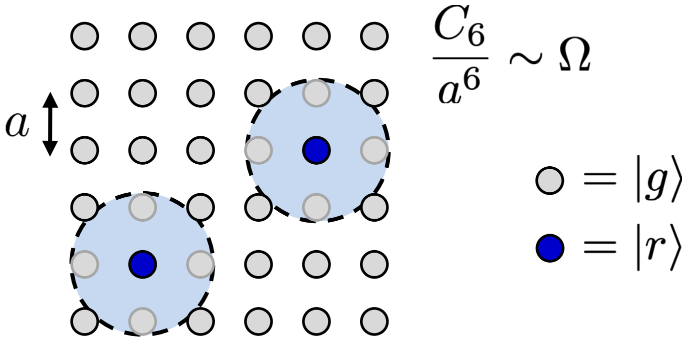
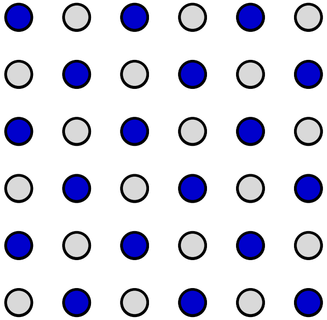


Anti-ferromagnetic ground state



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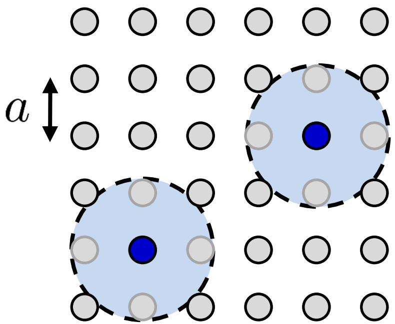
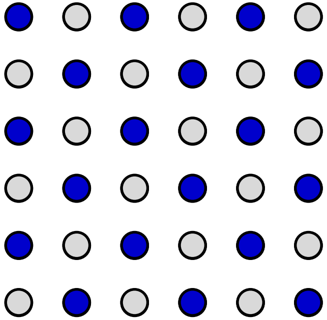


$$\frac{C_6}{a^6} \sim \Omega$$

○ $= |g\rangle = |\downarrow\rangle$

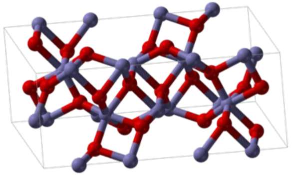
● $= |r\rangle = |\uparrow\rangle$

Anti-ferromagnetic ground state

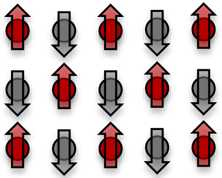


Ex of antiferromagnets:

MnO, FeO, CoO, NiO, $FeCl_2$...

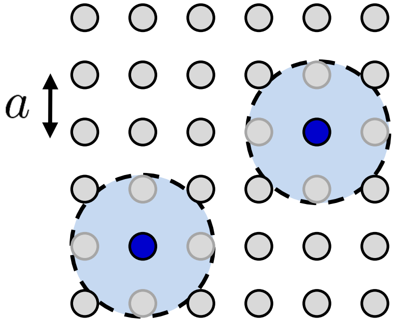


AFM (Néel) ordering (Z_2 phase)



2D Ising anti-ferromagnet on a square

Nearest neighb. interaction



$$\frac{C_6}{a^6} \sim \Omega$$

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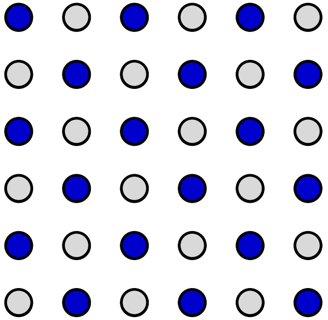
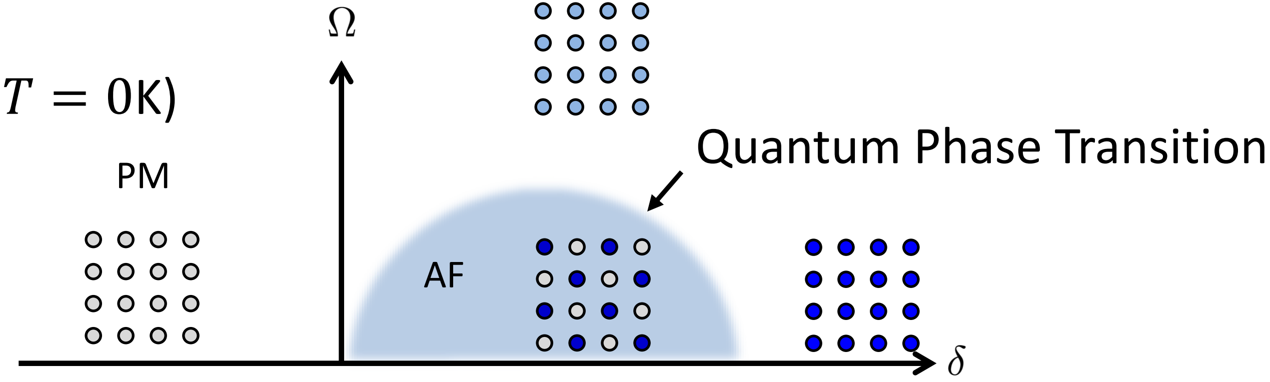
Anti-ferromagnetic ground state



2D phase diagram ($T = 0$K)
(1970)

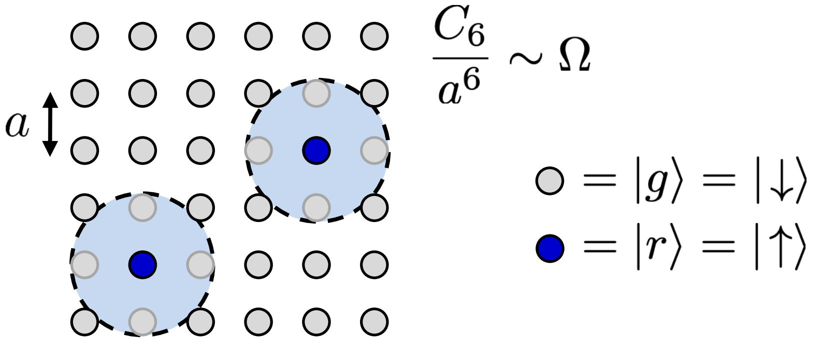


Known by Quantum Monte-Carlo

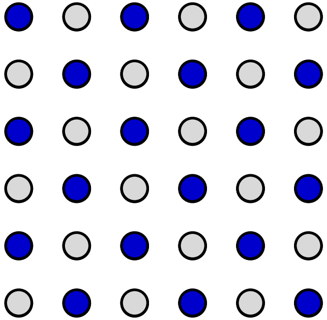
Never implemented and measured in 2D... (approximation in material)

