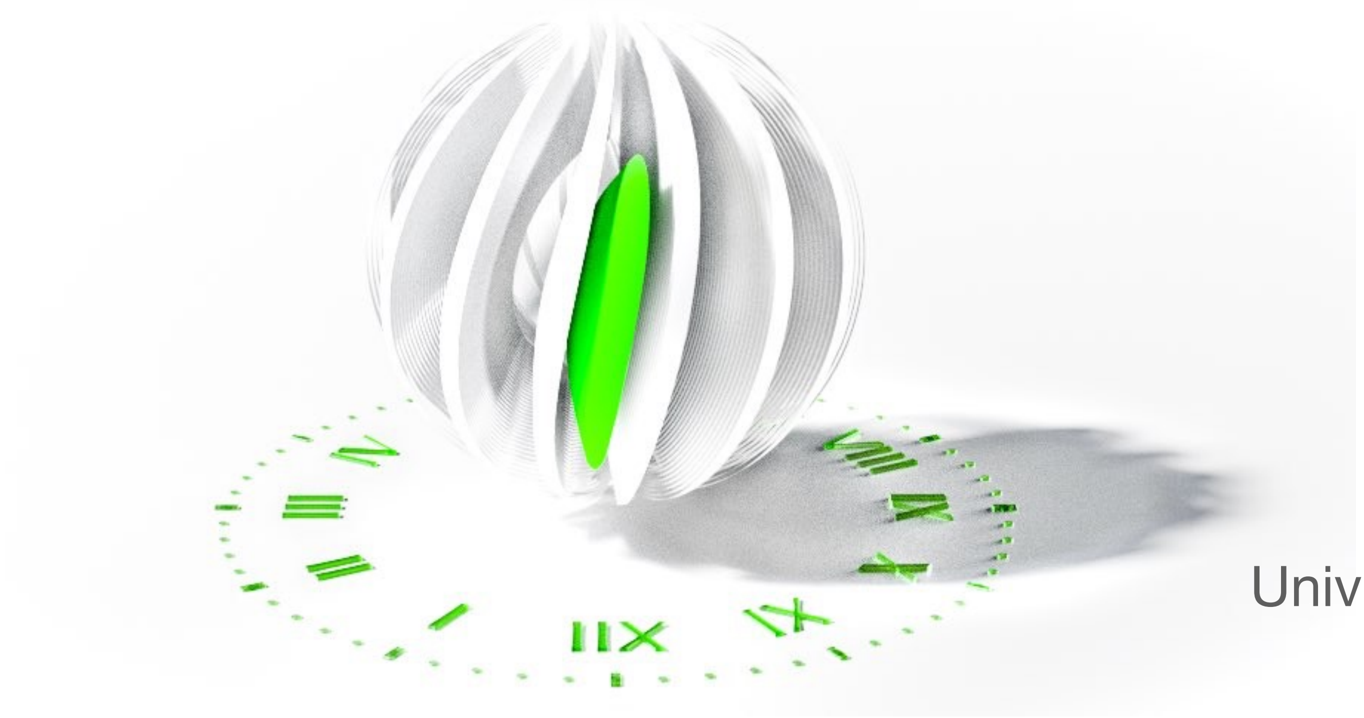
ICAP Summer School | June 13, 2026

Variational Quantum Metrology Beyond Spin Squeezing:

From Programmable Sensors to Quantum Tests of Gravity



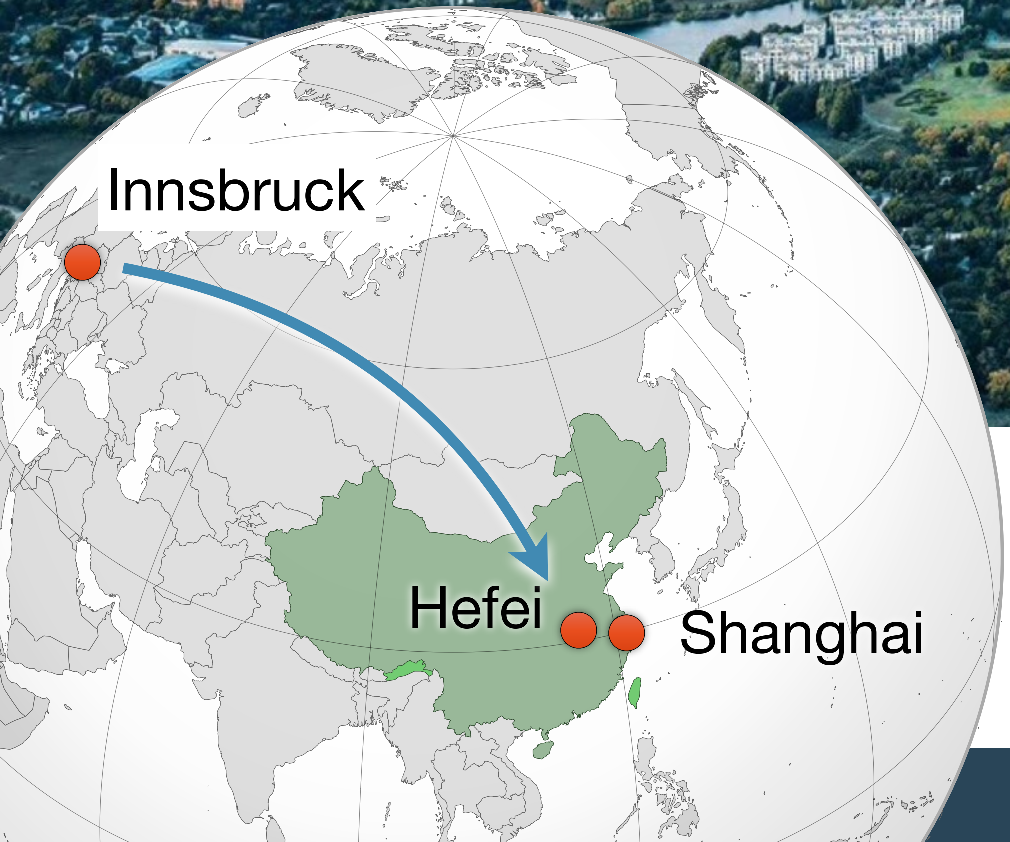
Denis Vasilyev

University of Innsbruck, IQOQI

Hefei National Lab, IMFP

Quantum Optics • Quantum Many-Body Systems • Quantum Metrology

Open positions:
- PhD students
- Postdocs

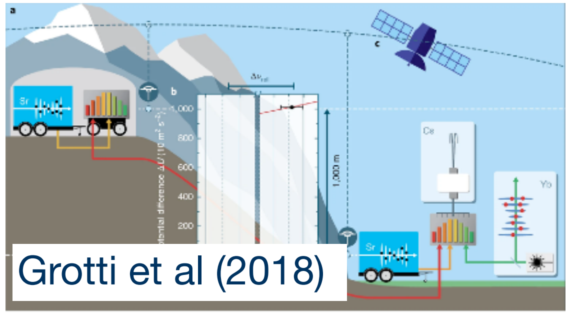


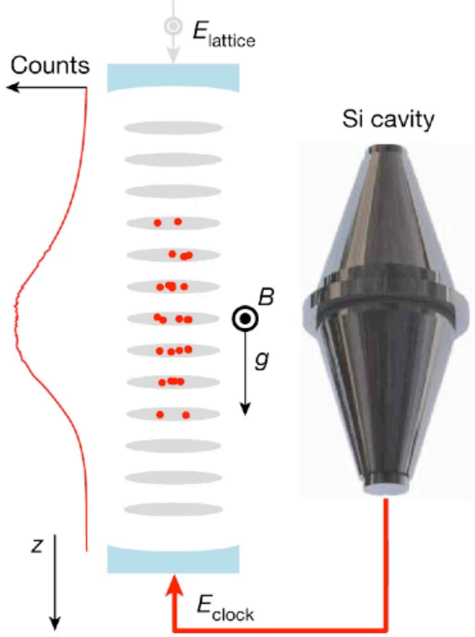
I start my new research group
at the **Institute for Mathematics and Fundamental Physics**,
in the superposition of **Hefei** and **Shanghai**

Quantum Technologies

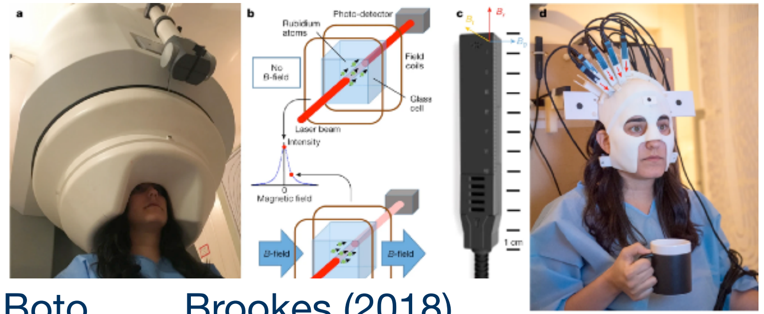
Quantum Sensors:


LIGO Labs


Grotti et al (2018)



Bothwell, …, Ye (2022)

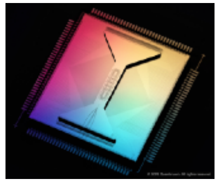


Boto, …., Brookes (2018)

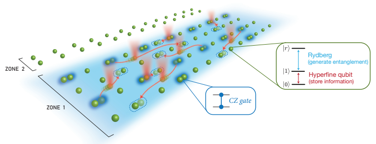
Programmable Quantum devices:

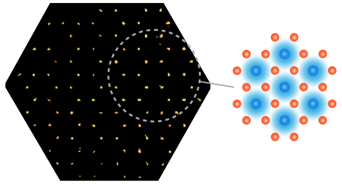
Trapped Ions:


Pogorelov, …, Monz (2023)

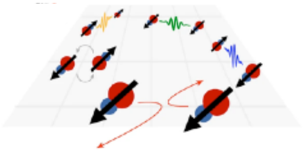

Qantinuum

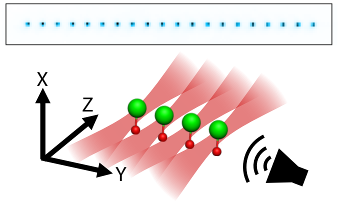
Rydberg atoms:


Bluvstein, …, Lukin (2022)


Scholl, …, Browaeys (2021)

Polar Molecules:


Li, …, Ye (2023)



Bao, …, Doyle (2023)

Quantum Technologies

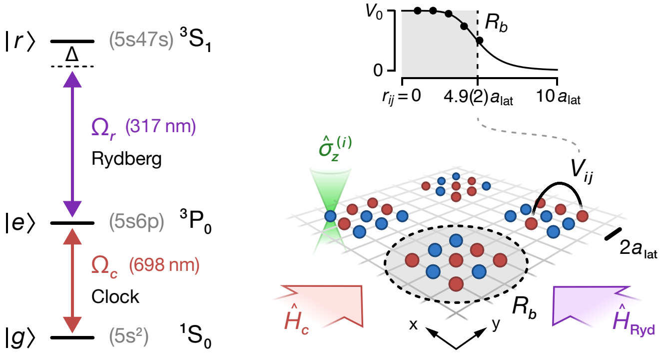
Quan


Grotti e


Boto, …

Programmable quantum sensors:

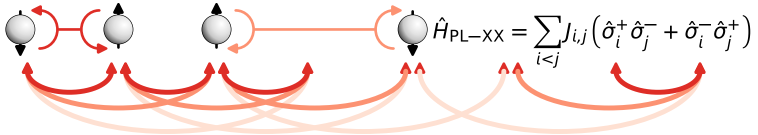
Rydberg Tweezer Arrays

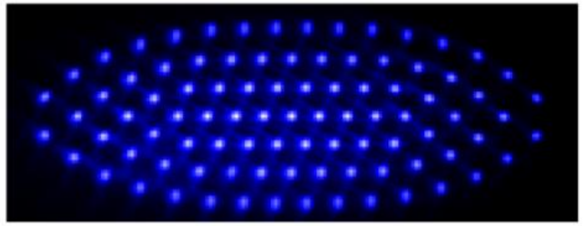


Eckner et al (2023)

Trapped Ions

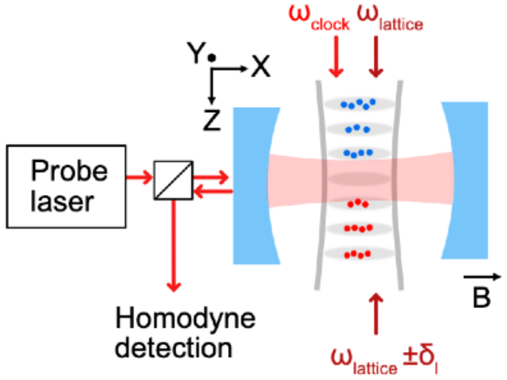
Franke et al (2023)



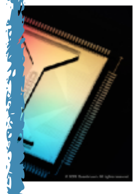


Kiesenhofer et al (2023)

Movable lattice clock in cavity:



Robinson et al (2022)



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023)

(2023)

Lectures Outline

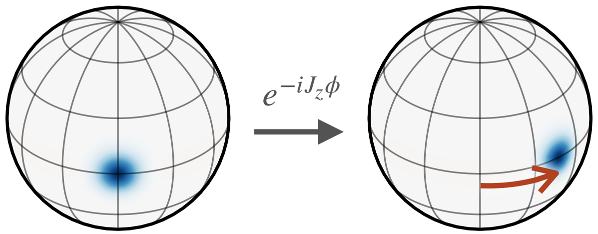
1. Beyond Spin Squeezing: Variational Quantum Metrology for Ramsey Interferometry and Atomic Clocks

2. Programmable Quantum Sensors and Quantum Compasses

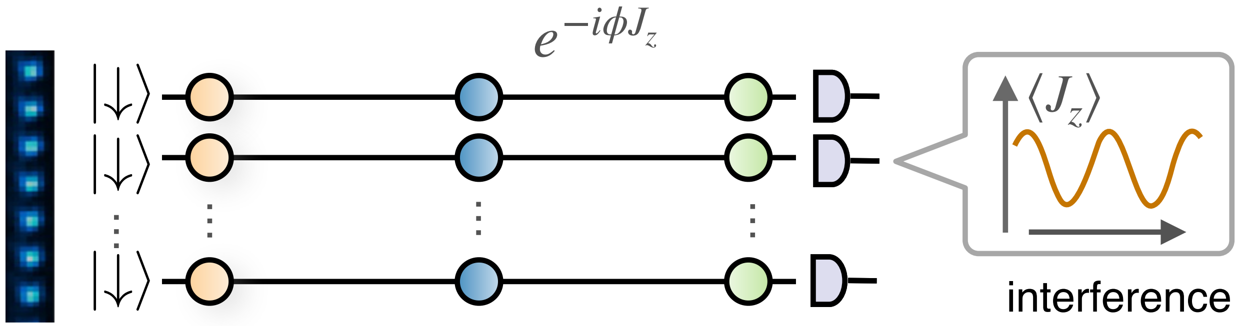
3. Quantum Sensing Networks for Tests of Quantum Mechanics and General Relativity

Quantum Interferometer

Ramsey interferometry



coherent spin state
on Bloch sphere

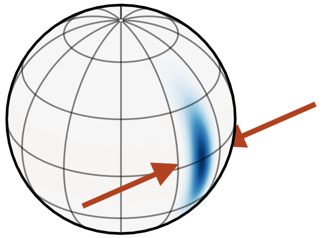


interference
fringes

application: atomic clocks, magnetometry….

