Quantum Thermalization, Chaos and Information Scrambling

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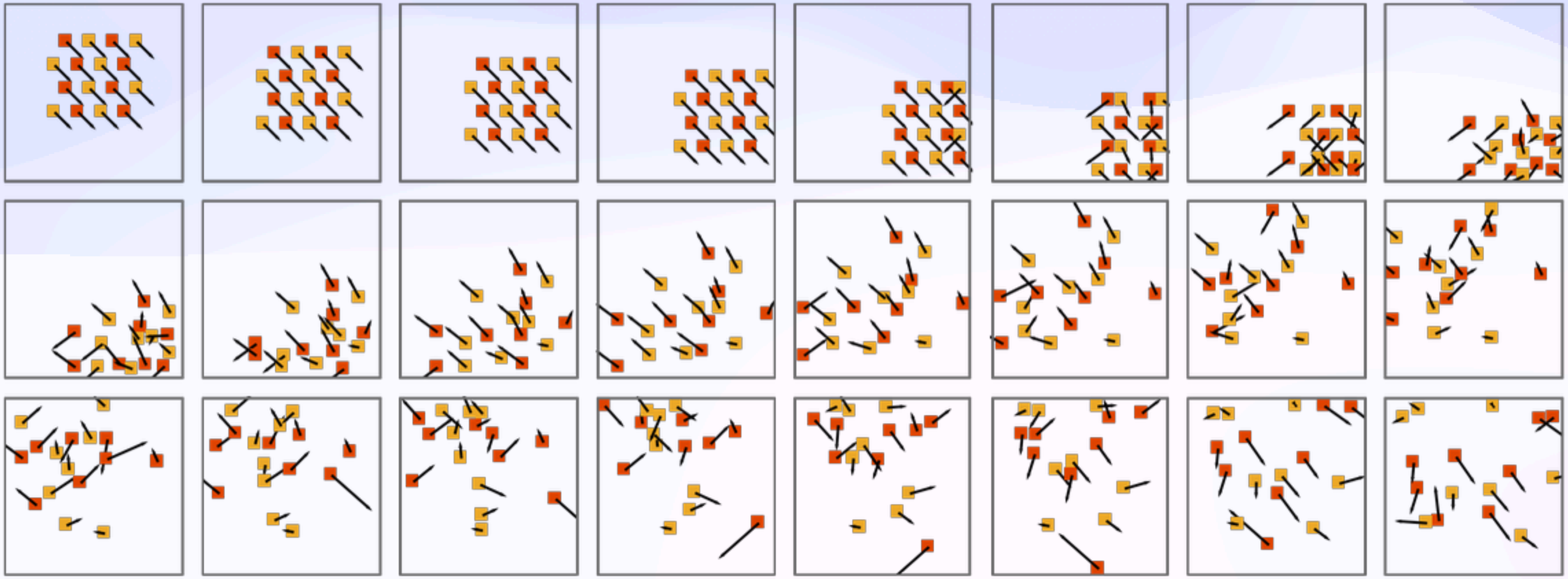
29th ICAP Summer School
2026. 6