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction



Anti-ferromagnetic ground state



2D phase diagram ($T = 0$K)
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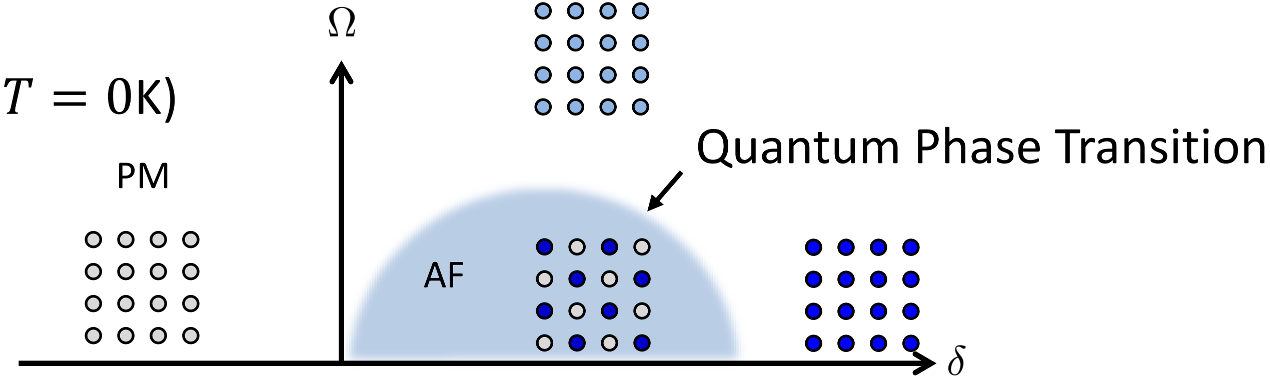
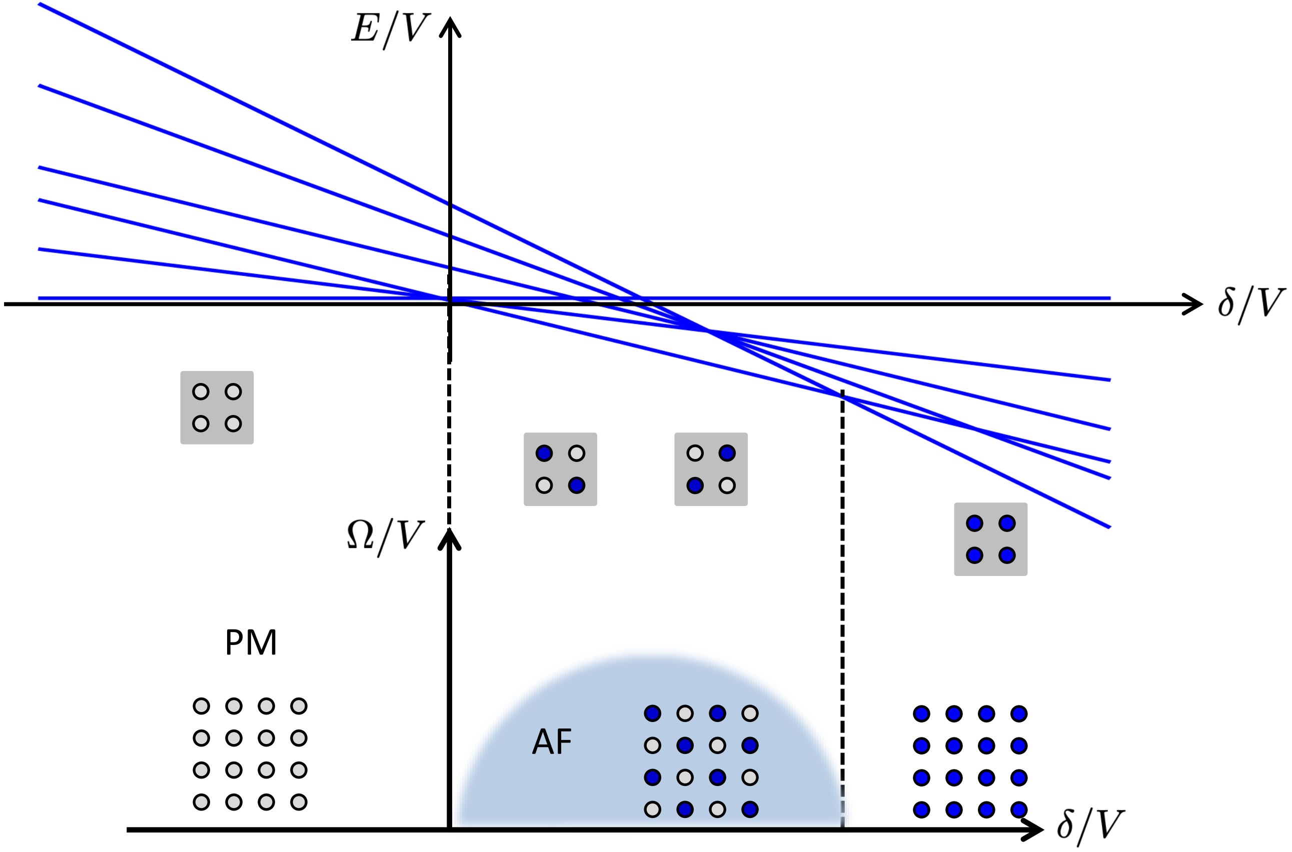