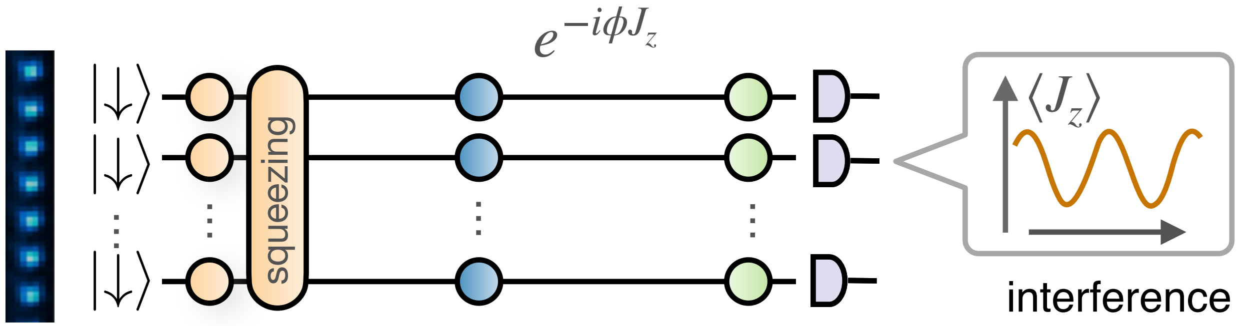
here: uncorrelated atom, Standard Quantum Limit (SQL)

Quantum Interferometer



spin squeezed state
on Bloch sphere

Ramsey interferometry



$T_z = e^{-i\chi J_z^2}$

One-Axis Twisting as
native q-gate for ions

interference
fringes

application: atomic clocks

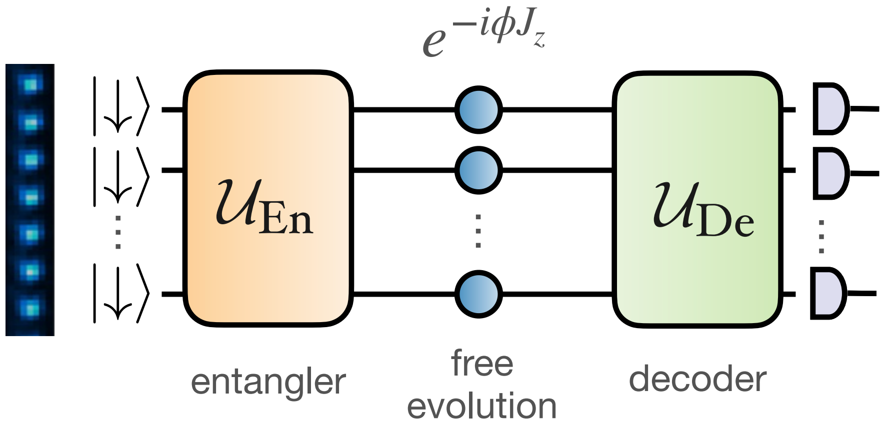
entangled states beyond SQL: spin-squeezing, GHZ, …

Nature 588, 414–418 (2020)
arXiv:2211.08621
arXiv:2303.08078

D. J. Wineland, et al., Phys. Rev. A **50**, 67 (1994).
J. Bollinger et al., Phys. Rev. A **54**, R4649 (1996).

Optimal Quantum Interferometer

Generalized Ramsey interferometry



beyond spin-squeezing …

Optimal quantum interferometer?

Identify cost function for the specific sensing task

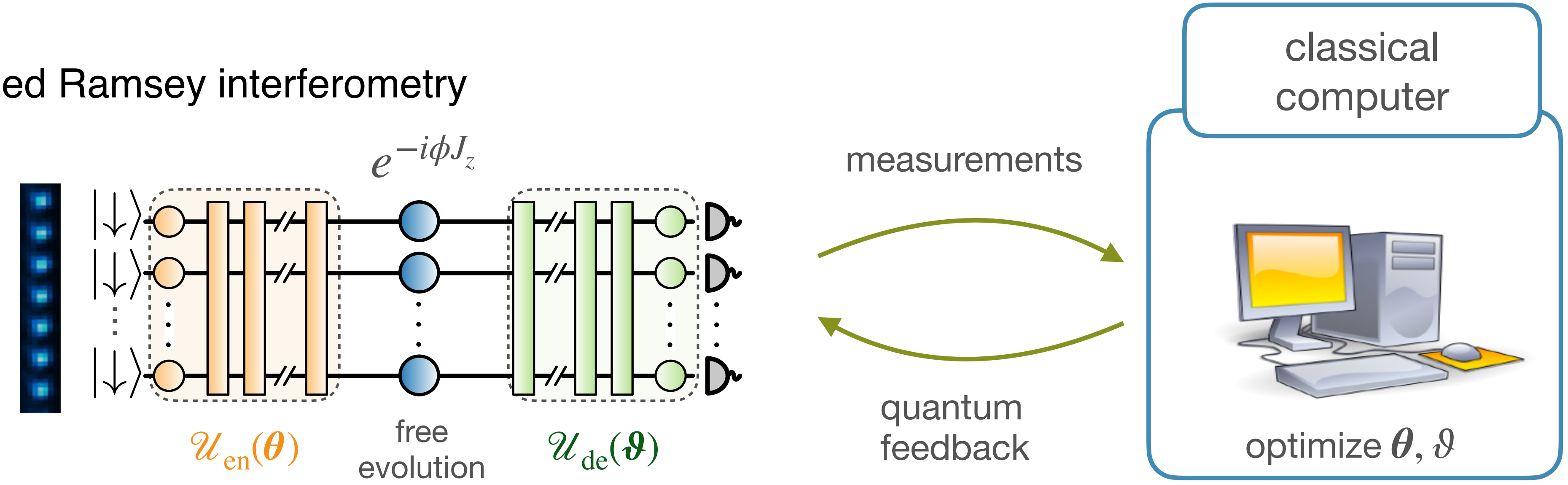
$$\mathcal{C} \to \max/\min \quad ?$$

How to implement?

R. Demkowicz-Dobrzański et al., in *Progress in Optics Vol 60* 2015) pp. 345; K. Macieszczak et al., New J. Phys. **16**, 113002 (2014)
L. Pezzè et al.,, Rev. Mod. Phys. **90**, 035005 (2018).

Optimal Quantum Interferometer — Variational Approach

Generalized Ramsey interferometry



beyond spin-squeezing …

Optimal quantum interferometer?

Identify cost function for the specific sensing task

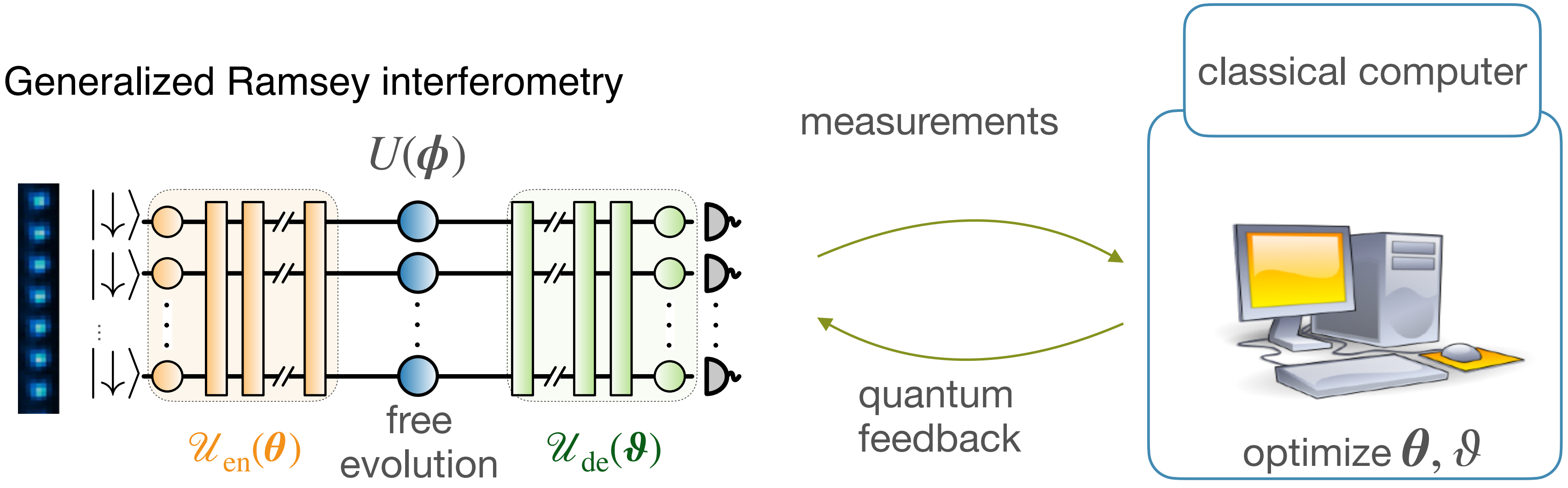
$$\mathcal{C} \rightarrow \max/\min \quad ?$$

How to implement?

variational optimization with
shallow-depth quantum circuits

R. Demkowicz-Dobrzański et al., in *Progress in Optics Vol 60* 2015) pp. 345; K. Macieszczak et al., New J. Phys. **16**, 113002 (2014)
L. Pezzè et al.,, Rev. Mod. Phys. **90**, 035005 (2018).

Three main questions of Variational Metrology



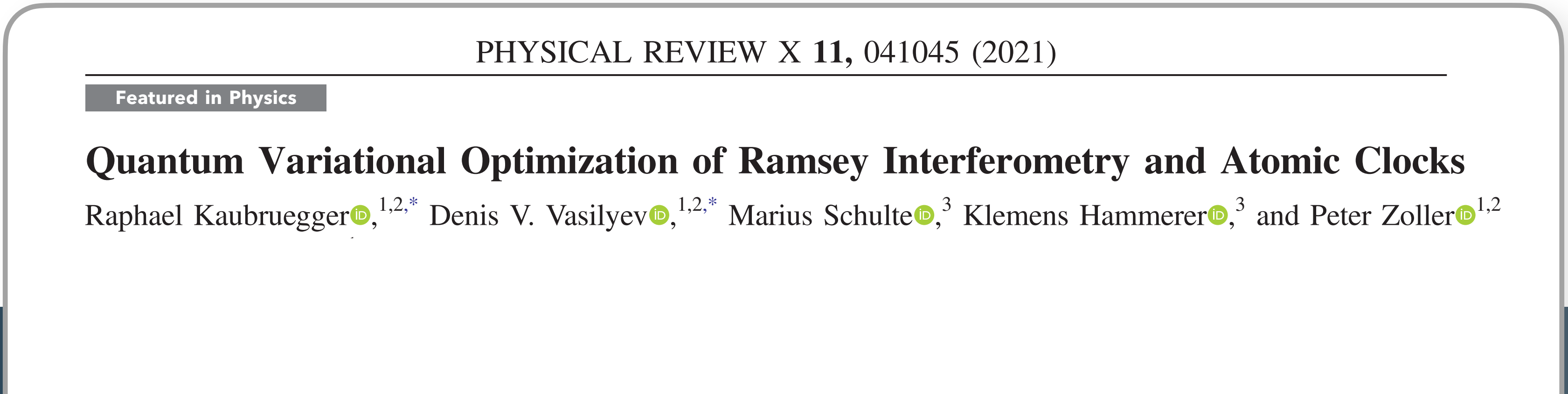
1. Which Cost Function?
    - **Challenge:** Different sensing tasks demand different figure-of-merit.
    - **Solution:** In single-shot estimation, the useful cost is the **Bayesian risk**, integrating prior information and measurement outcomes.
2. How to Realize the Ansatz?
    - Use a parameterized quantum circuit that captures the symmetry and structure of the true optimal sensor.
3. What Is the Optimal Quantum Sensor?
    - **Single-Parameter Case:** the known Demkovic–Dobrzansky solution, validating our framework.
    - **Multi-Parameter Extension:** We introduce and characterize **Optimal Quantum Sensors (OQSs)** for two- and three-parameter estimation, achieving fundamental precision bounds.

Optimal and Variational Quantum Metrology

Cost function

Fisher vs Bayes

Variational Quantum Optimization



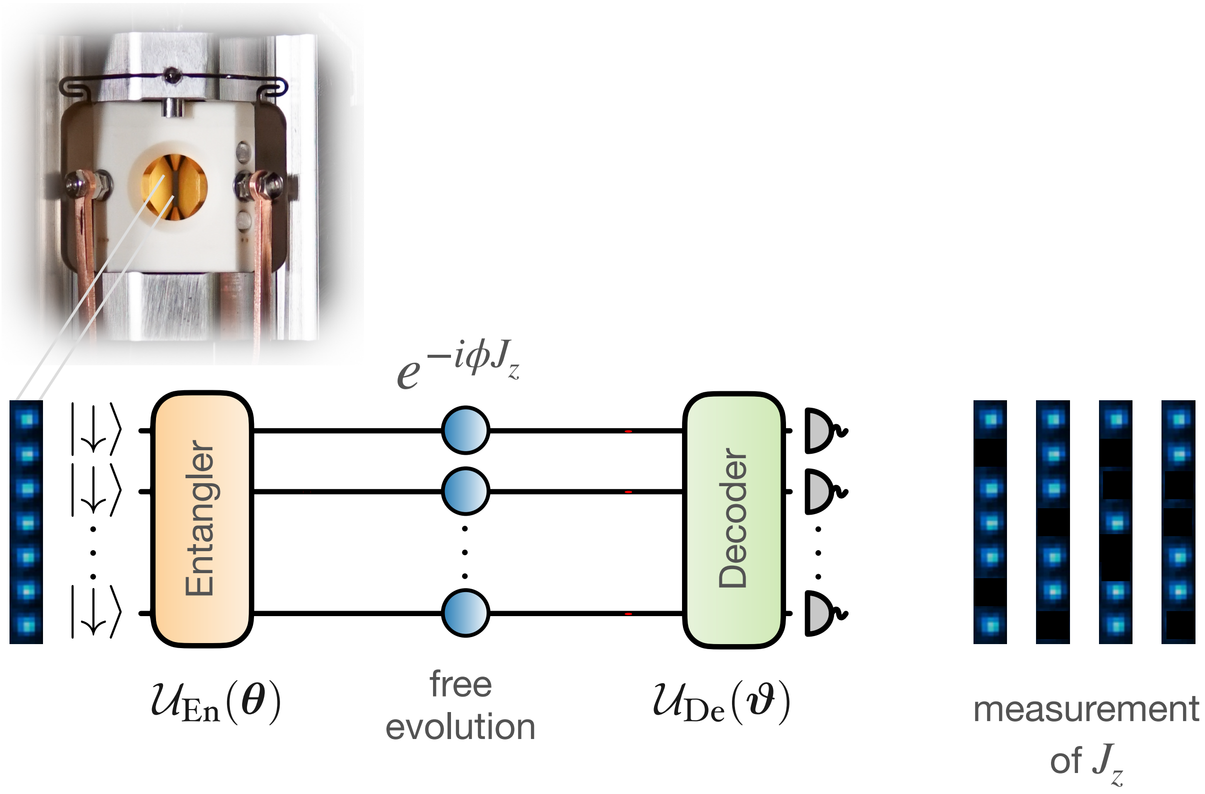
PHYSICAL REVIEW X **11,** 041045 (2021)