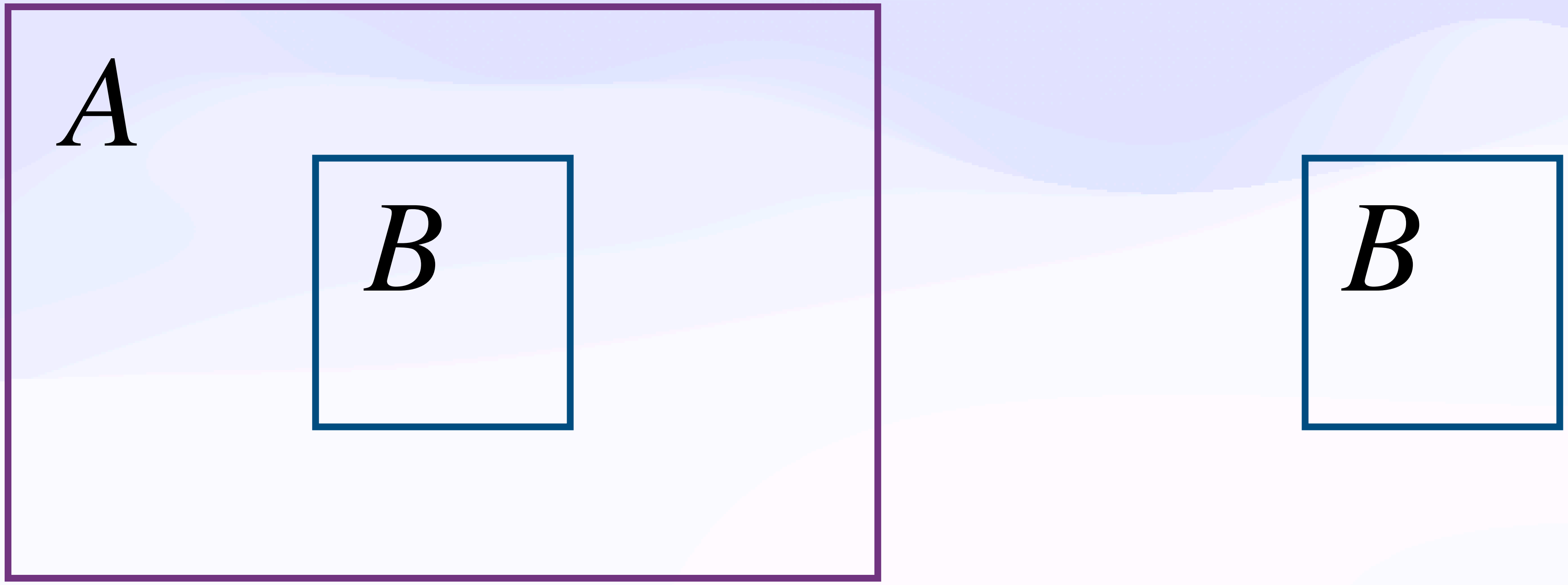
Second Law of Thermodynamics

Thermalization happens inevitably



Thermalization of a Quantum Many-body State

Entanglement Entropy = Thermal Entropy



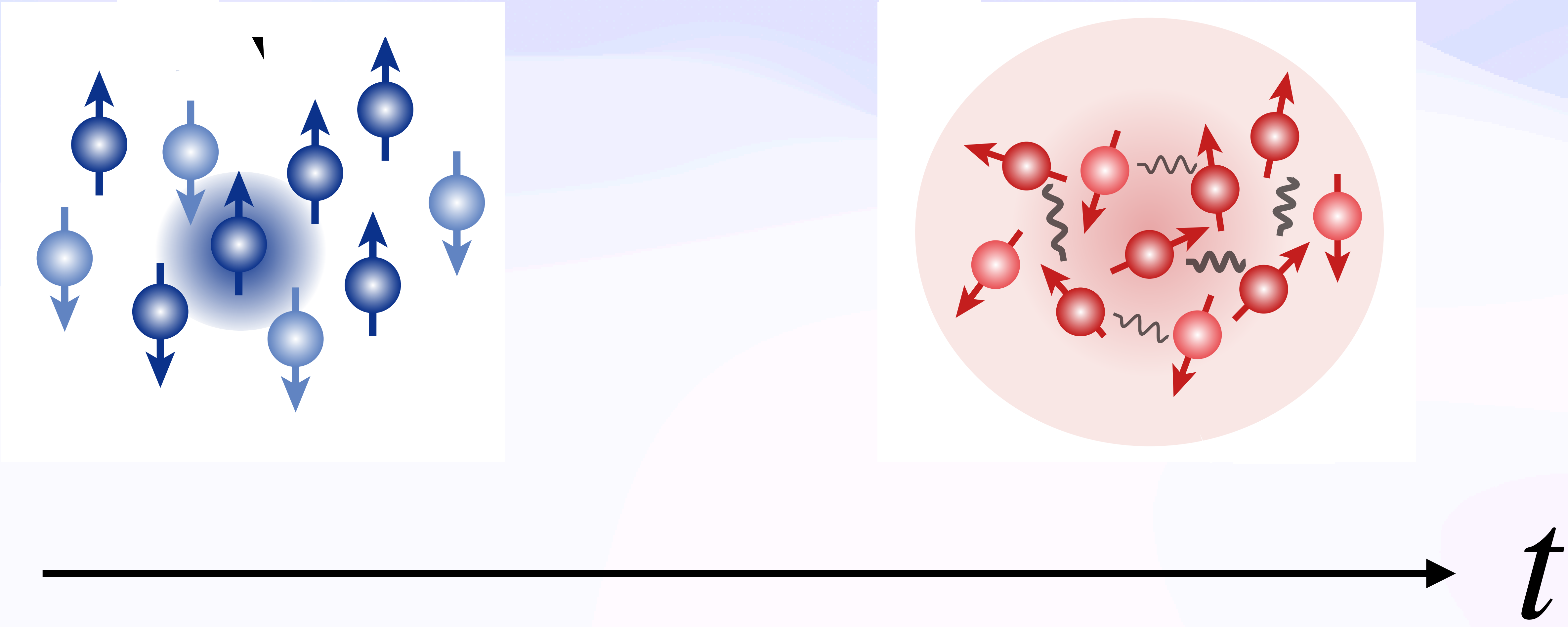
$$\rho_B^{ent} = Tr_A(|\Psi\rangle\langle\Psi|)$$