$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

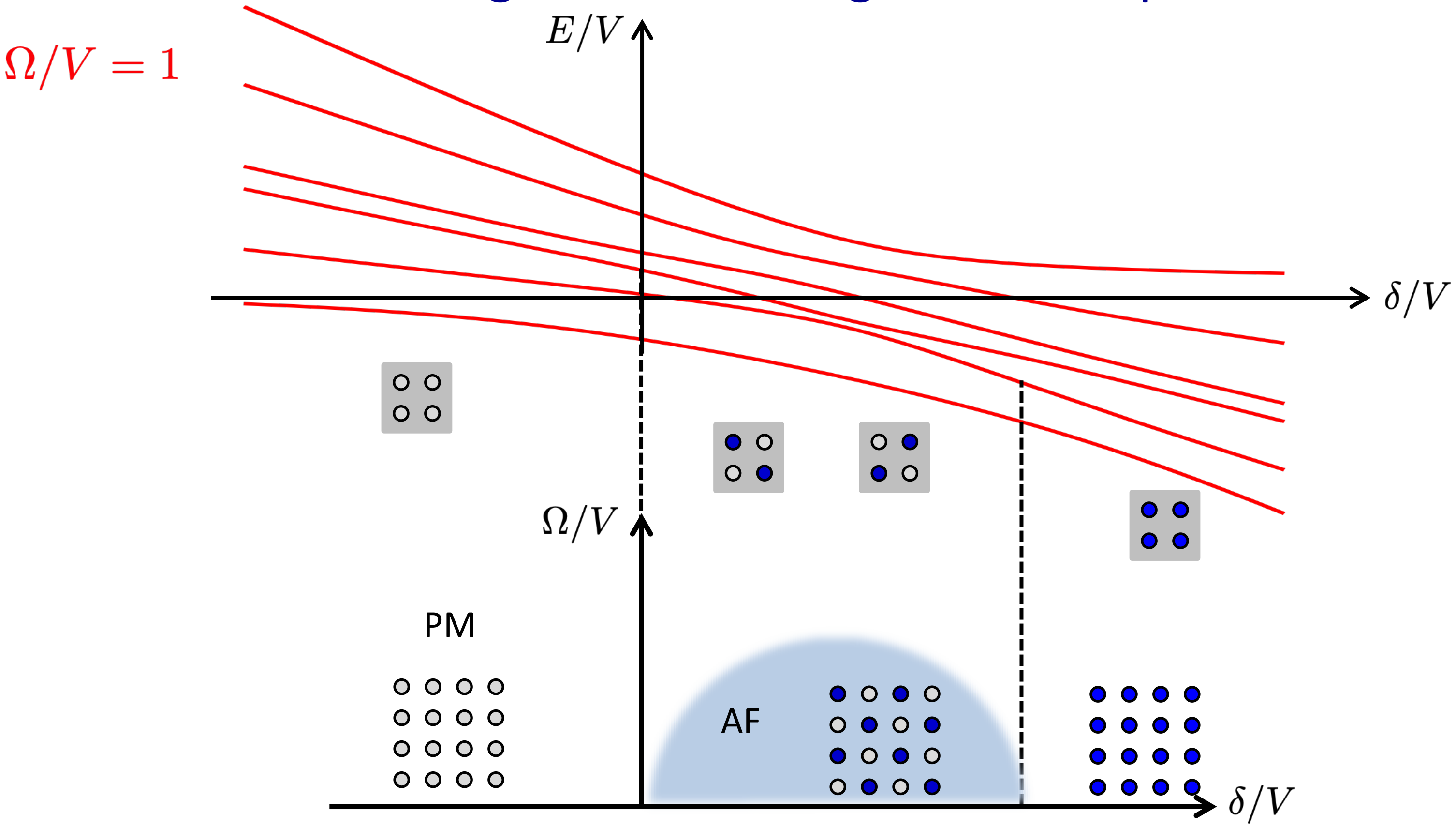
2D Ising anti-ferromagnet on a square

$\Omega/V = 0$



$$H = \quad -\hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

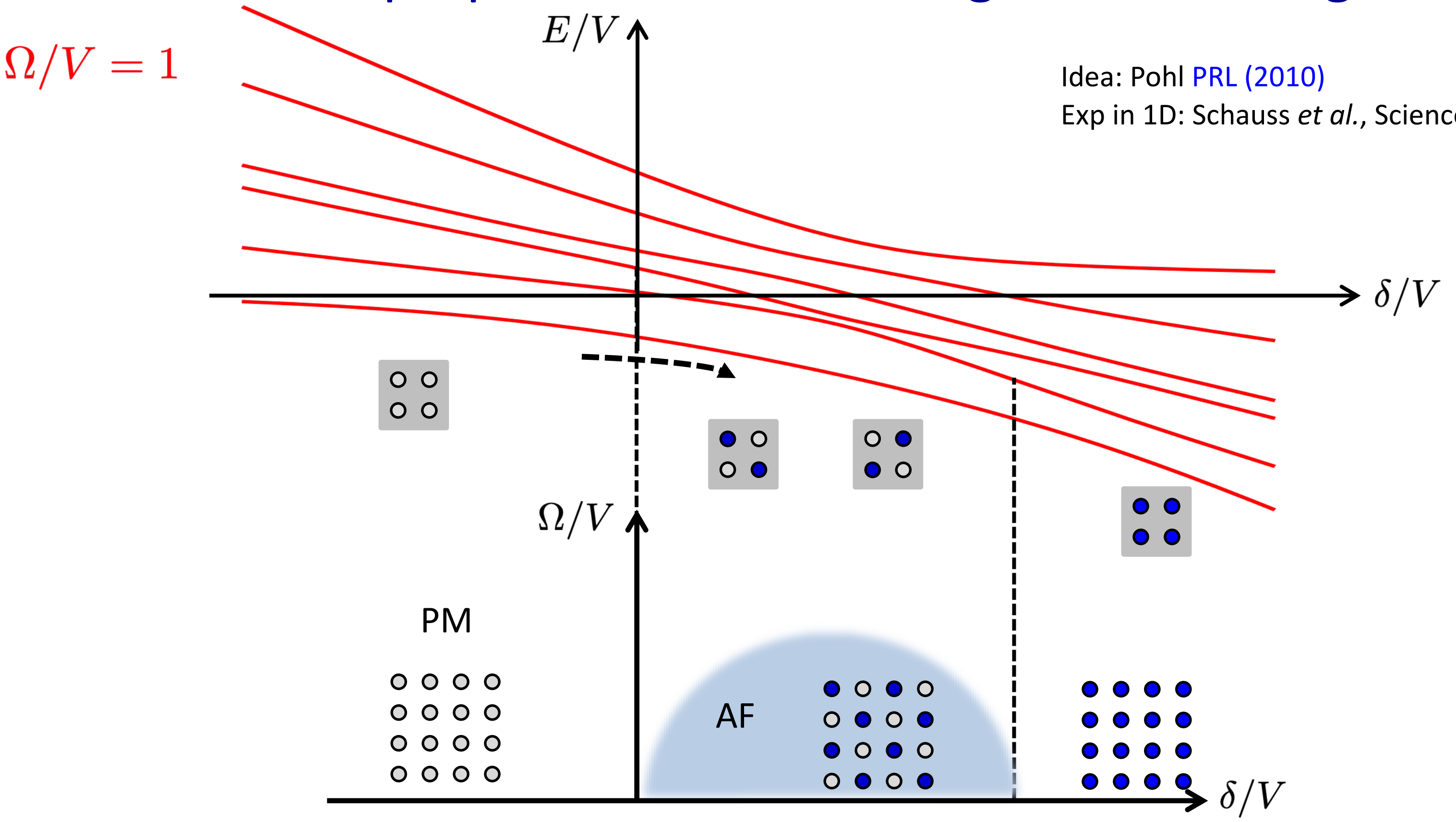
2D Ising anti-ferromagnet on a square



$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

Adiabatic preparation of a 2D Ising anti-ferromagnet

Idea: Pohl PRL (2010)
Exp in 1D: Schauss *et al.*, Science (2015)



$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

Adiabatic preparation of a 2D Ising anti-ferromagnet

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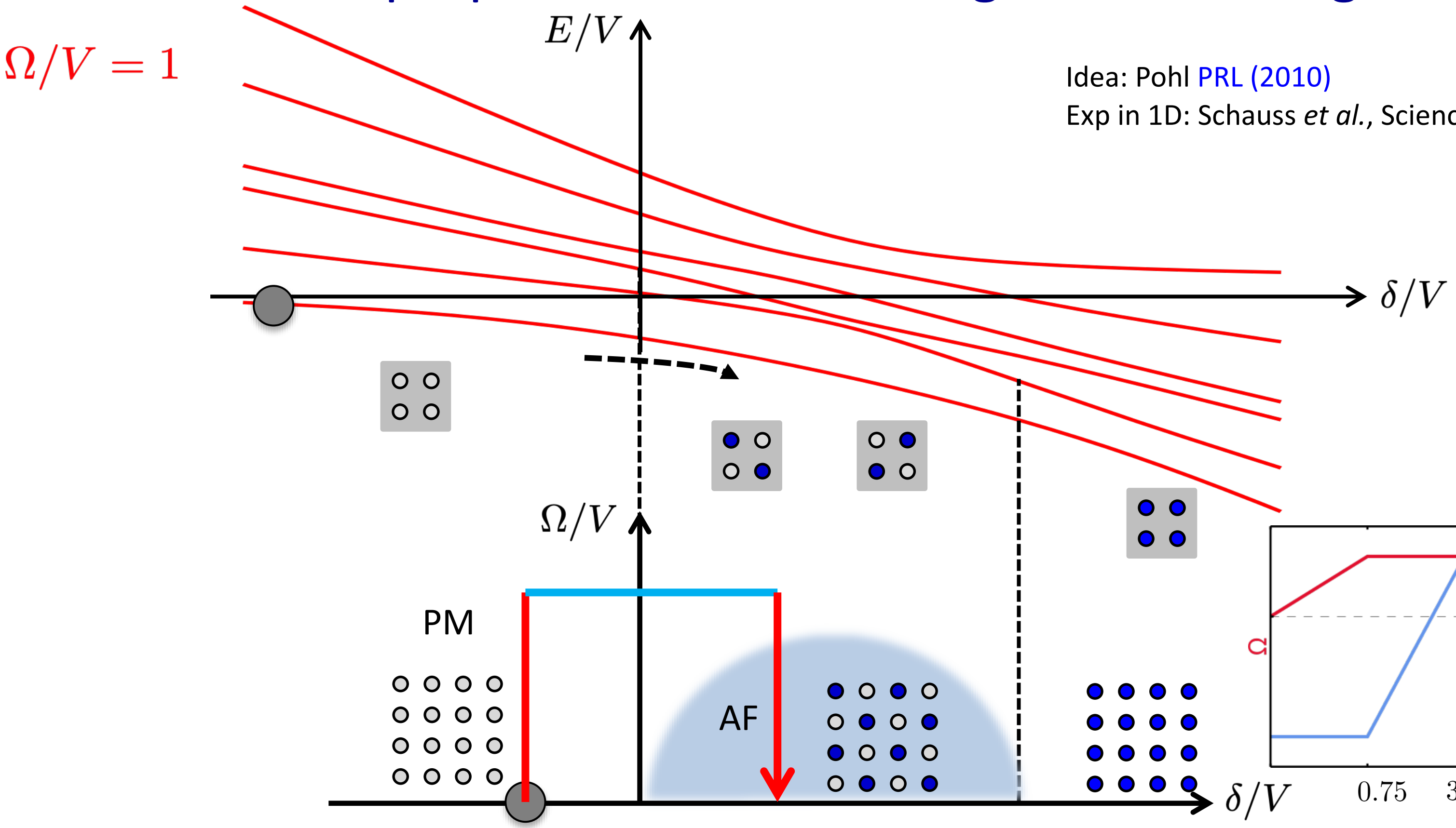
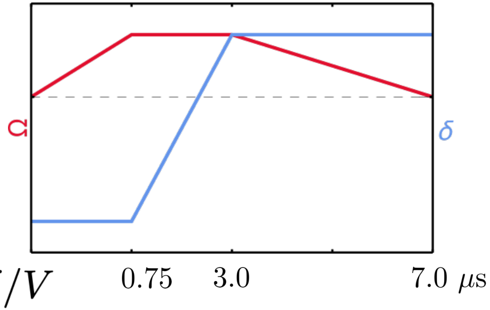
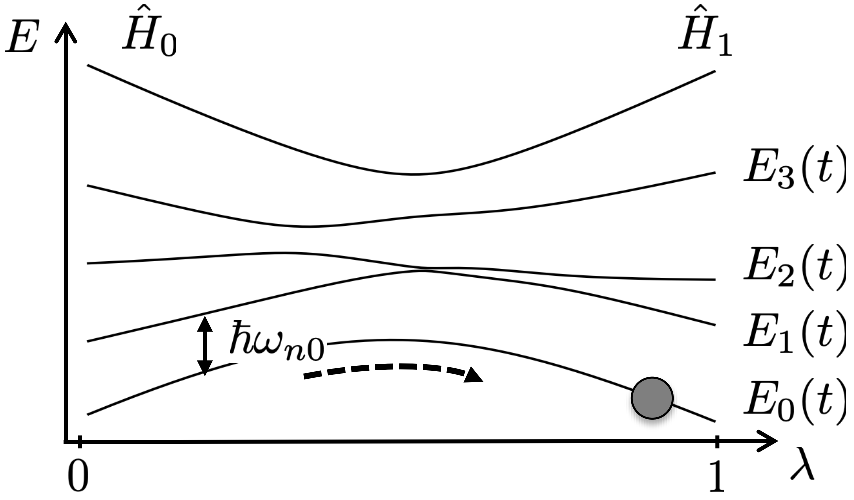