Featured in Physics

Quantum Variational Optimization of Ramsey Interferometry and Atomic Clocks

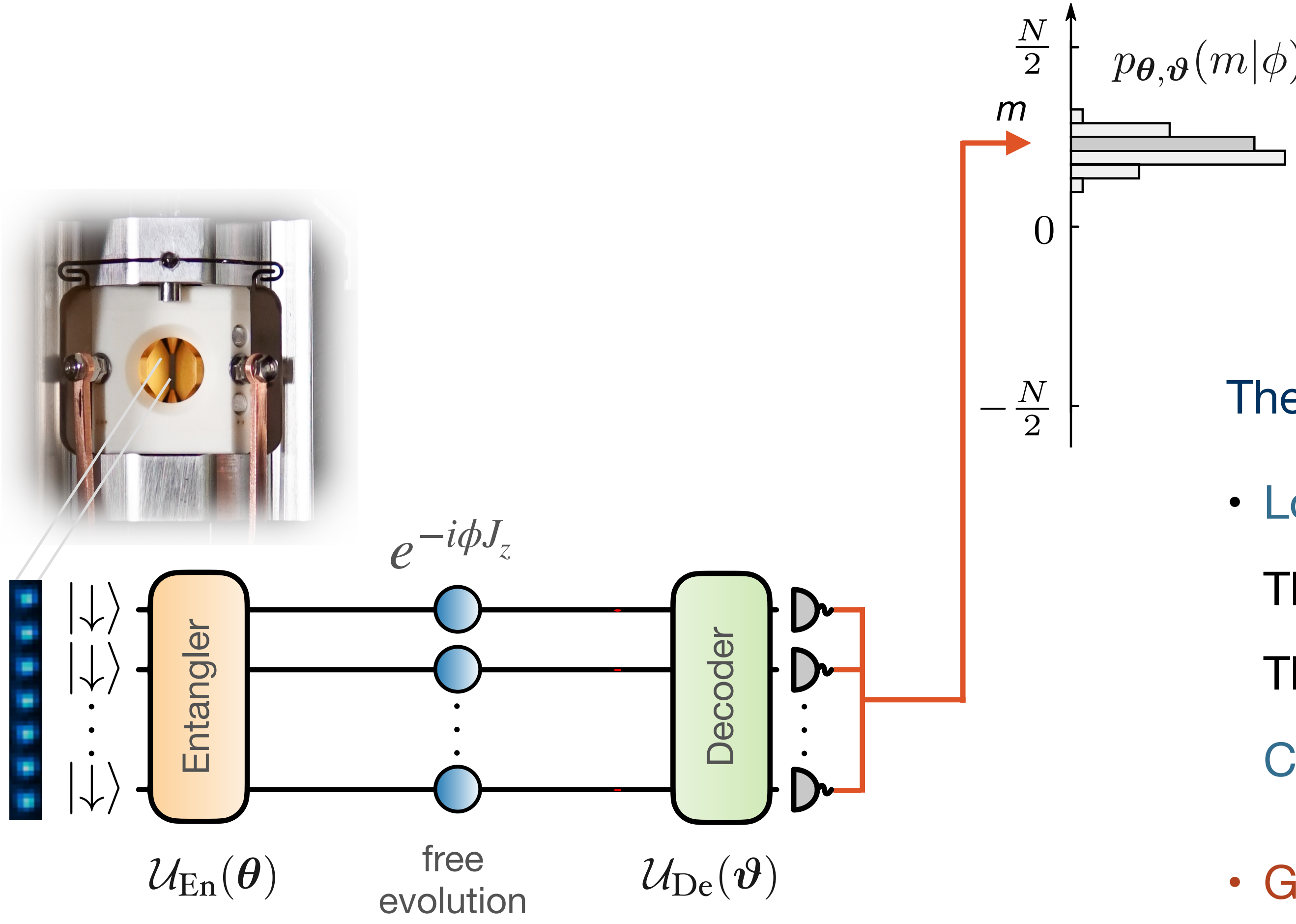
Raphael Kaubruegger,[1,2,*] Denis V. Vasilyev,[1,2,*] Marius Schulte,[3] Klemens Hammerer,[3] and Peter Zoller[1,2]

Cost function



$$m = \frac{1}{2}(\#\text{up} - \#\text{down})$$

Cost function



$$p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m|\phi) = \left|\langle m|U_{\text{de}}(\boldsymbol{\vartheta})e^{-i\phi J_z}U_{\text{en}}(\boldsymbol{\theta})\,|\downarrow\rangle^{\otimes N}\right|^2$$

How to estimate the phase ϕ given the noisy measurement outcome m?

There are two approaches:

- Local (Quantum Fisher Information)

  The estimator $\phi_{\text{est}}(m)$ is defined to be *locally unbiased.*

  Then the phase estimation uncertainty is limited by the quantum Cramer-Rao (CR) bound: $K\Delta^2\phi \geq K\Delta^2\phi_{\text{QCR}} = F_Q{}^{-1} = N^{-2}$

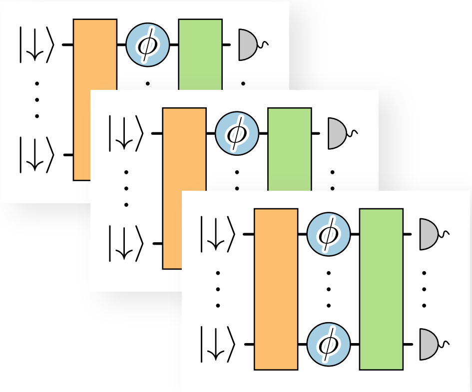
- Global (Bayesian)

  The estimator $\phi_{\text{est}}(m)$ minimizes error for a given *finite range* of phase values. π-corrected Heisenberg limit on phase uncertainty.

PRL **124**, 030501 (2020)

Local estimation — Many-Repetition Scenario

The aim is to estimate deviation of parameters from $\boldsymbol{\phi}_0$ using K independent measurements:



$$|\psi_{\boldsymbol{\phi}}\rangle = U(\boldsymbol{\phi})\,|\psi_{\text{in}}\rangle$$

$$\boldsymbol{\mu} = (\mu_1, \ldots, \mu_K)^T$$

vector of K measurement outcomes

Figure of merit is the mean squared error:

$$\text{MSE}(\phi) = \sum_{\boldsymbol{\mu}} (\phi - \xi_{\boldsymbol{\mu}})^2\, p(\boldsymbol{\mu}\,|\,\phi)$$

Fisher information

sets bounds on the achievable MSE

Conditional probability (Likelihood):

$$p(\boldsymbol{\mu}\,|\,\phi) = \prod_{k=1}^{K} p(\mu_k\,|\,\phi),$$

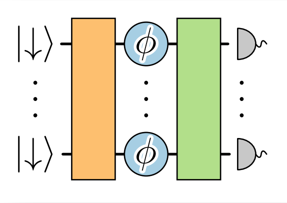
$$p(\mu\,|\,\phi) = \text{Tr}\{M_\mu\,|\psi_{\boldsymbol{\phi}}\rangle\langle\psi_{\boldsymbol{\phi}}\,|\,\}, \quad \sum_\mu M_\mu = \mathbb{I}, \quad M_\mu \geq 0$$

Matteo G. A. Paris, Int. J. Quant. Inf. 7, 125 (2009)
Jing Liu et al 2020 J. Phys. A: Math. Theor. 53 023001

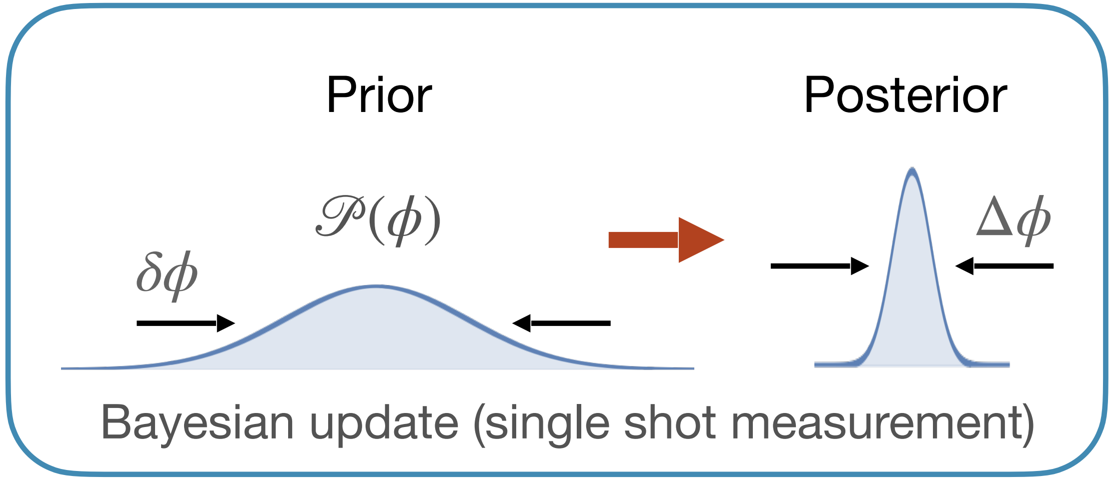
Global estimation — Single-Measurement Scenario

Bayesian approach:

The goal is to perform estimation using a single shot measurement



$\Big\} K = 1$



Bayesian update (single shot measurement)

$$C \equiv \Delta\phi = \sum_{\mu} \int d\phi\, (\phi - \zeta_{\nu})^{2} p(\mu\,|\,\phi) \mathscr{P}(\phi)$$

Minimize posterior variance C with respect to the state, measurement, and estimator

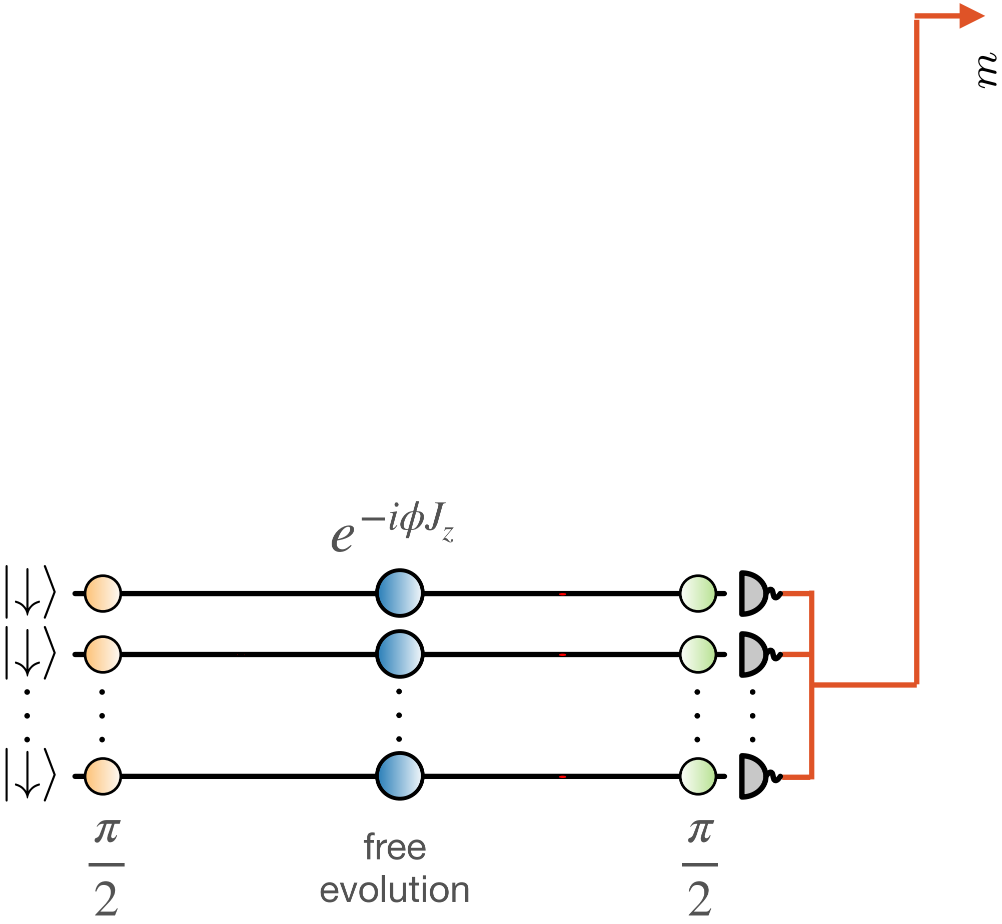
Prior knowledge $\delta\phi$

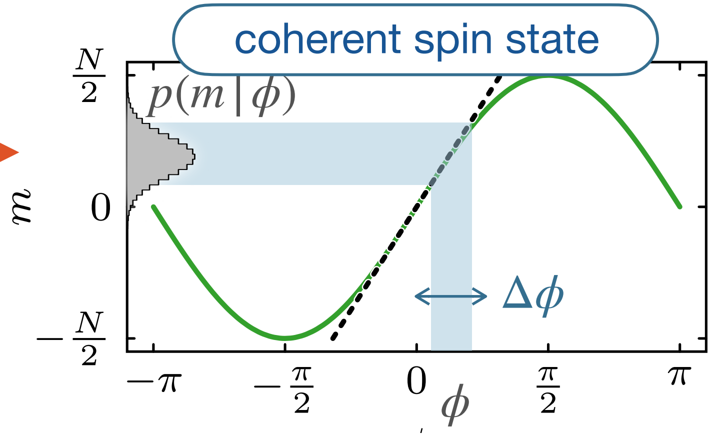
Posterior knowledge $\Delta\phi$

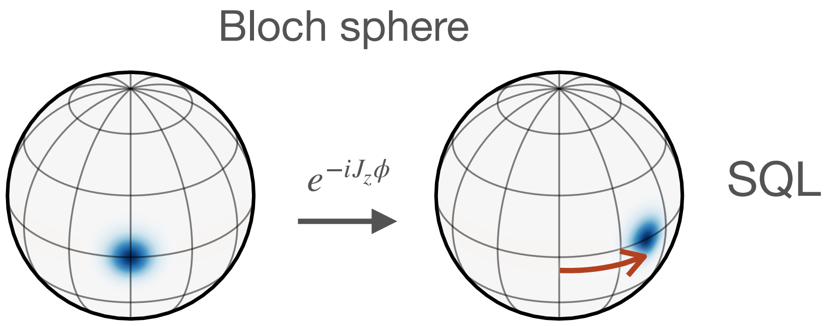
Rafal Demkowicz-Dobrzaski et al 2020 J. Phys. A: Math. Theor. 53 363001

Cost function: Local (QFI) strategy

Single parameter example





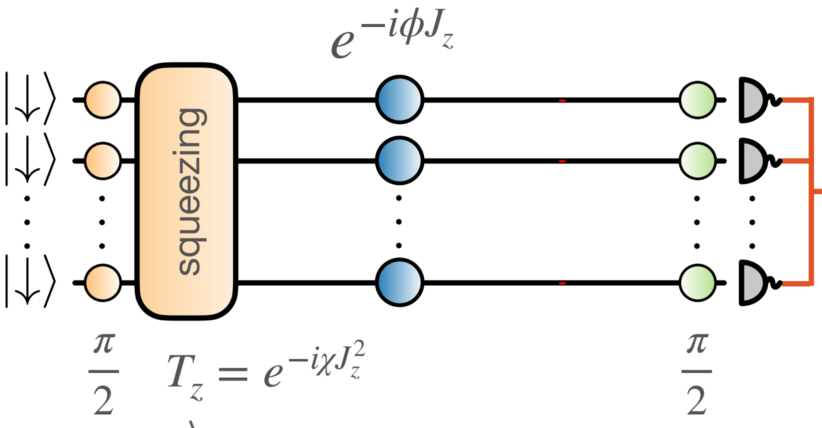


product state: $|CSS\rangle \sim \Big(|\uparrow\rangle + |\downarrow\rangle\Big)^{\otimes N}$