$$\rho_B^{the} = e^{-\beta H}$$

$$\boxed{\rho_B^{ent} = \rho_B^{the}}$$

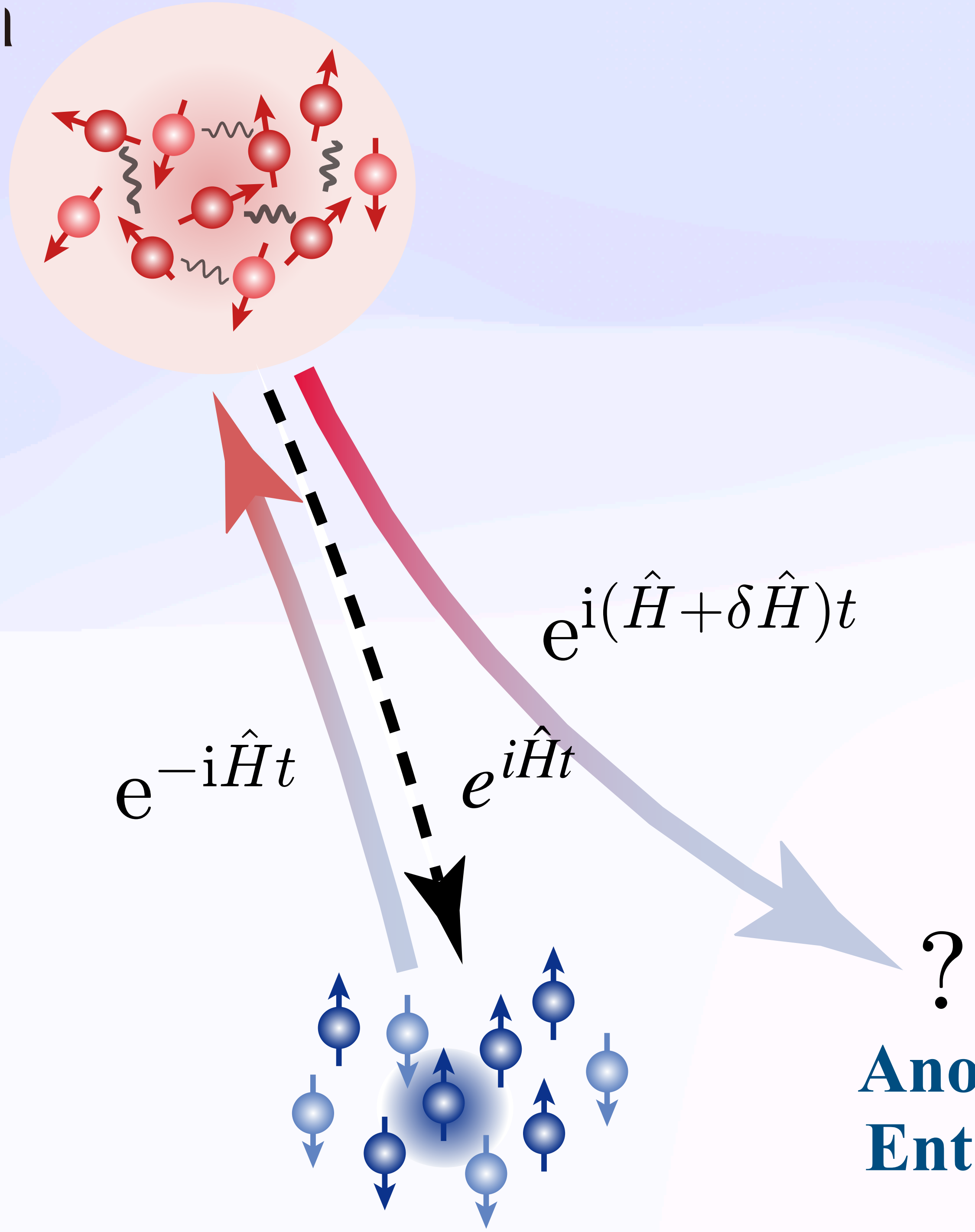
Second Law of Thermodynamics

Thermalization happens inevitably



Reverse Quantum Many-Body Dynamics

Highly-Entangled State



$\mathrm{e}^{\mathrm{i}(\hat{H}+\delta\hat{H})t}$

$\mathrm{e}^{-\mathrm{i}\hat{H}t}$

$e^{i\hat{H}t}$

?

Another Highly-Entangled State

Low-Entangled State

Quantum Information Scrambling

Quantum Many-body Chaos

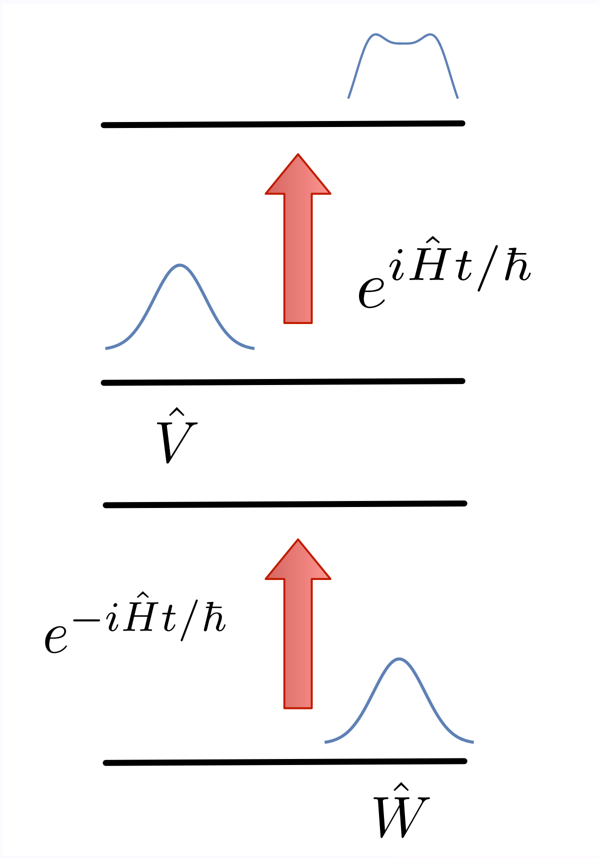
Non-Reversibility of Quantum Many-body Systems