$$H = \sum_i \left(\frac{\hbar\Omega(t)}{2}\sigma_x^i - \hbar\delta(t)\hat{n}_i \right) + \sum_{i<j} \frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

Adiabatic preparation of a ground state: quantum annealing



Sakurai, Quantum Mechanics & Wikipedia

$$\hat{H}(t) = (1-\lambda(t))\hat{H}_0 + \lambda(t)\hat{H}_1$$

Instantaneous eigenstates:

$$\hat{H}(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle$$

Solve: $i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$ with $|\psi(t)\rangle = \sum_n a_n(t)|\phi_n(t)\rangle,\ a_n(0) = \delta_{n,0}$

$$\Rightarrow |a_n(t)| \sim \frac{|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2}{\hbar\omega_{n0}^2}$$

Adiabatic following: $|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2 \ll \hbar\omega_{n0}^2$

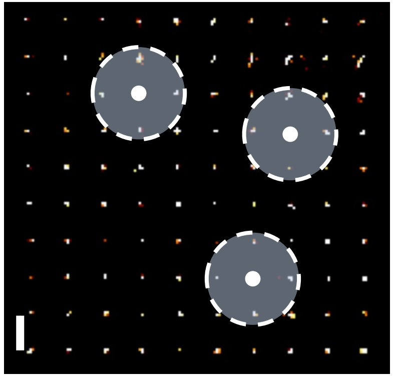
Rate of change of *H* slow with respect to the energy gaps…

Adiabatic preparation of an antiferromagnet on a square array

$$\frac{C_6}{a^6} \sim \Omega$$

Scholl et *al.* Nature (2021)

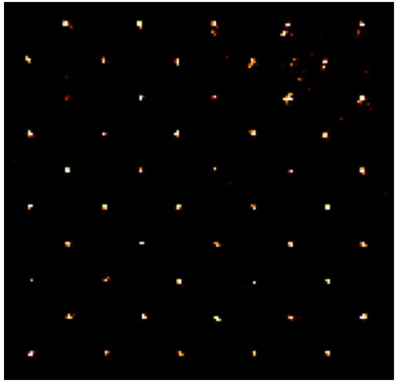
10×10



10 µm

$\Omega(t), \delta(t)$

ramp



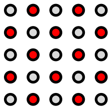
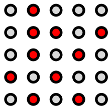
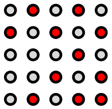
$= |f\rangle$ “bright”

$= |r\rangle$ “dark”

1D: Pohl PRL 2010; Bloch Science 2015; Lukin Nature 2017, 2019;
2D: Lienhard PRX 2018, Bakr PRX 2018; Lukin Nature 2021

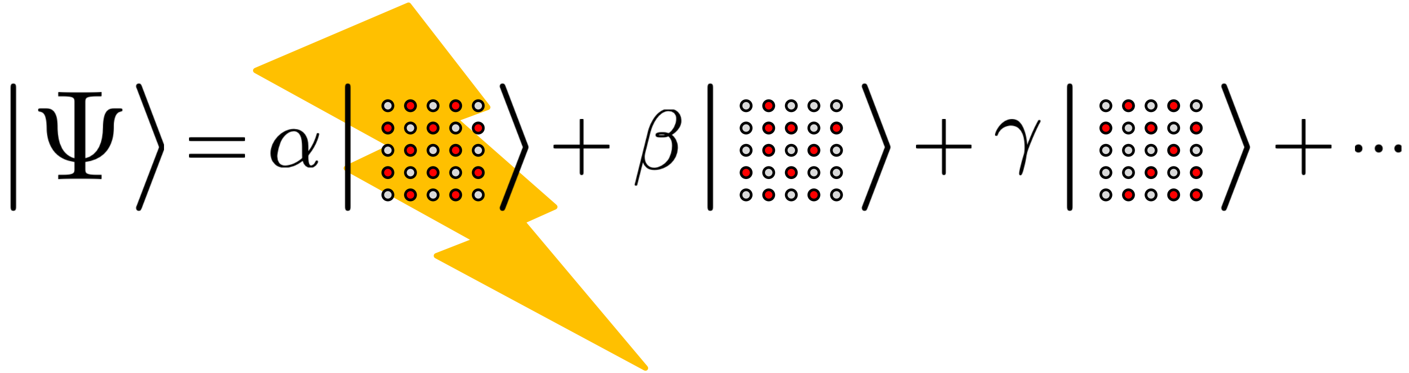
Seeing the many-body wavefunction…

At the end of experiment:

$|\Psi\rangle = \alpha\,|$  $\rangle + \beta\,|$  $\rangle + \gamma\,|$  $\rangle + \cdots$

Seeing the many-body wavefunction…

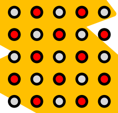
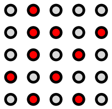
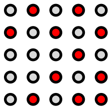
At the end of experiment:

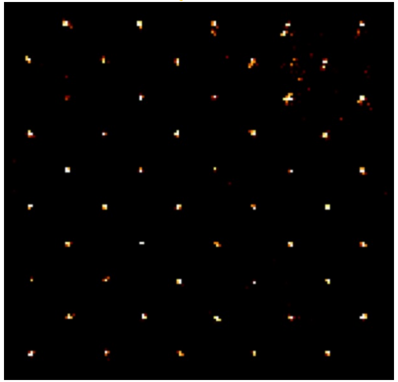


$$|\Psi\rangle = \alpha\,|\cdots\rangle + \beta\,|\cdots\rangle + \gamma\,|\cdots\rangle + \cdots$$

Seeing the many-body wavefunction…

At the end of experiment:

$|\Psi\rangle = \alpha\,|$  $\rangle + \beta\,|$  $\rangle + \gamma\,|$  $\rangle + \cdots$

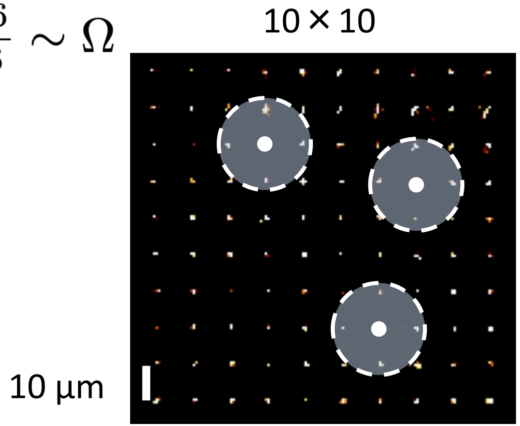
$|\Psi_{\mathrm{f}}\rangle =$ 

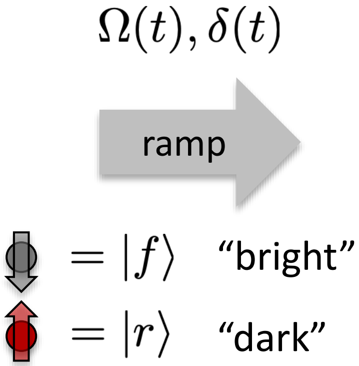
probability $|\alpha|^2$

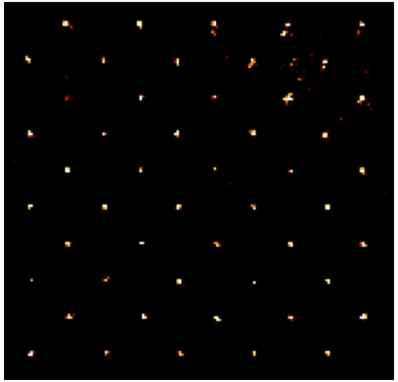
Adiabatic preparation of an antiferromagnet on a square array

$\frac{C_6}{a^6} \sim \Omega$

Scholl et *al.* Nature (2021)



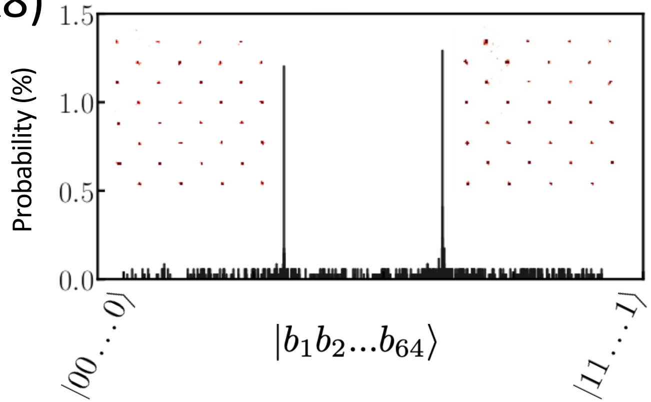


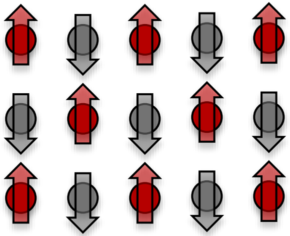


Perfect AF (Néel) ordering!
(proba < 1%)

(8x8)

2^{64} states!!!



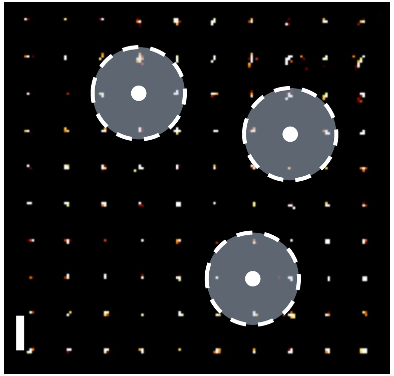


Adiabatic preparation of an antiferromagnet on a square array

Scholl et *al.* Nature (2021)

$$\frac{C_6}{a^6} \sim \Omega$$

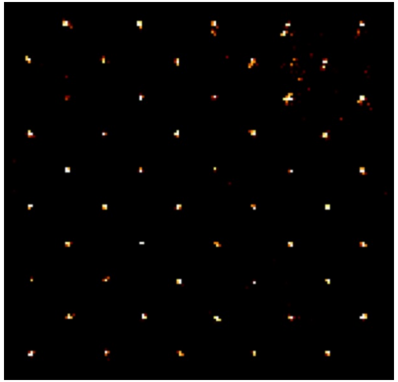
10×10



10 μm

$$\Omega(t), \delta(t)$$

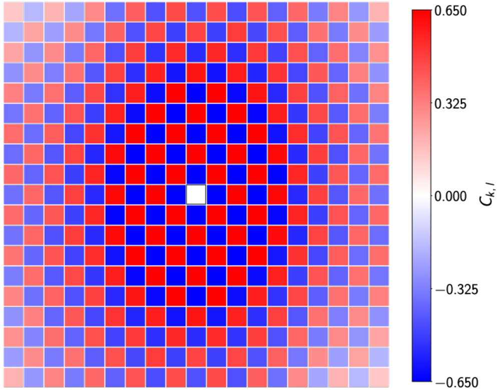
ramp



$= |f\rangle$ "bright"

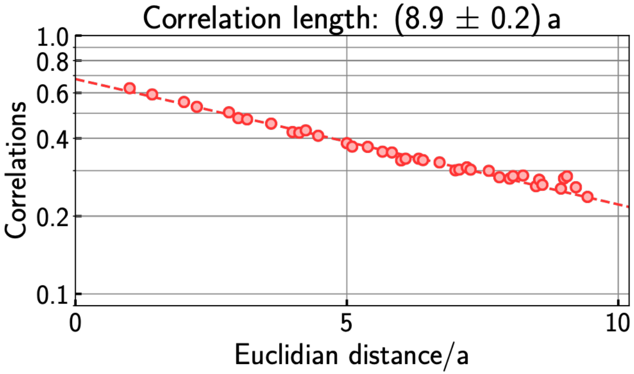
$= |r\rangle$ "dark"

$$C_{kl} \sim \langle n_k n_l \rangle - \langle n_k \rangle \langle n_l \rangle$$