Standard Quantum Limit (SQL): $\Delta\phi \sim \dfrac{1}{\sqrt{N}}$

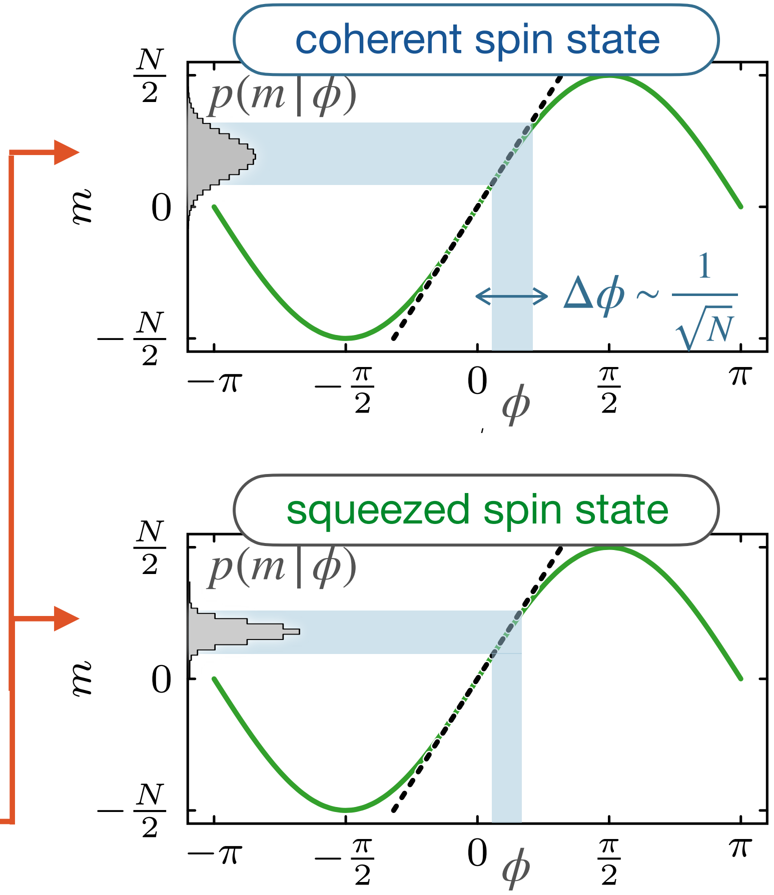
Cost function: Local (QFI) strategy

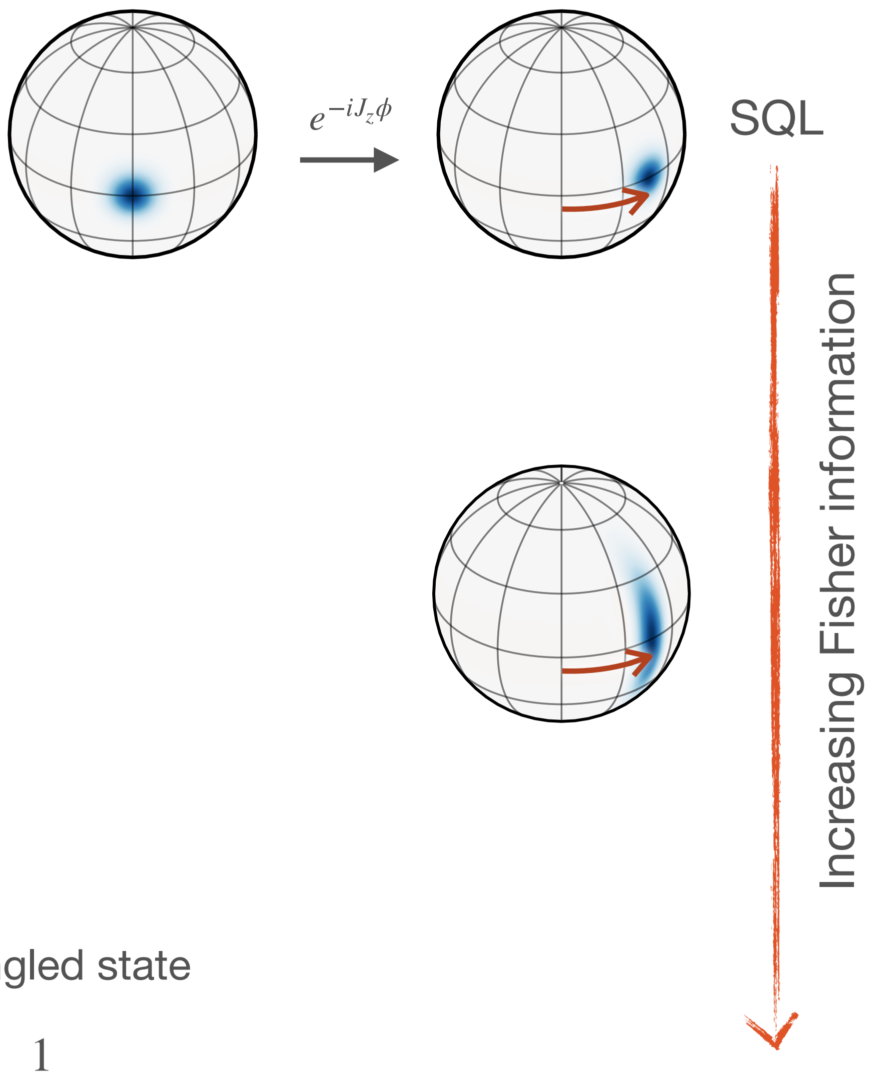
Single parameter example



One-Axis Twisting $J_z^2 = \sum_{i,j=1}^{N} \sigma_z^i \sigma_z^j$

infinite range interaction



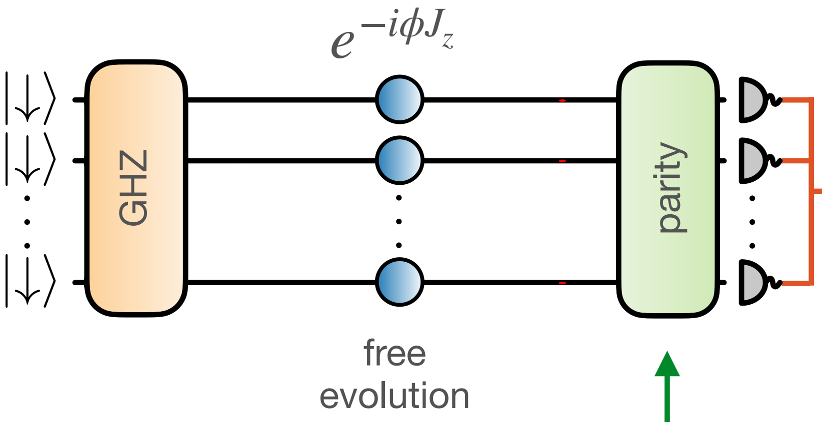


spin-squeezing ~ entangled state

below SQL: $\Delta\phi < \frac{1}{\sqrt{N}}$

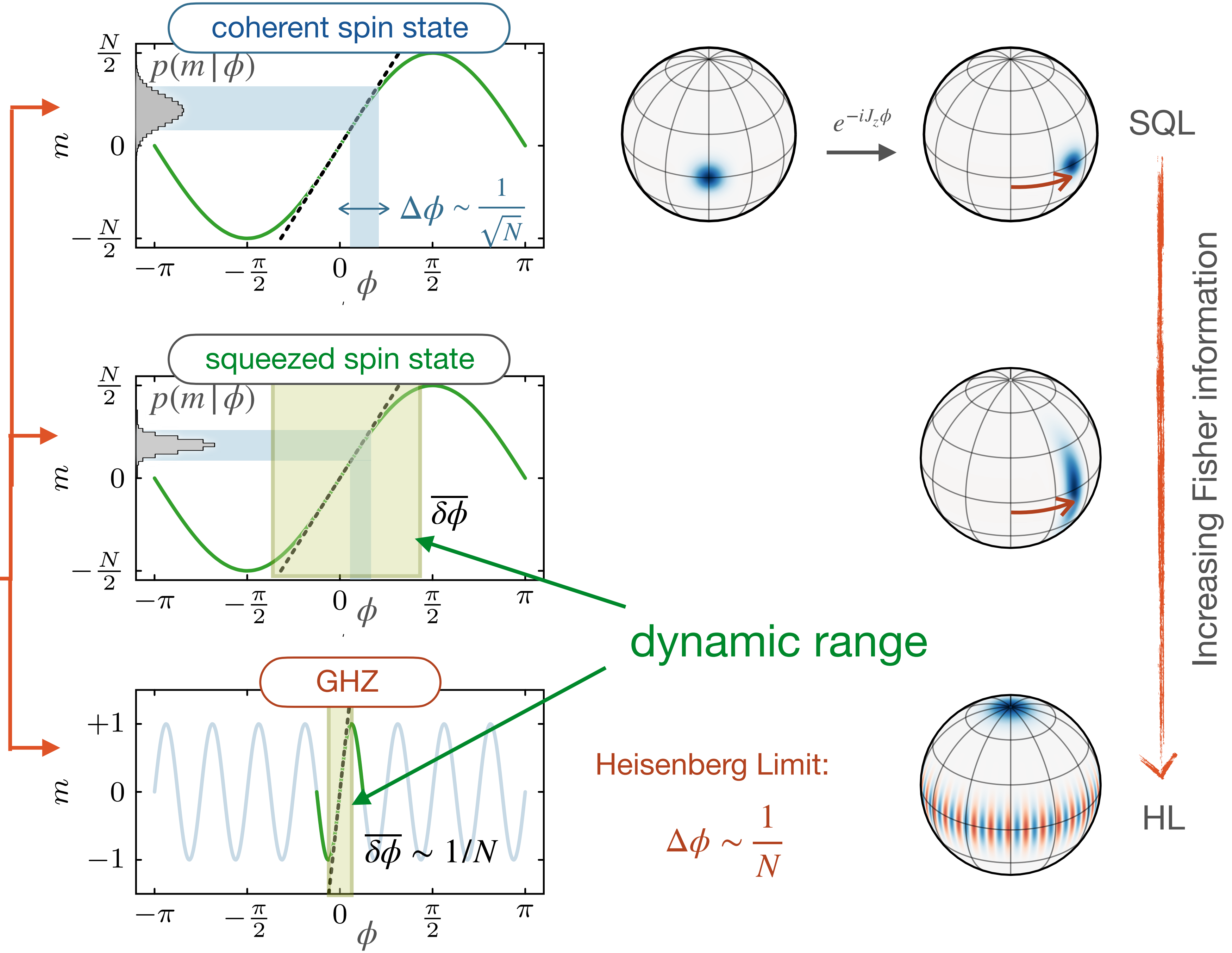
Cost function: Local (QFI) strategy

Single parameter example



$|GHZ\rangle \sim |\uparrow\rangle^{\otimes N} + |\downarrow\rangle^{\otimes N}$

No need for decoder,
optimal measurement is *local*



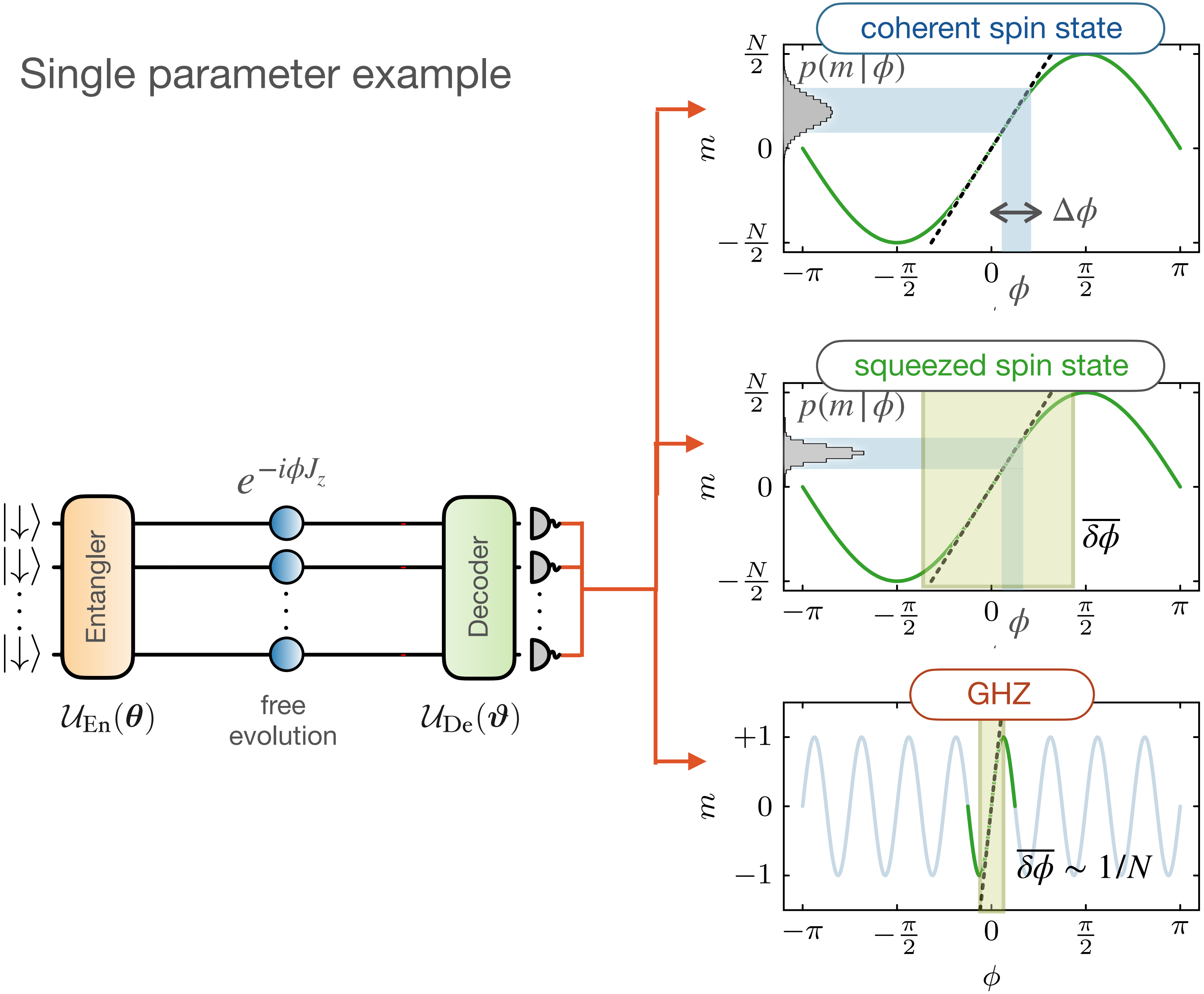
dynamic range

Heisenberg Limit:

$$\Delta\phi \sim \frac{1}{N}$$

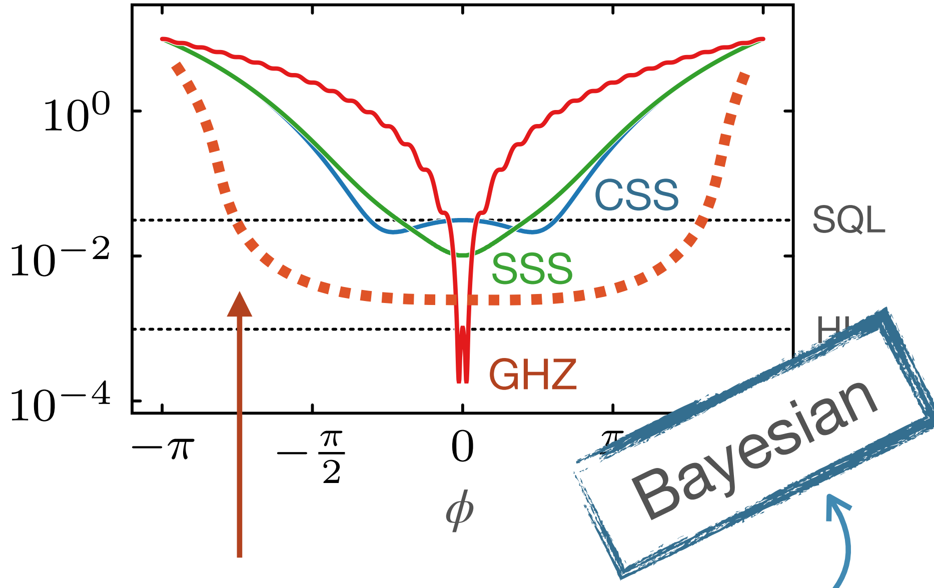
Cost function: QFI vs Bayesian

Single parameter example



- mean square error with respect to phase ϕ

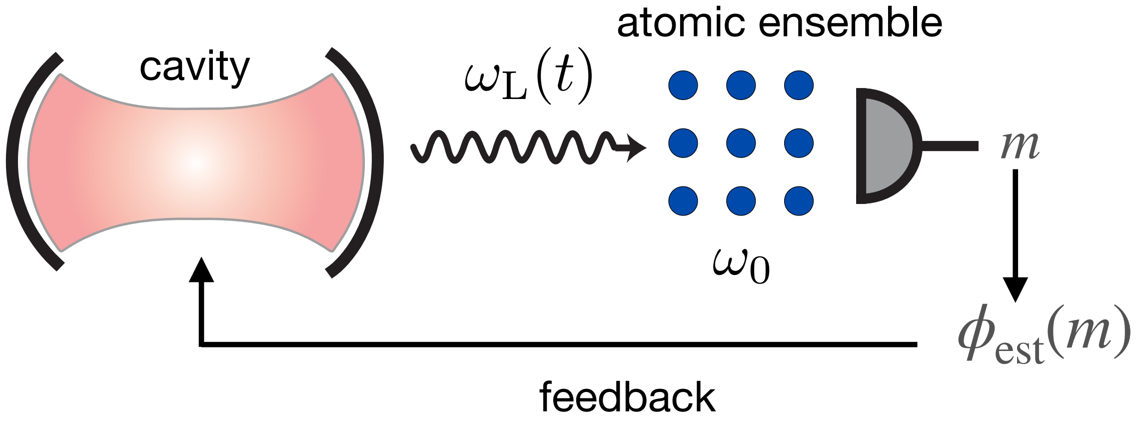
$$\mathrm{MSE}(\phi) = \sum_m \left[\phi - \phi_{\mathrm{est}}(m)\right]^2 p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m|\phi)$$



interferometer we wish to have

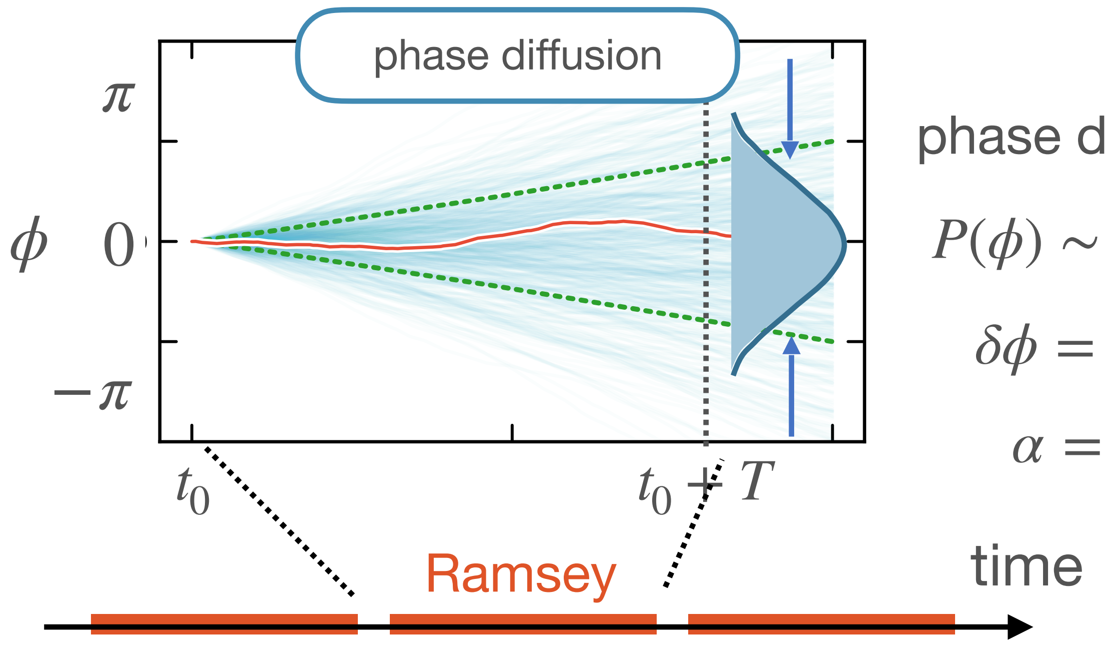
- best Signal /Noise ratio
- for given dynamic range $\delta\phi$