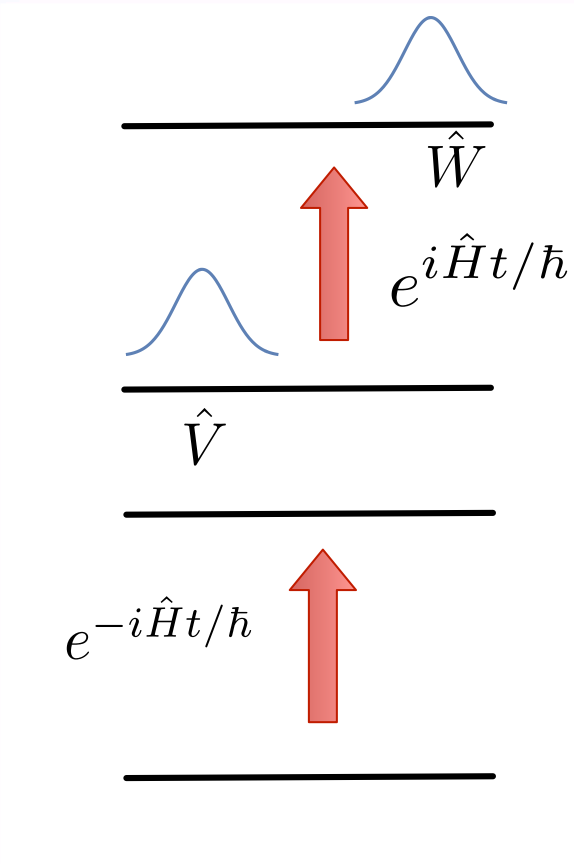
Out-of-Time-Ordered Correlator

$$Tr[\hat{W}(0)\hat{V}(t)\hat{W}(0)\hat{V}(t)]$$

The overlap between these two outcomes:

$$\hat{V}(t)\hat{W}\,|\Psi\rangle \qquad \hat{W}\hat{V}(t)\,|\Psi\rangle$$





Out-of-Time-Ordered Commutator

$$Tr[[\hat{W}(0), \hat{V}(t)]^2]$$

containing two correlates with normal order and two correlators with out-of-time-order

$$\langle [\hat{W}(0), \hat{V}(t)][\hat{W}(0), \hat{V}(t)] \rangle$$

$$= \langle \hat{W}(0)\hat{V}(t)\hat{V}(t)\hat{W}(0) \rangle + \langle \hat{V}(t)\hat{W}(0)\hat{W}(0)\hat{V}(t) \rangle$$

$$- \langle \hat{W}(0)\hat{V}(t)\hat{W}(0)\hat{V}(t) \rangle - \langle \hat{V}(t)\hat{W}(0)\hat{V}(t)\hat{W}(0) \rangle$$

Out-of-Time-Ordered Commutator

$$Tr[[\hat{W}(0), \hat{V}(t)]^2]$$

containing two correlates with normal order and two correlators with out-of-time-order