l

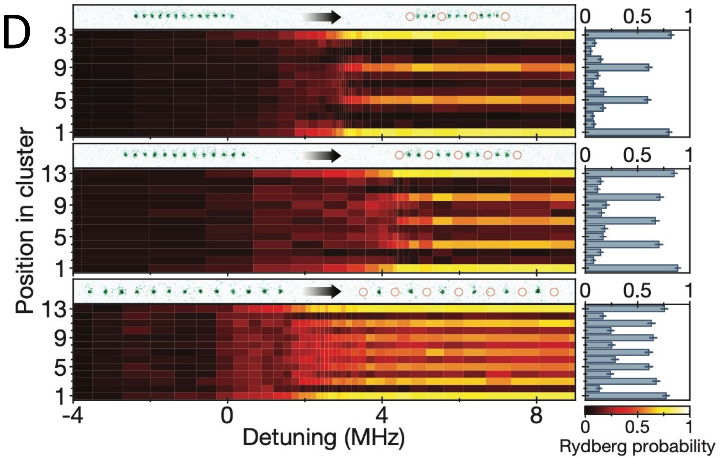
k



Also: Lukin Nature 2021

Ising model in other geometries

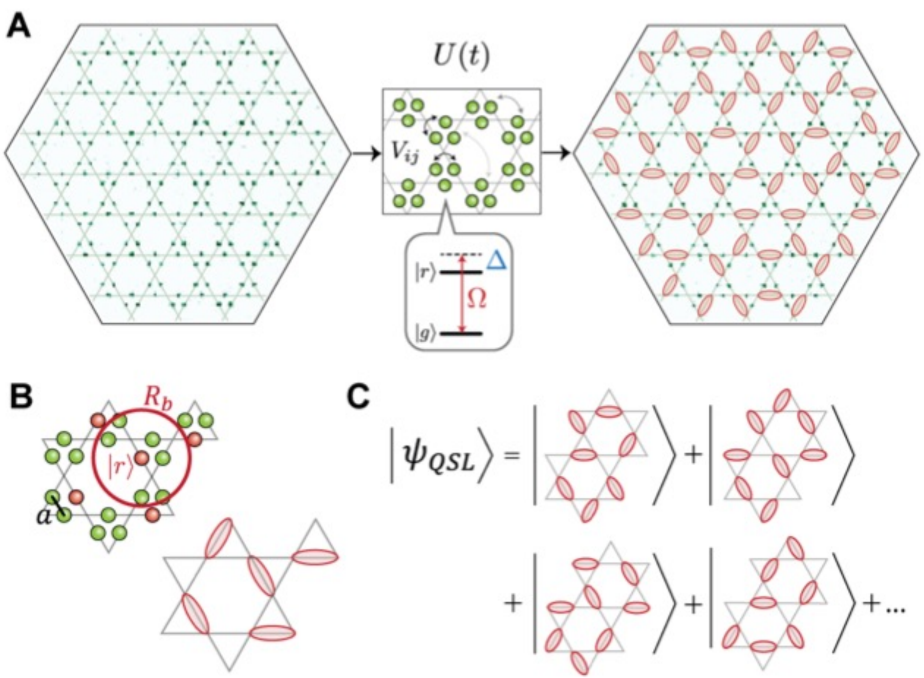
1D



Bernien, Nature 2017

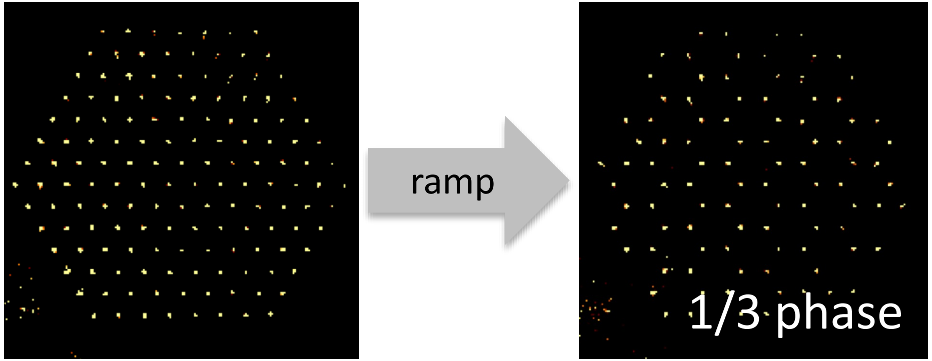
Ruby lattice: spin liquid?

Lukin, Science 2021



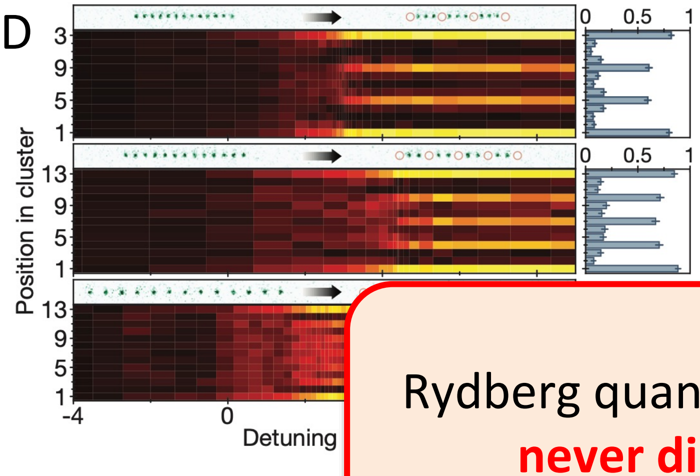
Triangle (frustration)

Scholl et *al.* Nature (2021)



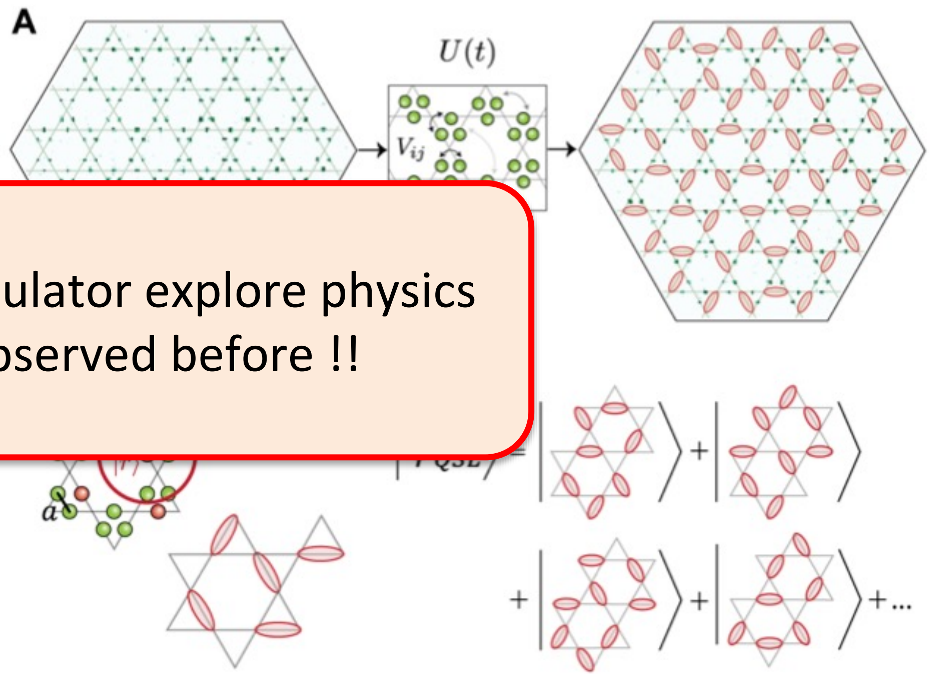
Ising model in other geometries

1D



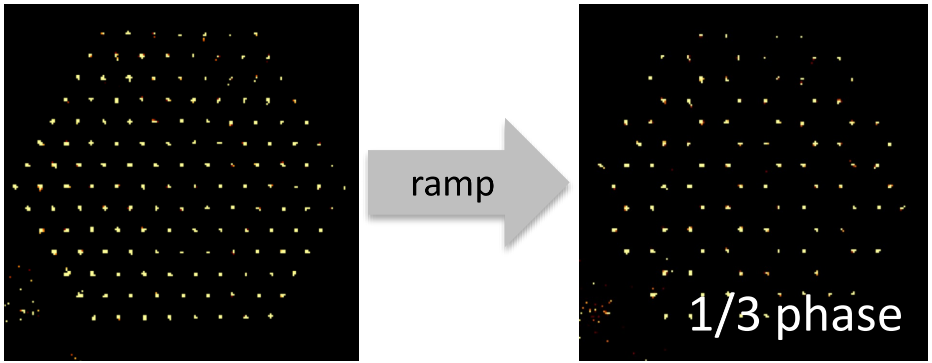
Ruby lattice: spin liquid?