Atomic Clock



Goal: lock noisy laser frequency $\omega_L(t)$ to the atomic transition ω_0 by observing accumulated phase difference ϕ with a Ramsey interferometer

$$\phi = \int_{t_0}^{t_0+T} \left(\omega_L(t) - \omega_0\right) dt$$



phase distribution

$$P(\phi) \sim e^{-\phi^2/2\delta\phi^2}$$

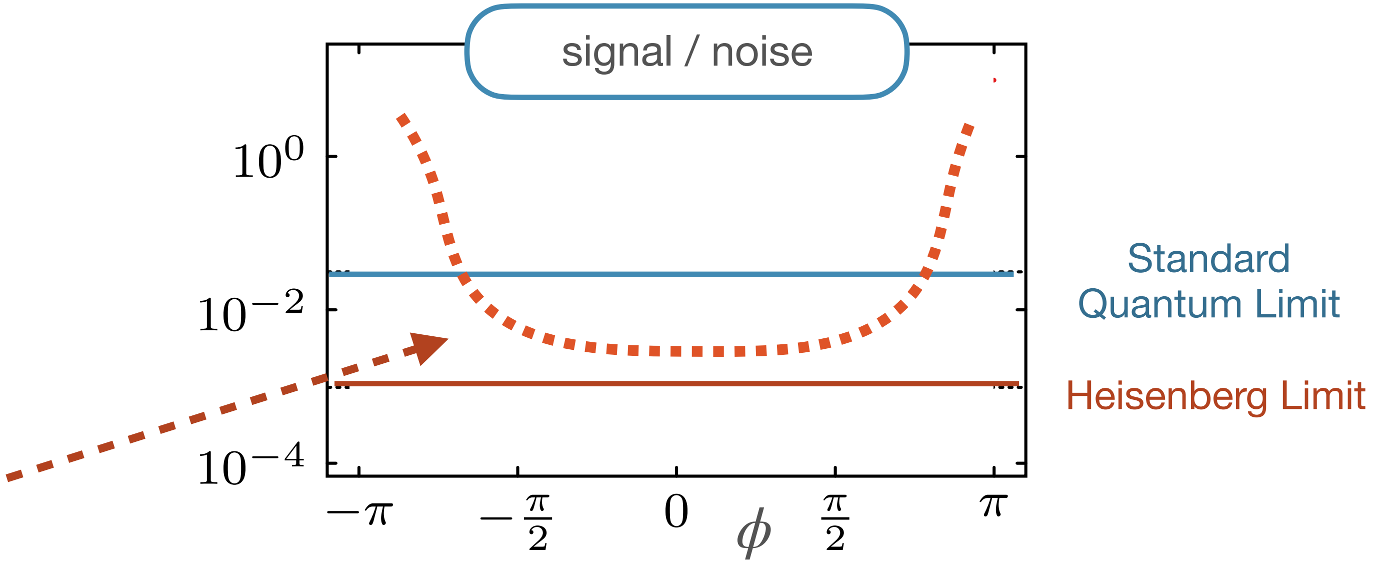
$$\delta\phi = (b_\alpha T)^{\alpha/2}$$

$$\alpha = 1{,}2{,}3$$

Variational Classical-Quantum Algorithms

$$\mathscr{C}_{\text{metrological}} \to \text{opt}$$

wishlist:
- ✓ signal / noise ratio
- ✓ finite dynamic range $\delta\phi$



A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, *Optical Atomic Clocks*, Rev. Mod. Phys. **87**, 637 (2015).

Optimal and Variational Quantum Metrology

Cost function

Variational Quantum Optimization

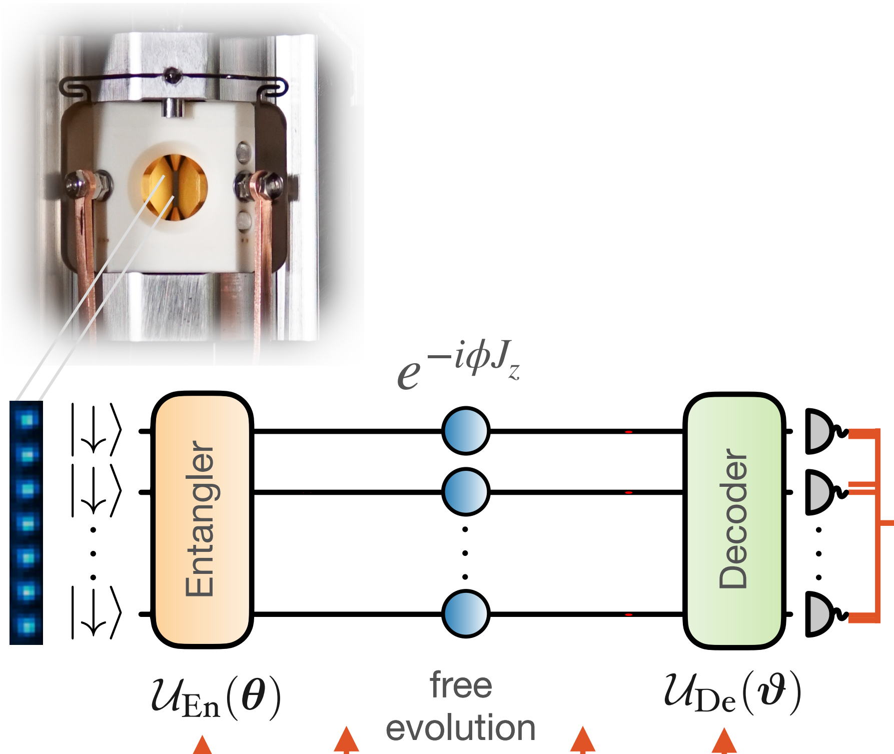
Optimal metrology with programmable quantum sensors

604 | Nature | Vol 603 | 24 March 2022

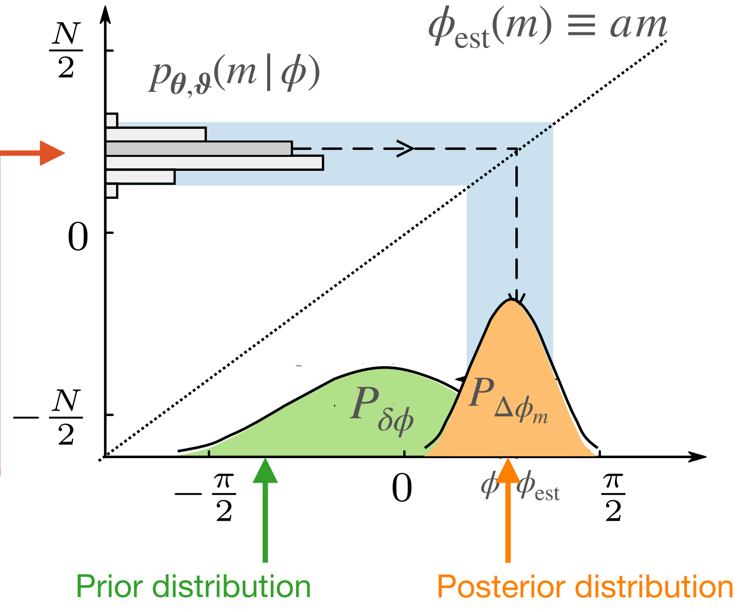
Christian D. Marciniak[1,5], Thomas Feldker[1,5], Ivan Pogorelov[1], Raphael Kaubruegger[2,3], Denis V. Vasilyev[2,3], Rick van Bijnen[2,3], Philipp Schindler[1], Peter Zoller[2,3], Rainer Blatt[1,2] & Thomas Monz[1,4]✉

Variational Quantum Algorithm for Optimal Ramsey Interferometry

Ramsey interferometer



classical post processing



Mean squared error with respect to the true phase ϕ

$$\mathrm{MSE}(\phi) = \sum_m \left(\phi - \phi_{\mathrm{est}}(m)\right)^2 p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m\,|\,\phi)$$

Cost function (Bayesian approach)

$$\mathscr{C}(\boldsymbol{\theta},\boldsymbol{\vartheta}) \equiv (\Delta\phi)^2 = \int d\phi\, \mathrm{MSE}(\phi)\, P_{\delta\phi}(\phi)$$

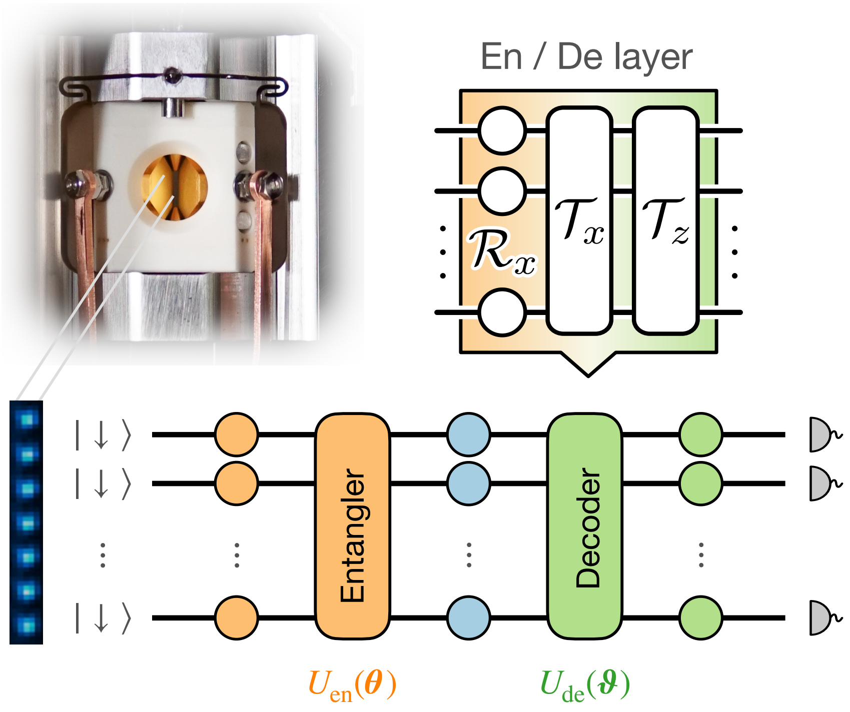
Minimization of cost function $\mathscr{C}(\boldsymbol{\theta},\boldsymbol{\vartheta})$ defines optimal interferometer in variational class

max information gain on ϕ in a single measurement

updated variational parameters

Variational Circuit Decomposition

Ramsey interferometer

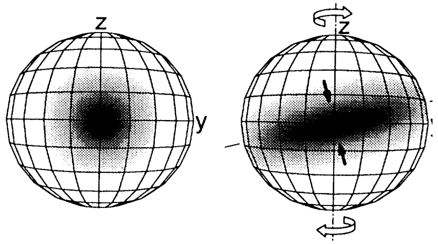


Ion quantum computer - native gates

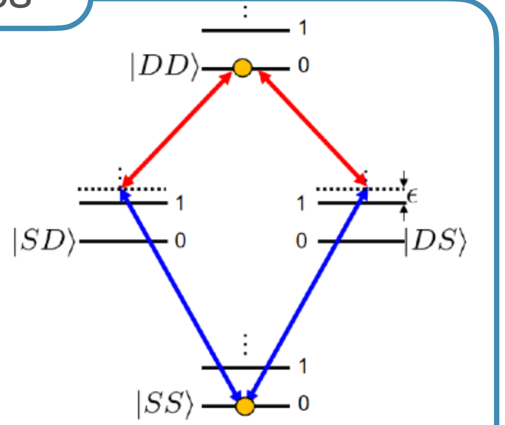
Resource operations

$R_\alpha(\theta) = e^{-i\theta J_\alpha}$ Rotations

$T_\alpha(\theta) = e^{-i\theta J_\alpha^2}$ One-axis twisting



Kitagawa, Ueda PRA (1993)

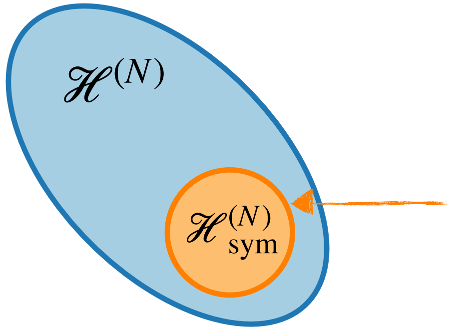


Sorensen, Molmer PRL (1999)

AQTION

AQT

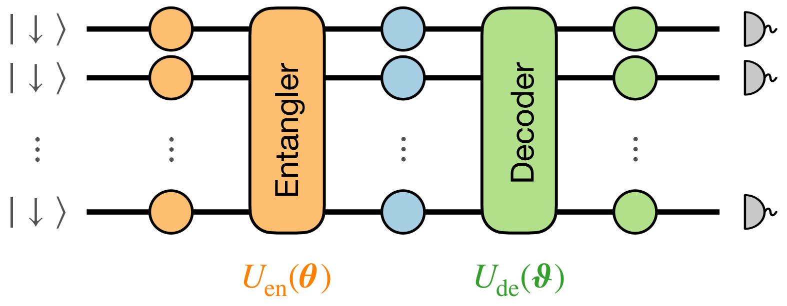
Pogorelov PRX Quantum (2021)



Dynamics restricted to symmetric subspace
$\dim = N + 1$

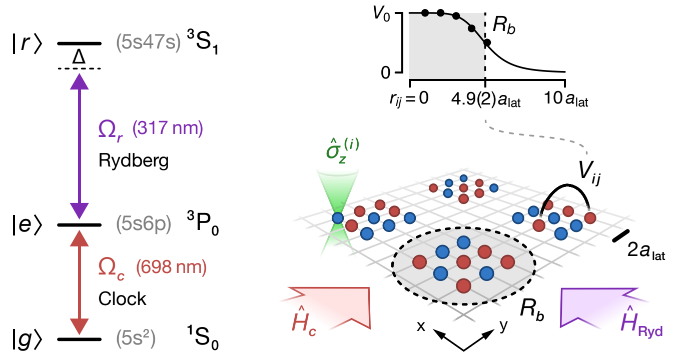
Variational Circuit Decomposition

Ramsey interferometer



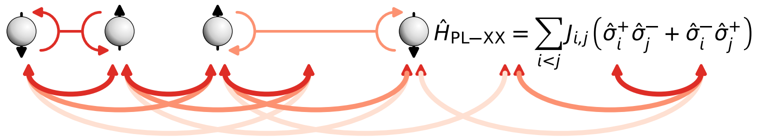
Other platforms (finite range interactions)