$$\hat{V}(t) = e^{i\hat{H}t}\hat{V}e^{-i\hat{H}t}$$

operator complexity increases

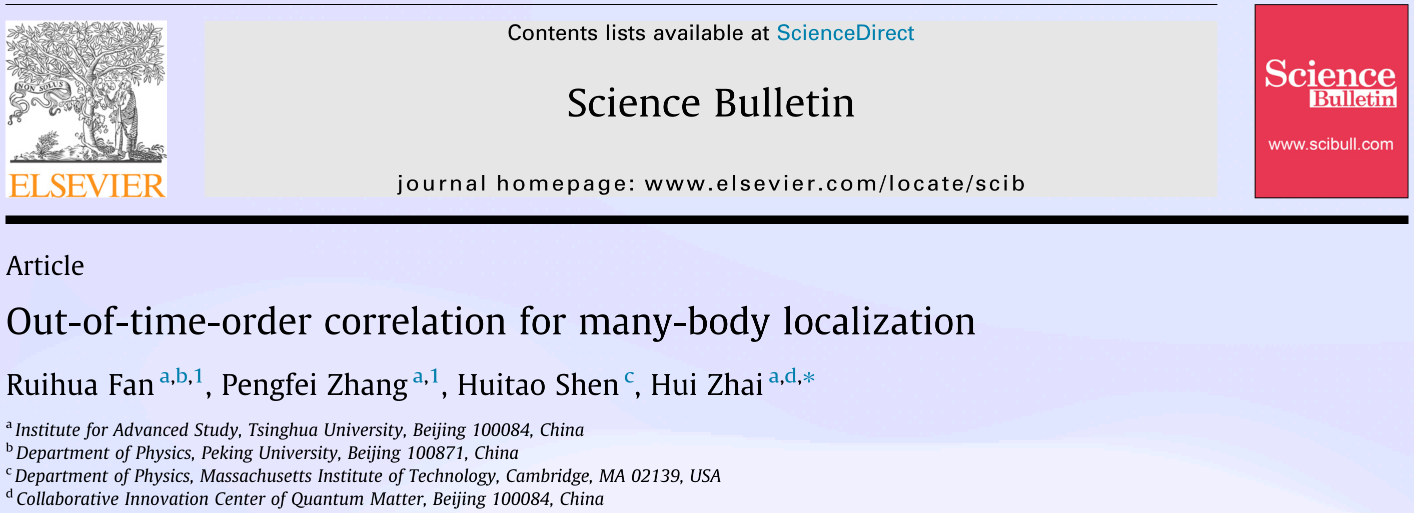
$$= \hat{V} + [\hat{H}, \hat{V}](it) + \frac{1}{2}[\hat{H}, [\hat{H}, \hat{V}](it)^2 + \dots$$

$$Tr[[\hat{W}(0), \hat{V}(t)]^2] \sim e^{\lambda t}$$

↑

Lyapunov exponent

OTOC and Entropy Growth



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Article

Out-of-time-order correlation for many-body localization

Ruihua Fan [a,b,1], Pengfei Zhang [a,1], Huitao Shen [c], Hui Zhai [a,d,*]

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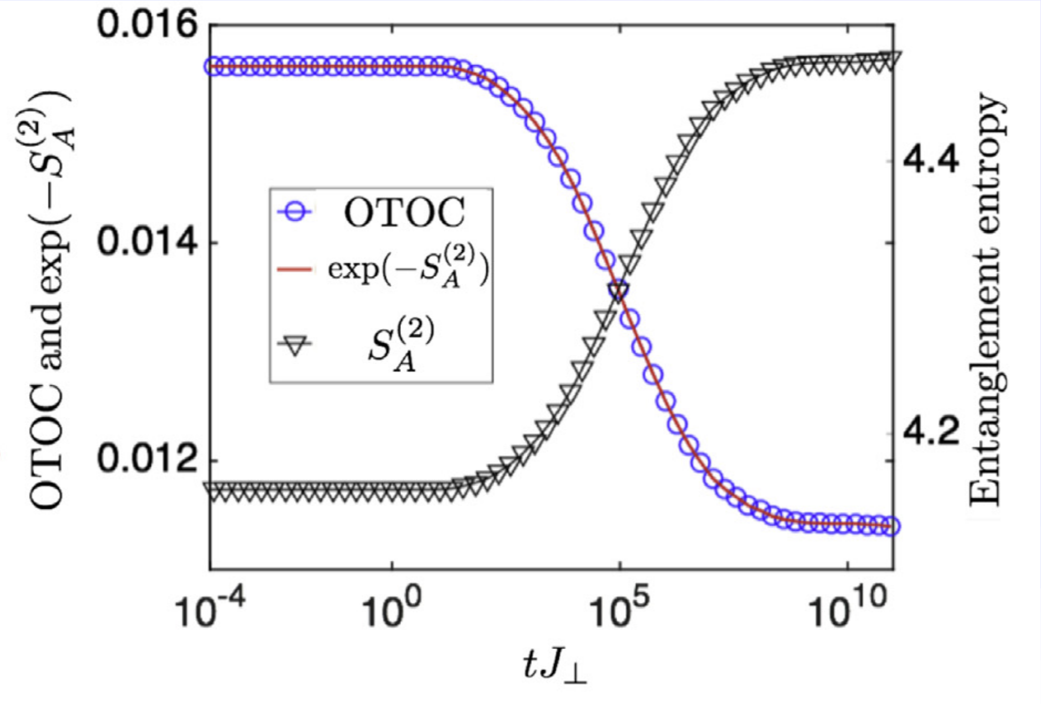
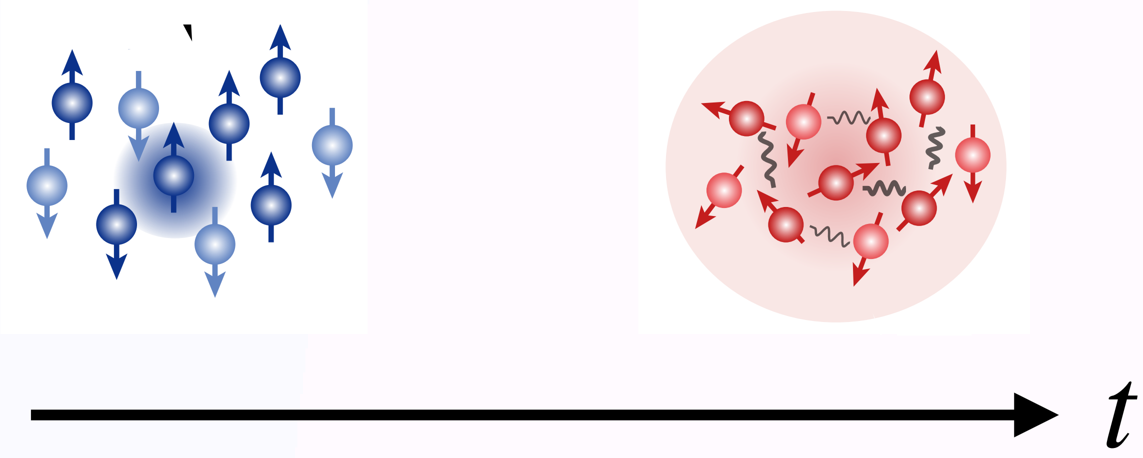
[b] Department of Physics, Peking University, Beijing 100871, China
[c] Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[d] Collaborative Innovation Center of Quantum Matter, Beijing 100084, China