Lukin, Science 2021

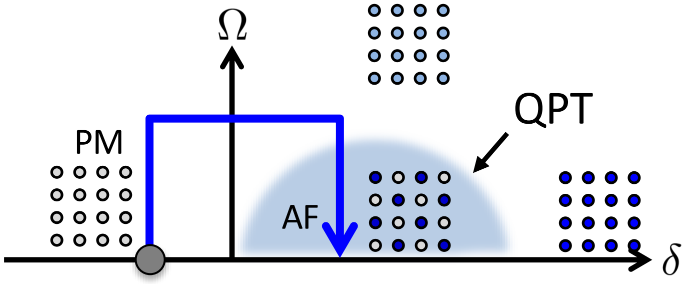


Rydberg quantum simulator explore physics **never directly** observed before !!

Triangle (frustratio



Use failure of adiabaticity to study quantum phase transition



Adiabaticity criteria:

$$H(t) = (1-\lambda(t))H_0 + \lambda(t)H_{\mathrm{MB}}$$

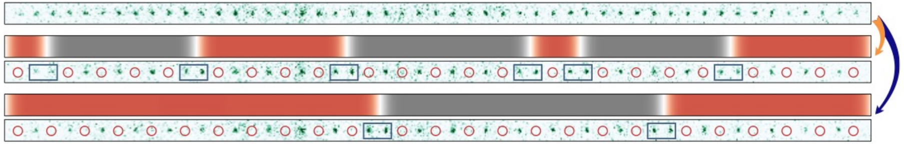
$$|\langle\psi_1(t)|\frac{dH}{dt}|\psi_0(t)\rangle| \ll \frac{\Delta E(t)^2}{\hbar}$$

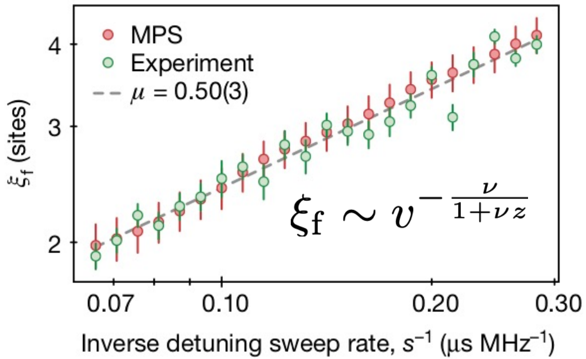
But…gaps close at the QPT!!

Sweeping too fast ⇒ create defects

1D: Keesling, Nature (2019), 2D: arXiv.2012.12281

$R_b \sim a$ 51 atoms





$$\xi_f \sim v^{-\frac{\nu}{1+\nu z}}$$

Kibble-Zurek mechanism:

statistics of defects ⇒ critical exponent

ν_{1D} = 0.50(3) (ν_{MF} = 1/3)

$\nu_{2D,square}$ = 0.62(4) (ν_{MF} = 1/2)

Outline – Lecture 3

1. Quantum simulation = many-body physics with synthetic matter

2. From Rydberg interactions to spin models

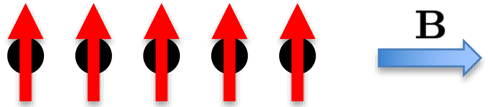
3. Examples of ground-state preparation

4. Examples of out-of-equilibrium dynamics

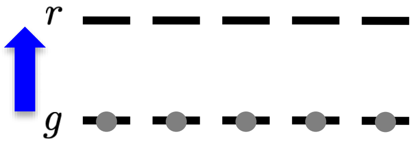
5. Outlook: what we did not discuss… & beyond

Thermalization of closed Many-Body systems

Question: do closed systems always reach equilibrium?

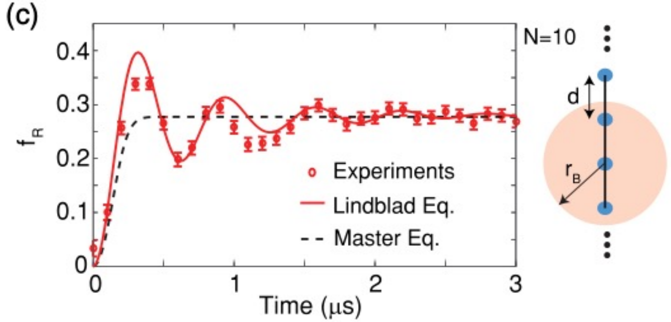


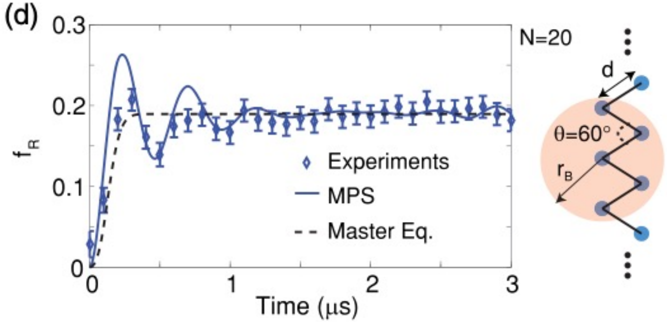
$\Rightarrow ??$



$\Rightarrow ??$

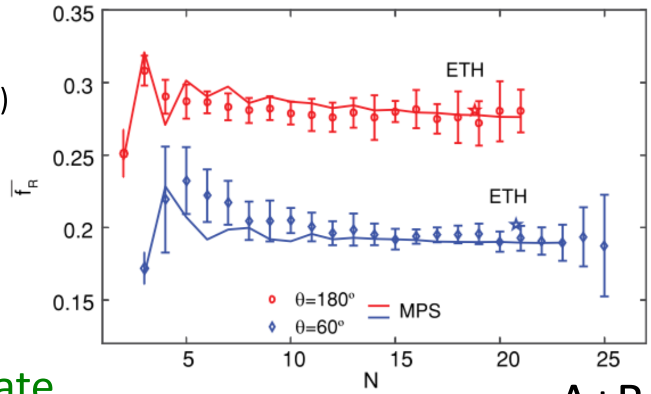
Quenched dynamics in Ising model



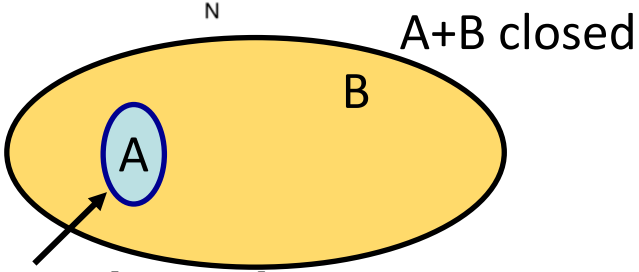


Kim,… Ahn,
PRL **120**, 180502 (2018)