Rydberg Tweezer Arrays

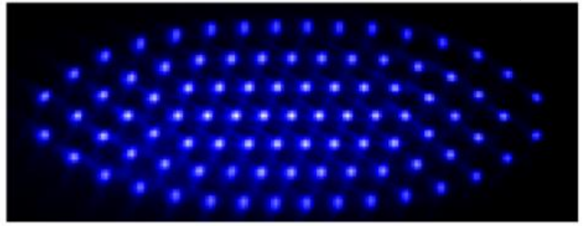


Eckner et al (2023)

Trapped Ions

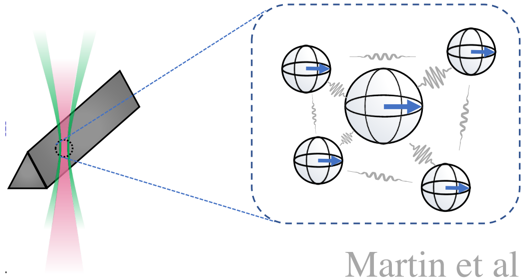


Franke et al (2023)

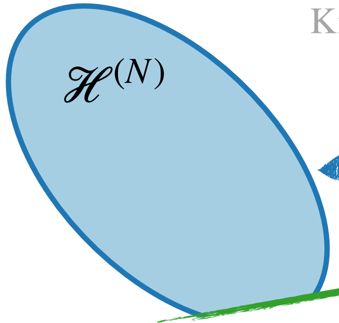


Kiesenhofer et al PRX Quantum (2023)

NV Centers & Dipolar Interactions



Martin et al PRL (2023)

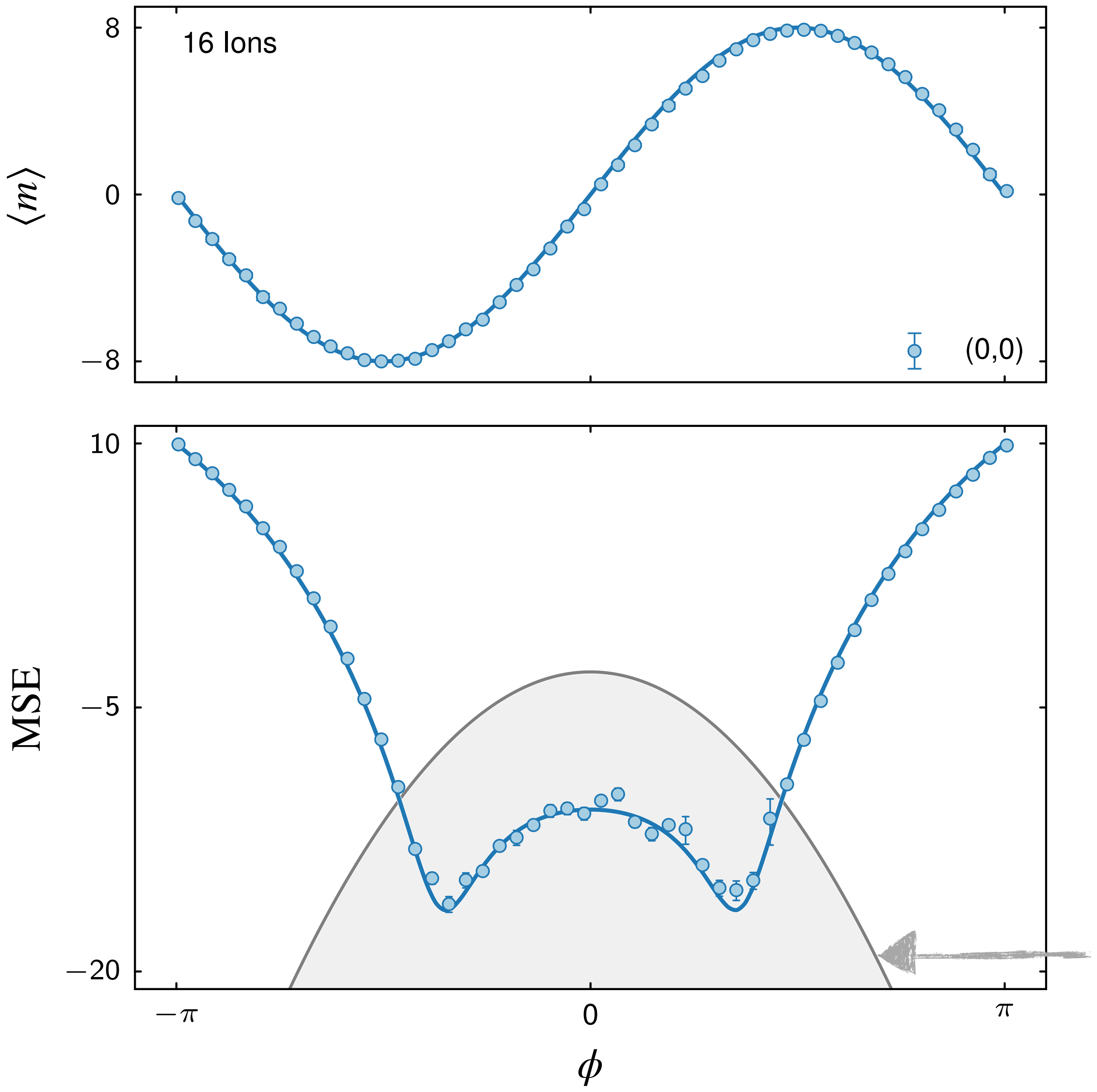


Dynamics the full
Hilbert space
dim = 2^N

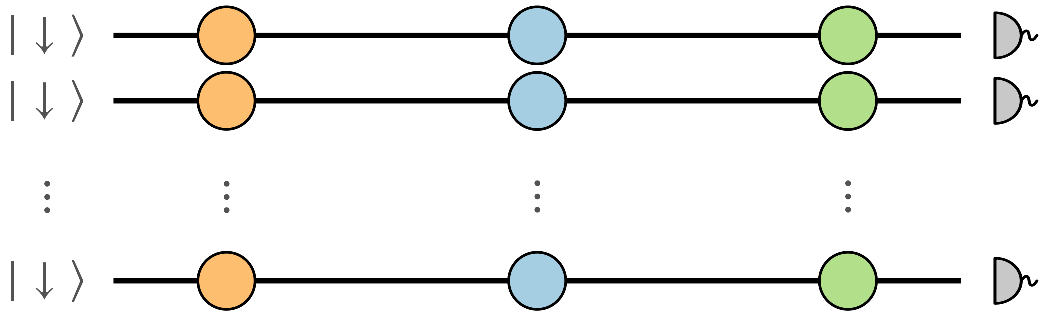
many-body complexity
('relevant' quantum advantage)

Trapped Ion Experiment

(0, 0): Coherent spin state, uncorrelated measurement



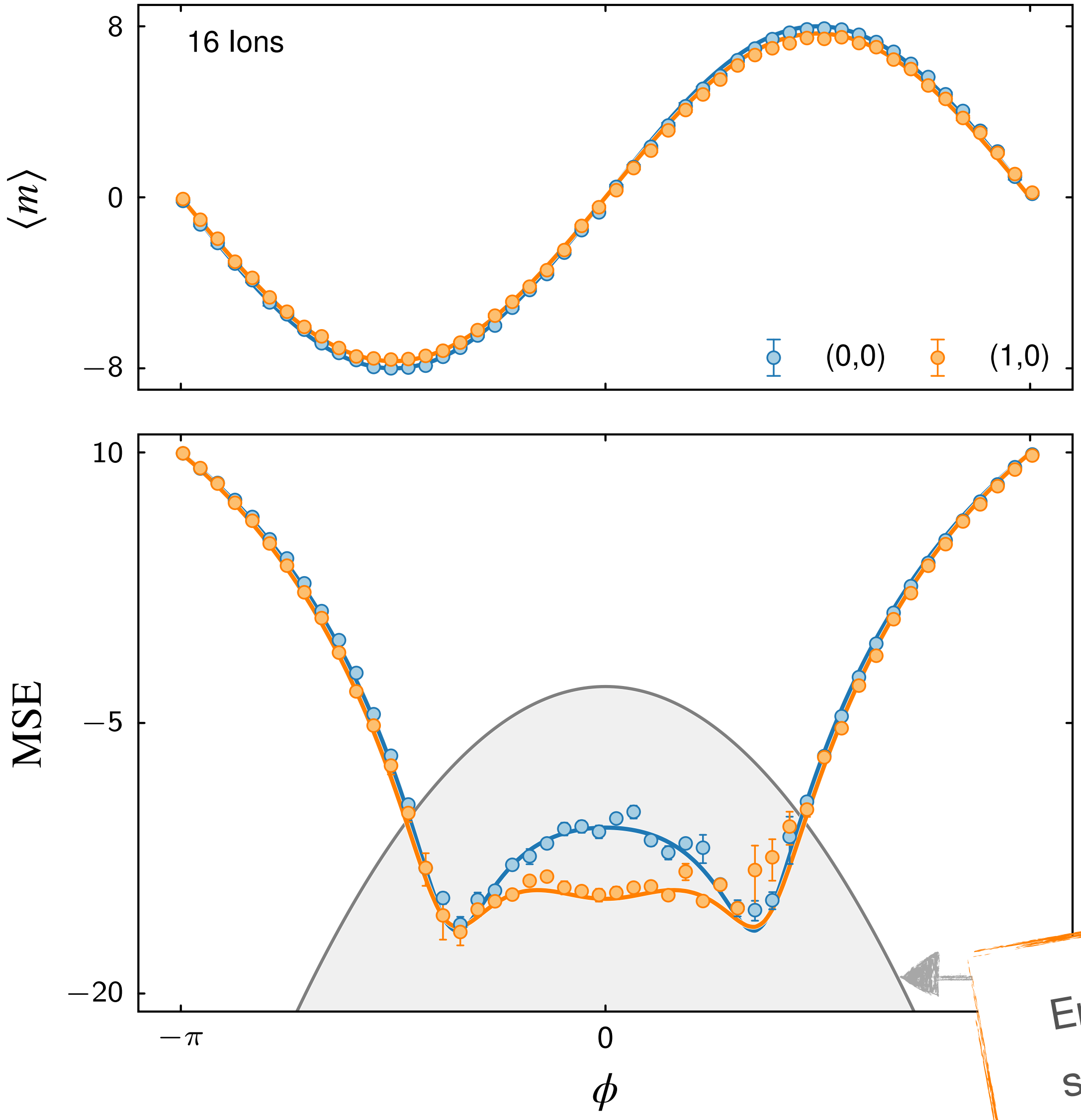
Prior distribution $P_{\delta\phi}(\phi)$
with $\delta\phi \approx 0.7$

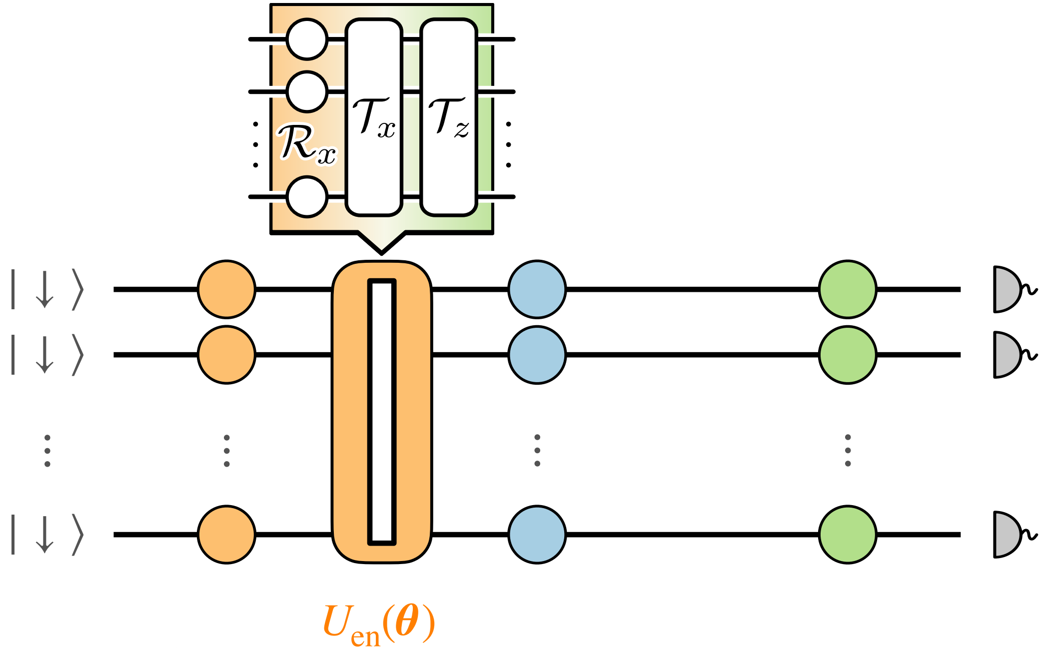


Trapped Ion Experiment

(0, 0): Coherent spin state, uncorrelated measurement

(1, 0): Spin squeezed state, uncorrelated measurement





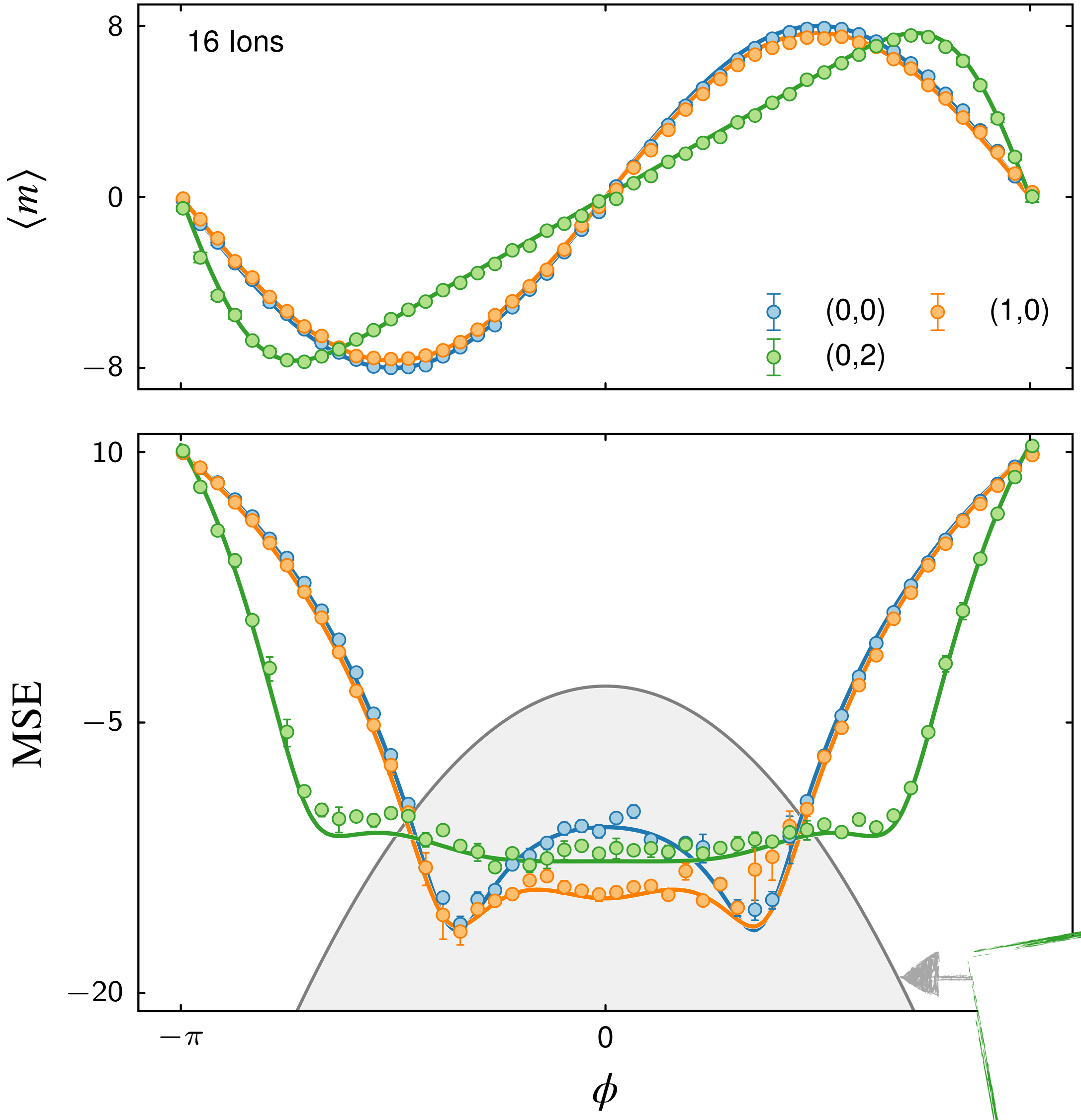
Entangling increases sensitivity around $\phi = 0$