$$\exp(-S_A^{(2)}) = \sum_{\hat{M}\in B} \langle \hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)\rangle_{\beta=0}$$

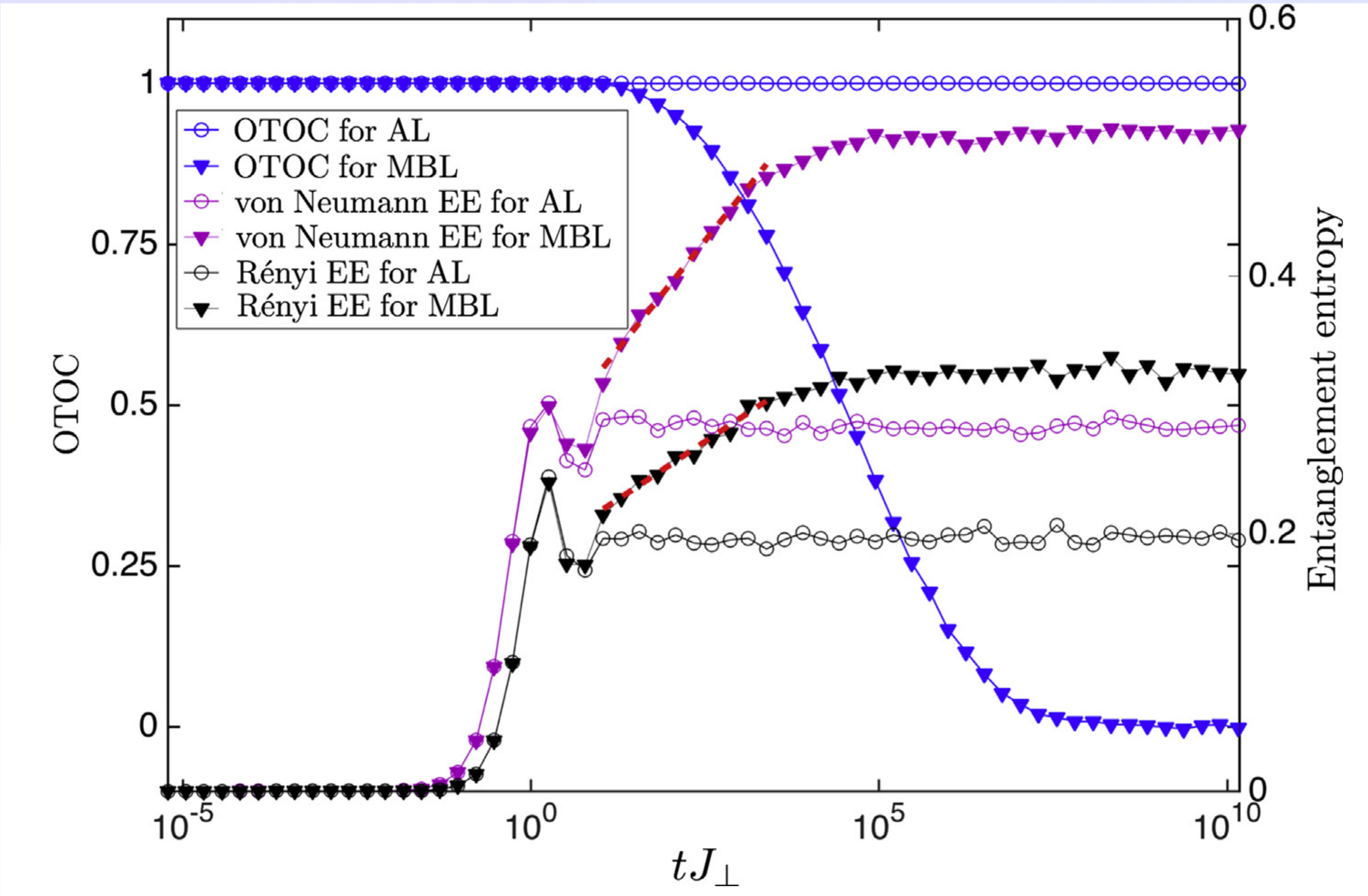
linear growth

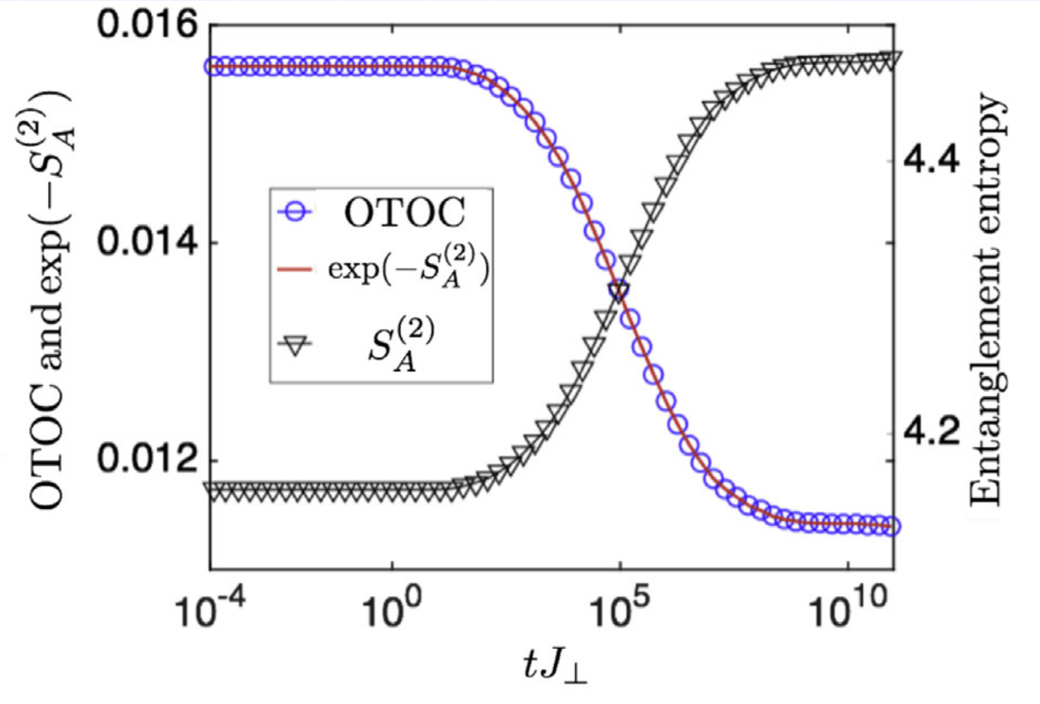