$$f_r = \frac{\langle N_r \rangle}{N}$$

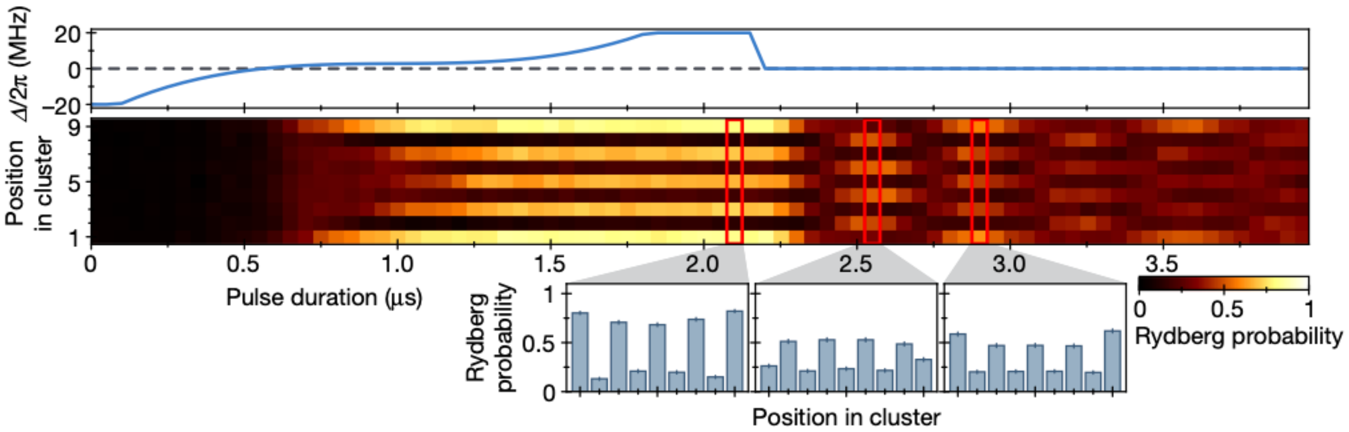


Eigenstate Thermalization Hypothesis (ETH)



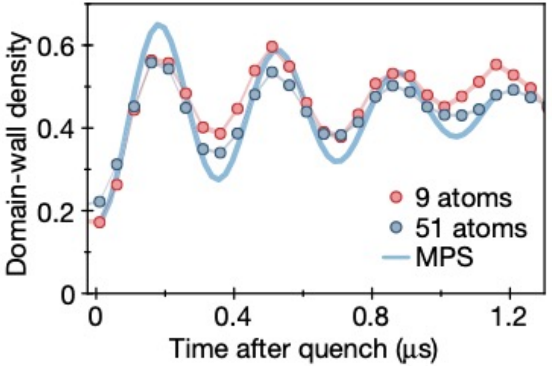
$$\rho_A = \mathrm{Tr}[\rho_{AB}] \propto \exp[-\beta H_A]$$

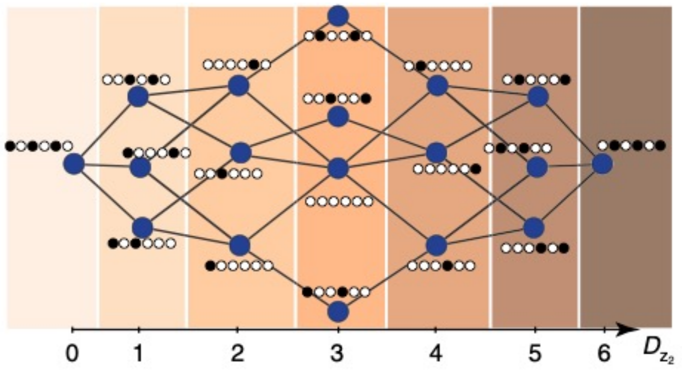
But sometimes it does not thermalize…: quantum scars in 1D



Bernien *et al.*,
Nature **551**, 579 (2017)

Blockade constraint breaks ergodicity





Systems keeps memory of initial state!

Turner *et al.*,
Nat. Phys. 2018

Quantum scars

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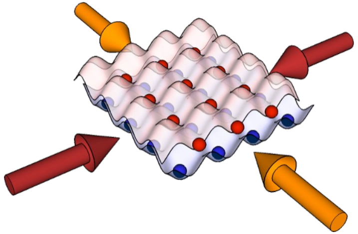
Analog versus digital quantum simulation

Analog

The platform implements directly H_{model}



e.g.: Fermi Hubbard, spin models, electrons in B-fields…

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t H_{\text{mod}}(t')dt'\right)|\psi(0)\rangle$$

Non-universal

Georgescu, Rev. Mod. Phys. (2014)

Digital

H_{model} synthesized digitally

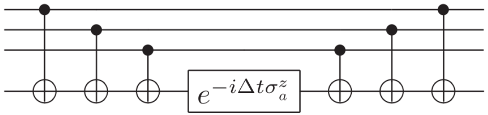
$$H_{\text{mod}} = \sum_{n=1}^{N} H_n$$

e.g. single & 2-qbit operations

$$e^{-iH_{\text{mod}}t} \approx \left(e^{-iH_1 t/n} e^{-iH_2 t/n} \ldots e^{-iH_3 t/n}\right)^n$$

= “universal” quantum simulation

Ex:



$$H_{\text{mod}} = \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$$

Digital quantum simulation: resource estimates…

Quantum Science and Techno. **7**, 045025 (2022)

Number of *perfect* gates to reproduce current *imperfect* analog simulation

Gate	Gate Count	Depth
CNOT	1.7×10^5	8.4×10^3
$R_Z(\theta)$	6.8×10^4	6.7×10^2

TABLE I. Gate count and depth estimates for digital quantum simulation of the Hubbard model with $J\tau = 2.7$, $M = 100$ and $tJ = 10$.

M sites

Gate	Gate Count	Depth
CNOT	1.6×10^3	5.5×10^2
$R_Z(\theta)$	2.1×10^4	3.5×10^2

TABLE III. Gate count and depth estimates for digital quantum simulation of the nearest neighbour Ising model with $J\tau = 2.6$, $M = 100$, $tJ = 10$.

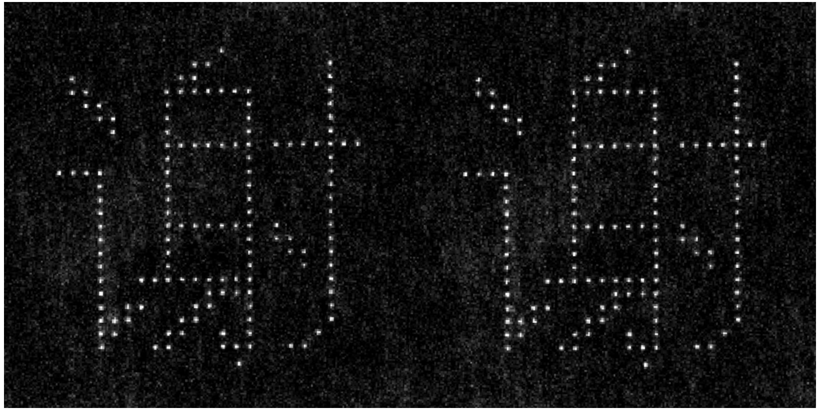
Gate	Gate Count	Depth
CNOT	6.9×10^5	1.4×10^4
$R_Z(\theta)$	3.5×10^5	7.0×10^3

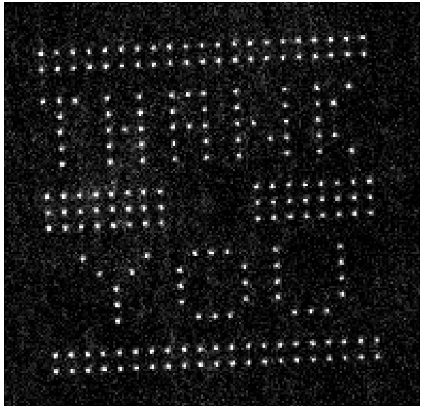
TABLE II. Gate count and depth estimates for digital quantum simulation of the long-range Ising model with $J\tau = 2.6$, $M = 100$ and $tJ = 10$.

Numbers explode when analog errors → 0

Application: Area	Application: Task	Target runtime	Logical gates	Logical qubits	Minimum physical qubits needed (surface code) 10^{-3}	10^{-4}	10^{-6}	Desired logical gate execution rate
Physics	Dynamical sim. of spin system	10 sec	1M	100	100k	23k	5k	0.1 MHz
Physics	Fermi–Hubbard (10 × 10) ground-state sim.	1 hr	100M	250	500k	100k	25k	0.03 MHz

Assembled arrays…





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