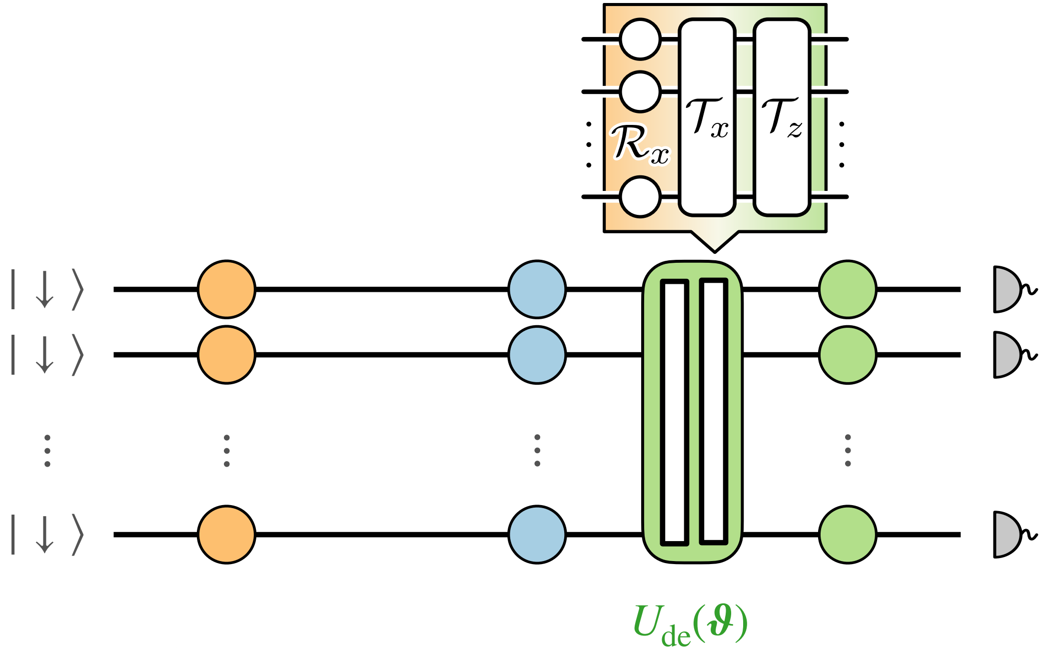
Trapped Ion Experiment

(0, 0): Coherent spin state, uncorrelated measurement

(1, 0): Spin squeezed state, uncorrelated measurement

(0, 2): Coherent spin state, decoded measurement

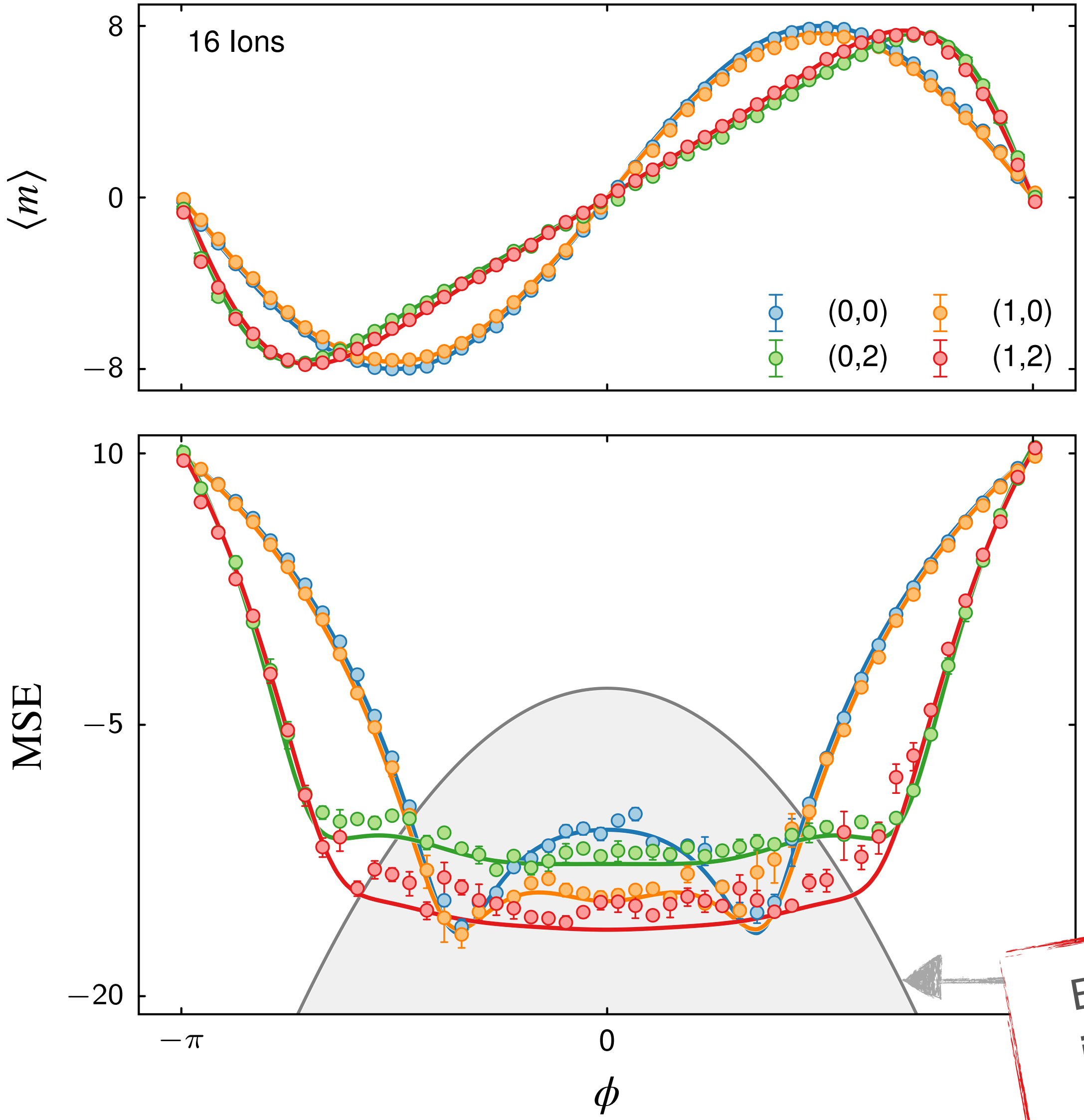


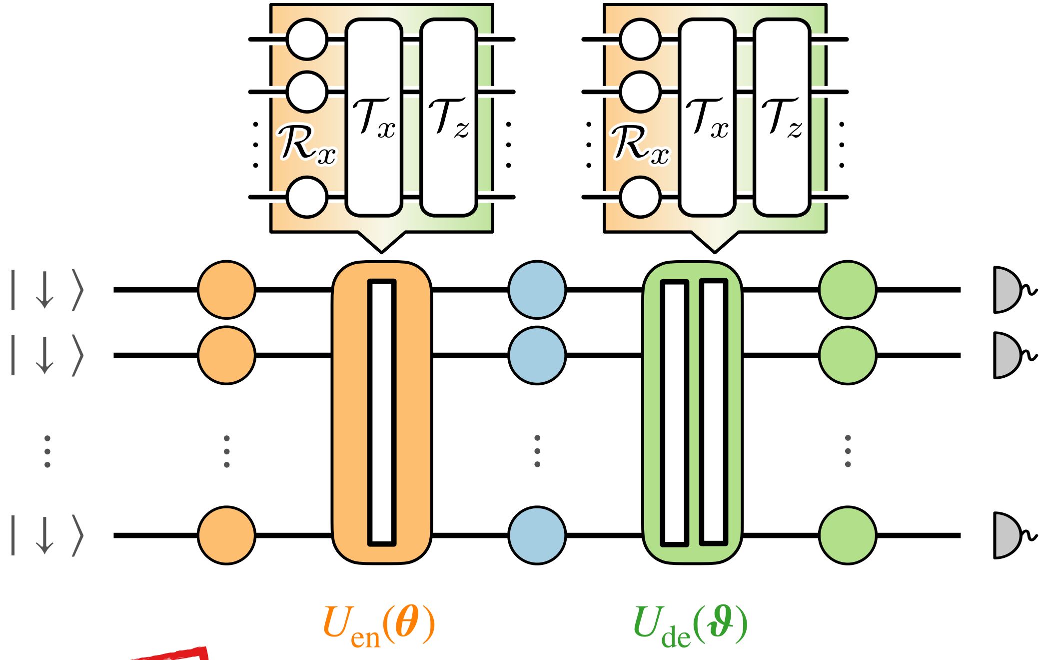


Decoding increases dynamic range

Trapped Ion Experiment

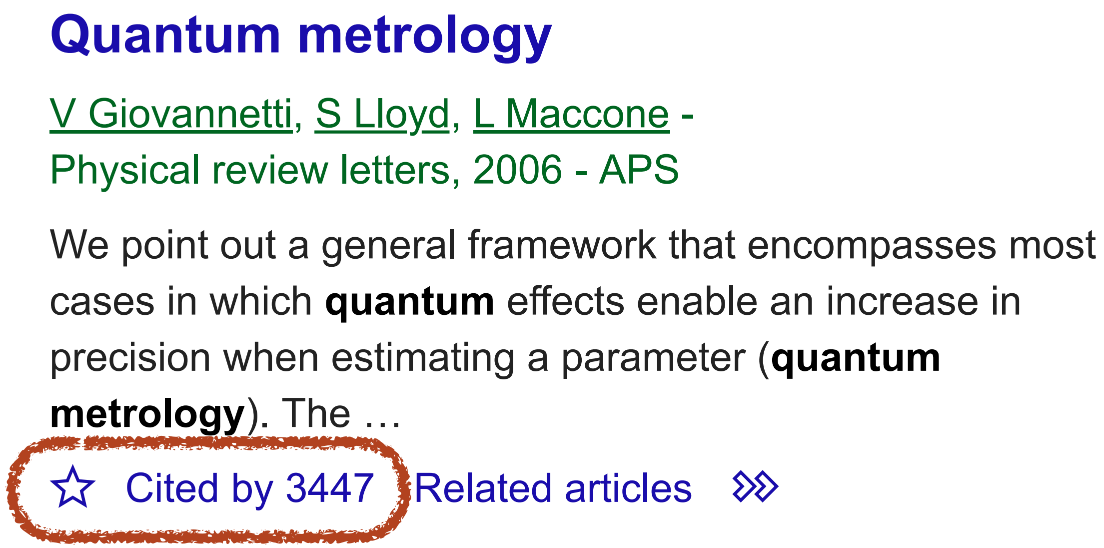
(0, 0): Coherent spin state, uncorrelated measurement
(1, 0): Spin squeezed state, uncorrelated measurement
(0, 2): Coherent spin state, decoded measurement
(1, 2): Entangled state, decoded measurement





Entangling and decoding increases sensitivity and dynamic range

An apparent contradiction



Quantum metrology

V Giovannetti, S Lloyd, L Maccone -
Physical review letters, 2006 - APS

We point out a general framework that encompasses most cases in which **quantum** effects enable an increase in precision when estimating a parameter (**quantum metrology**). The …

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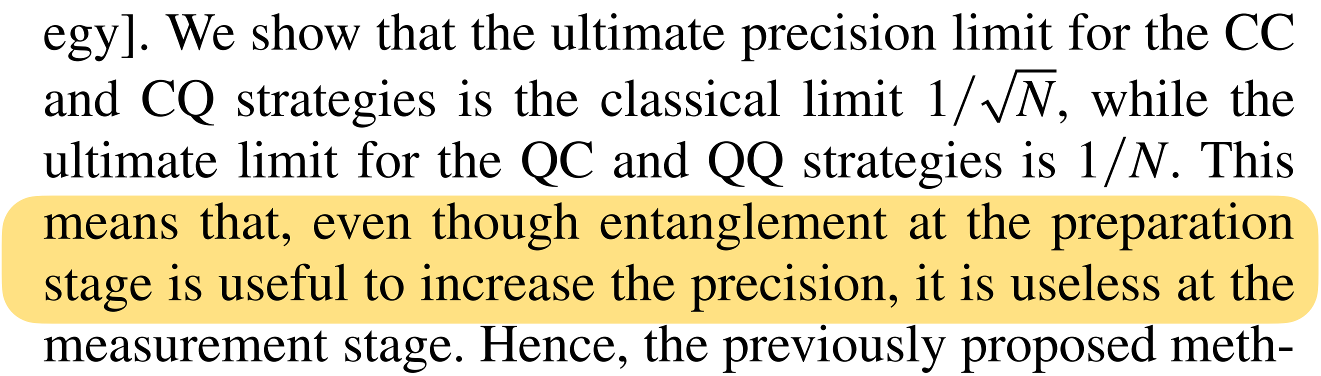
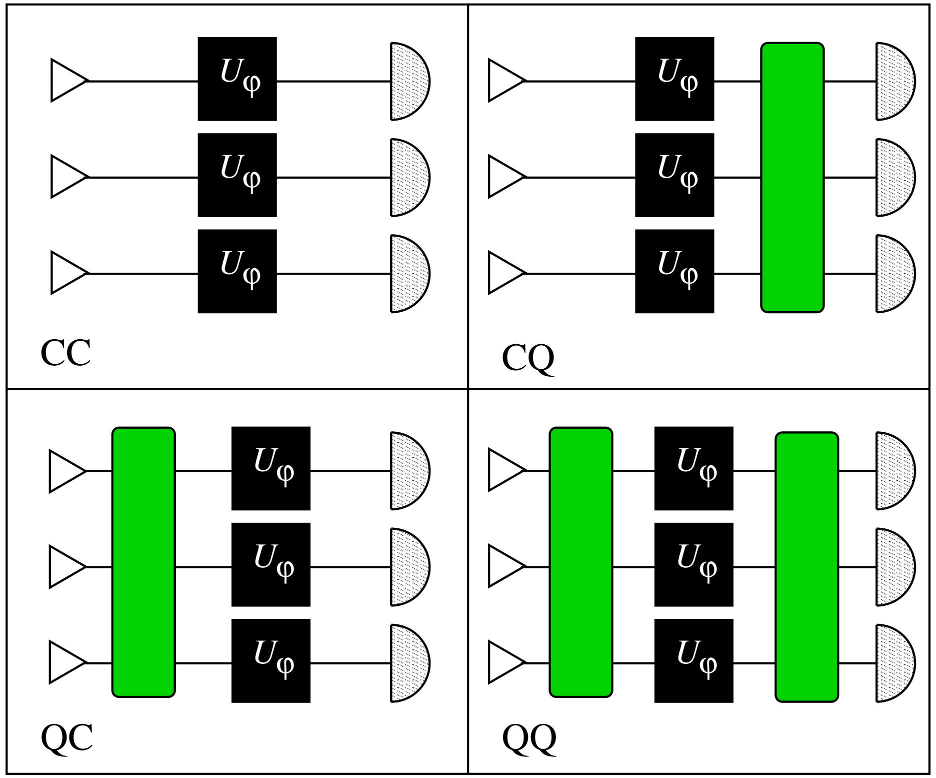


egy]. We show that the ultimate precision limit for the CC and CQ strategies is the classical limit $1/\sqrt{N}$, while the ultimate limit for the QC and QQ strategies is $1/N$. This means that, even though entanglement at the preparation stage is useful to increase the precision, it is useless at the measurement stage. Hence, the previously proposed meth-

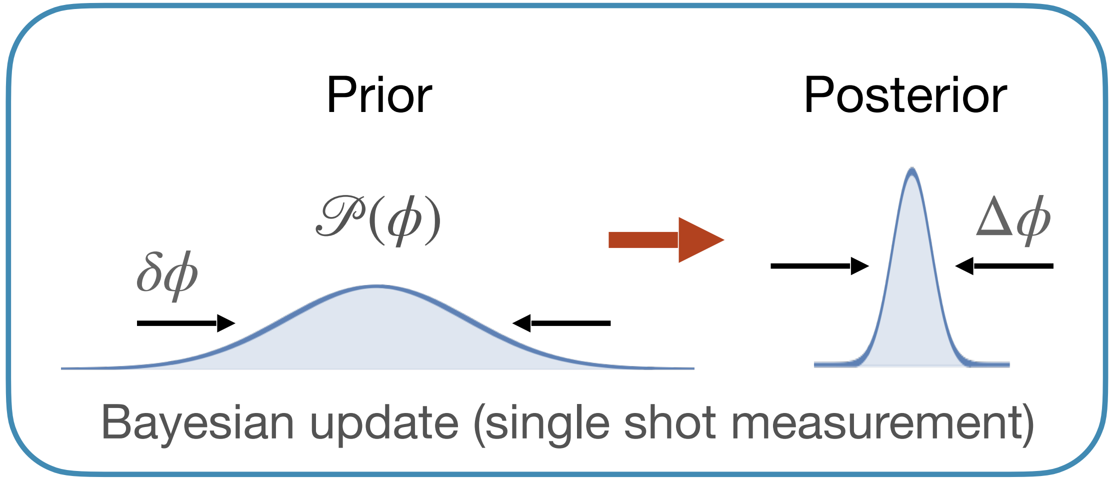


This multi-shot local estimation conclusion is not applicable to our single-shot Bayesian scenario!