exponential decrease



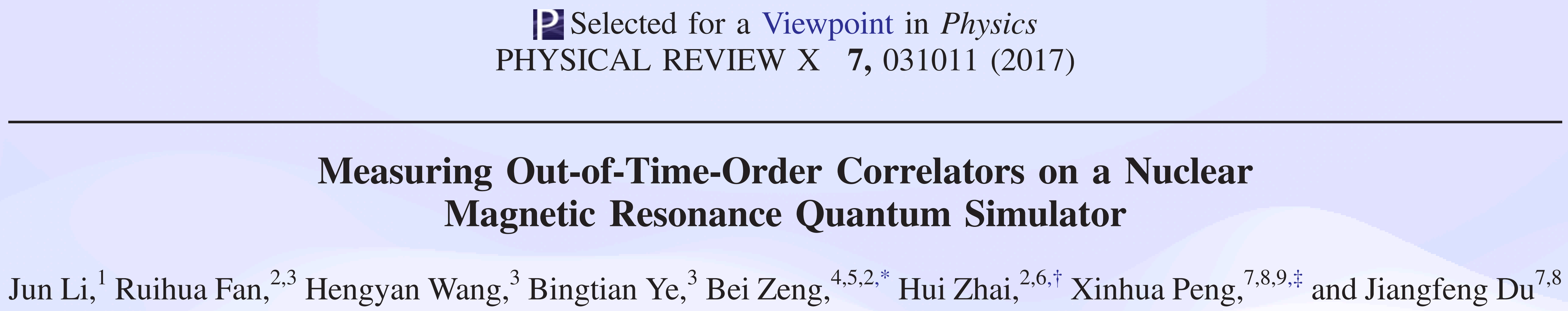
Citation >350

Many-body Localization versus Thermalization





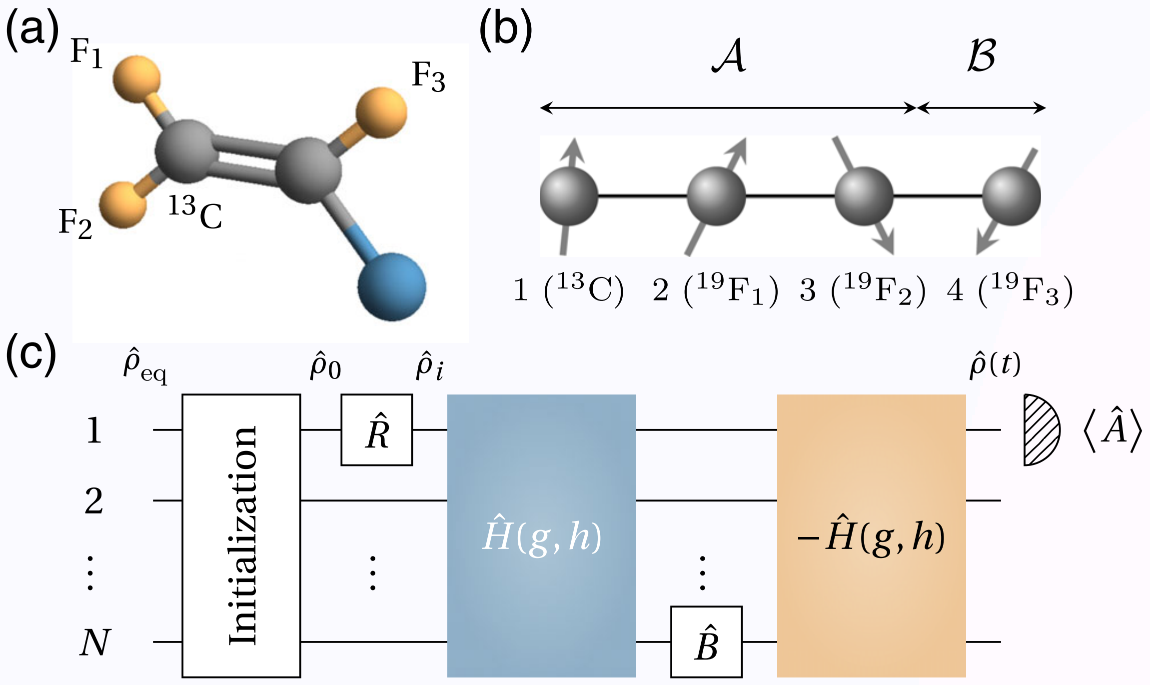
First Experimental Measurements of OTOC