1. Experiment vs. Theory: ‘Reducing Ignorance’ in Bayesian Update

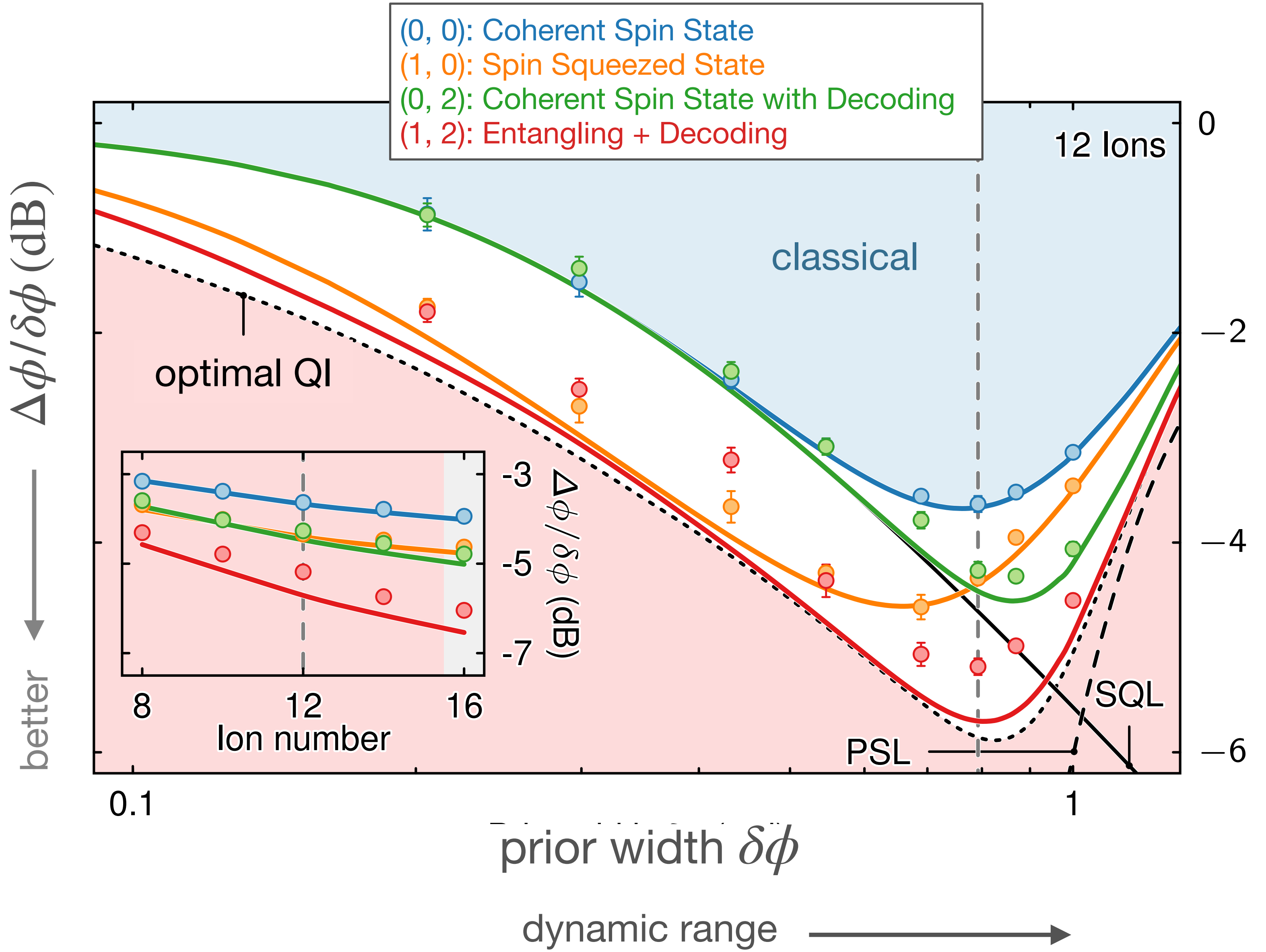
Prior knowledge $\delta\phi$

Posterior knowledge $\Delta\phi$

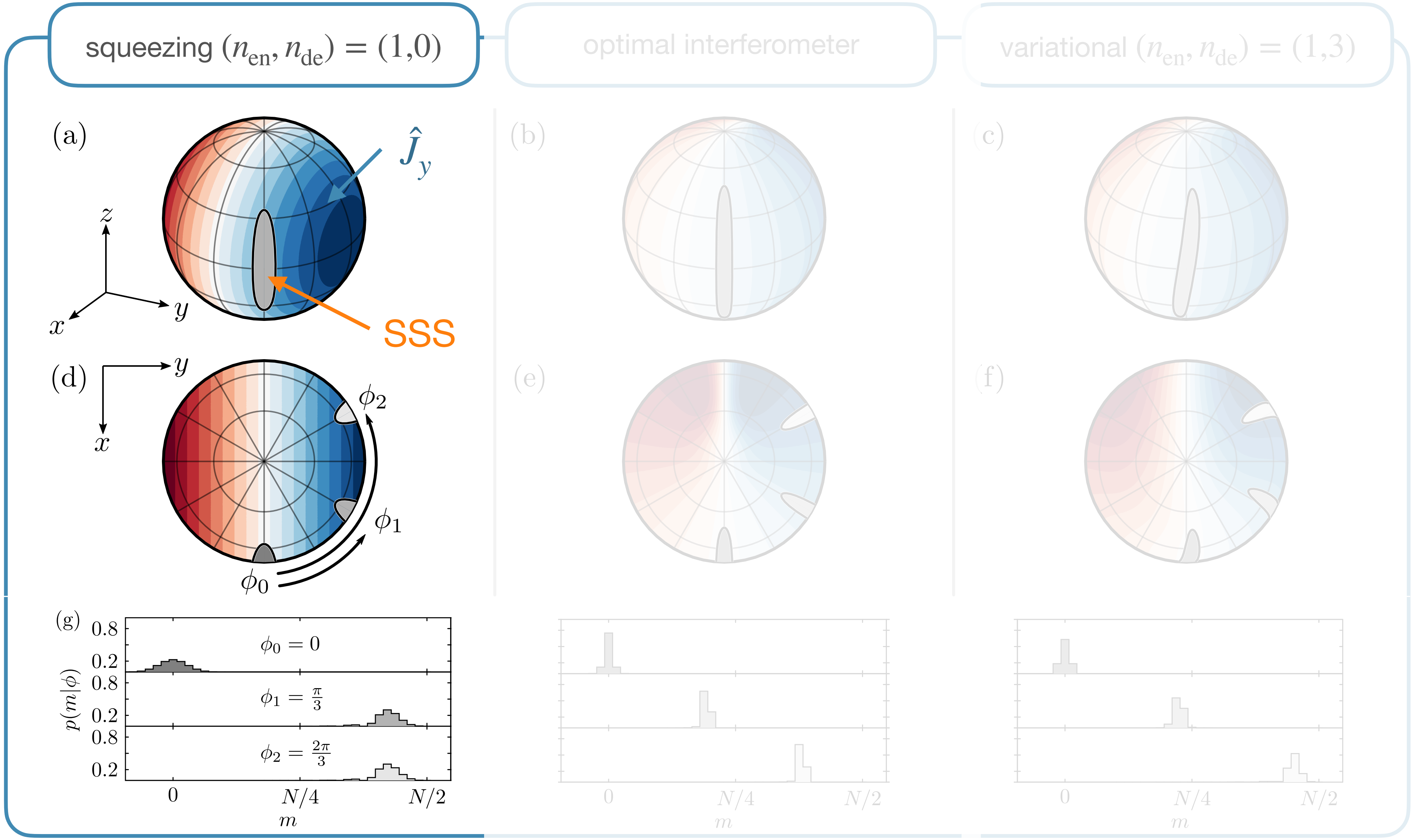


Bayesian update (single shot measurement)

Uncertainty reduction in single measurement $\Delta\phi/\delta\phi$



Theory: Interpretation of Results



- Wigner plots of input states

$$|\psi_{\mathrm{in}}\rangle = \mathcal{U}_{\mathrm{en}} |\downarrow\rangle^{\otimes N}$$

and measurement operators

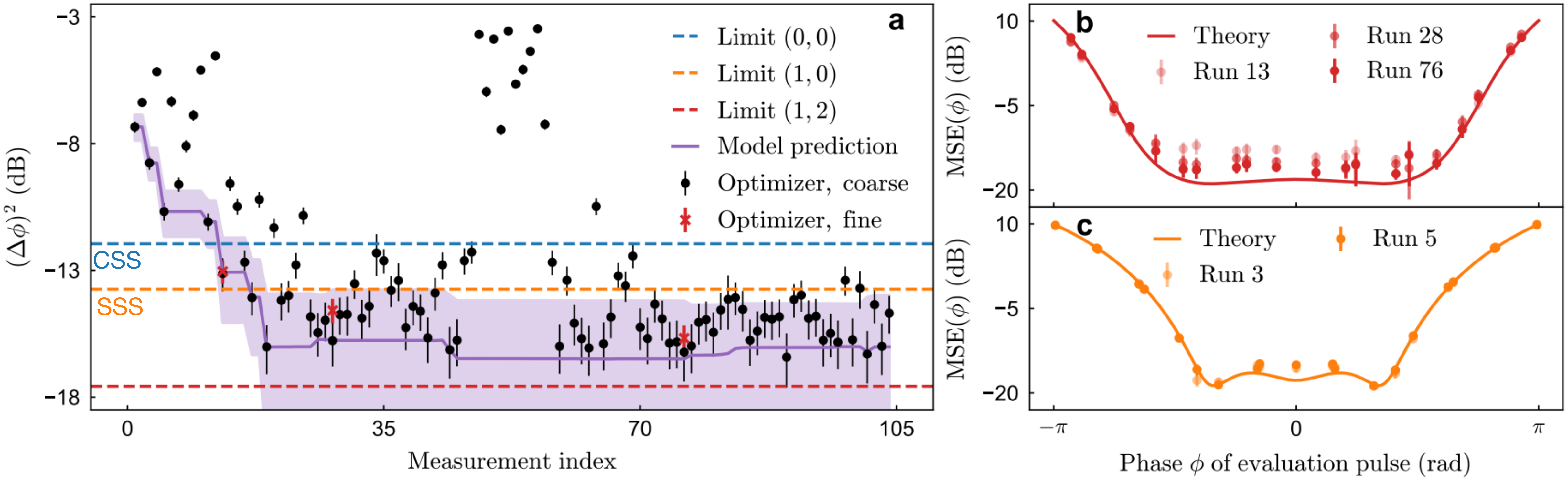
$$\mathcal{U}_{\mathrm{de}} \hat{J}_y \mathcal{U}_{\mathrm{de}}^\dagger$$

- contour lines of input states and measurement operators match for broad range $\delta\phi$

Macieszczak, Fraas, Demkowicz-Dobrzański
NJP 16, 113002 (2014)

2. `On-device’ optimization for $\boldsymbol{\theta}_{\mathrm{opt}}$, $\boldsymbol{\vartheta}_{\mathrm{opt}}$ in experiment

26 ion optimizer* run of (1, 2) sequence, 7 free parameters, twisting angles <u>not calibrated</u>



*Modified DIRECT global optimizer with trigonometric covariance kernel and GP meta-model [R. van Bijnen; and C Kokail et al., Nature 2019]

\`On-device' optimization in regime of quantum advantage

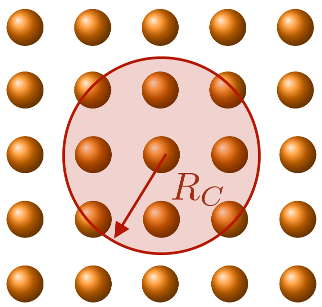
- Quantum Sensors benefit from scaling to large particle # N

Science **366**, 93–97 (2019)   4 October 2019

ATOMIC PHYSICS

Seconds-scale coherence on an optical clock transition in a tweezer array

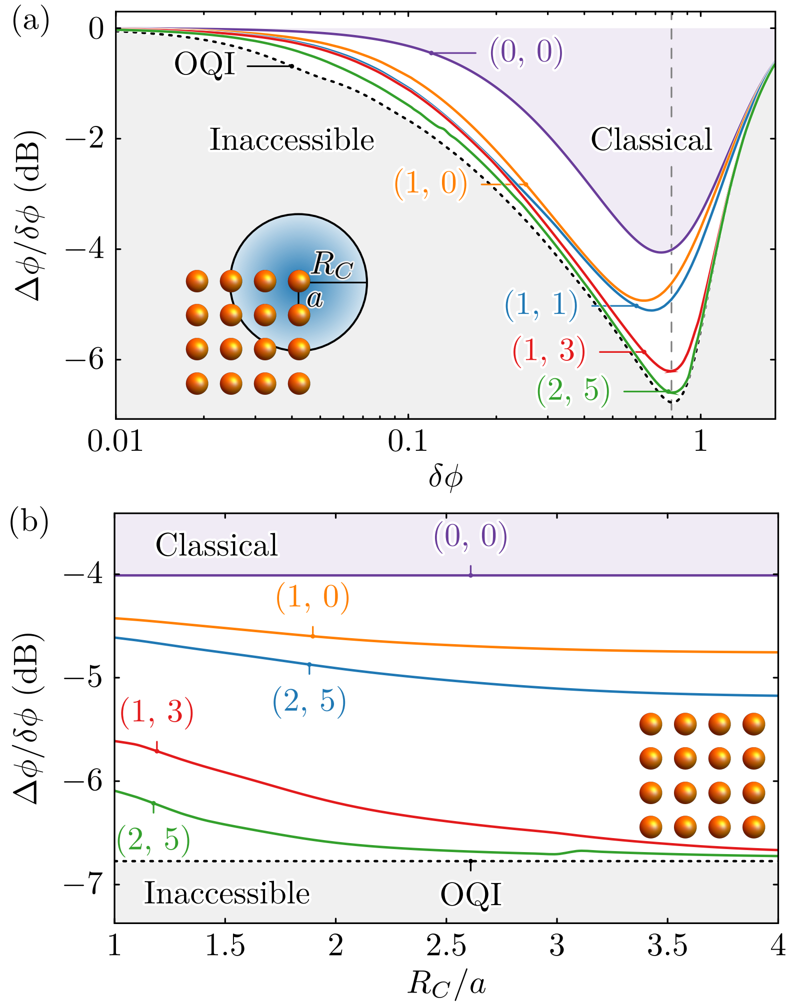
Matthew A. Norcia, Aaron W. Young, William J. Eckner, Eric Oelker, Jun Ye, Adam M. Kaufman*



$$V_{ij} = \frac{R_C^6}{R_C^6 + |\boldsymbol{r}_i - \boldsymbol{r}_j|^6}$$

finite range Rydberg interactions

see also Endres (Caltech), …



- Classical optimization of variational entangler and decoder is challenging in regime $N > 50$ spins, and in 2D etc.
- \`On-device' optimization to solve \`many-body problem' in quantum advantage regime

Next lecture

1. Beyond Spin Squeezing: Variational Quantum Metrology for Ramsey Interferometry and Atomic Clocks

2. Programmable Quantum Sensors and Quantum Compasses

3. Quantum Sensing Networks for Tests of Quantum Mechanics and General Relativity

Open positions:
- PhD students
- Postdocs