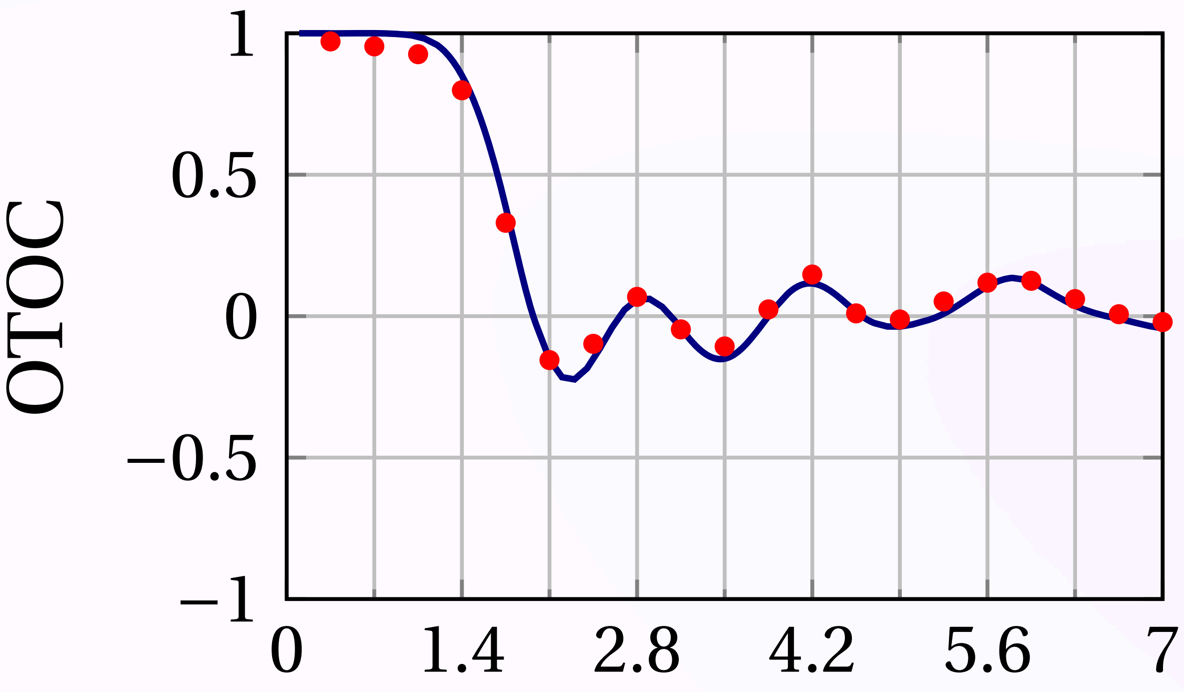


Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]





Citation >500

First Experimental Measurements of OTOC

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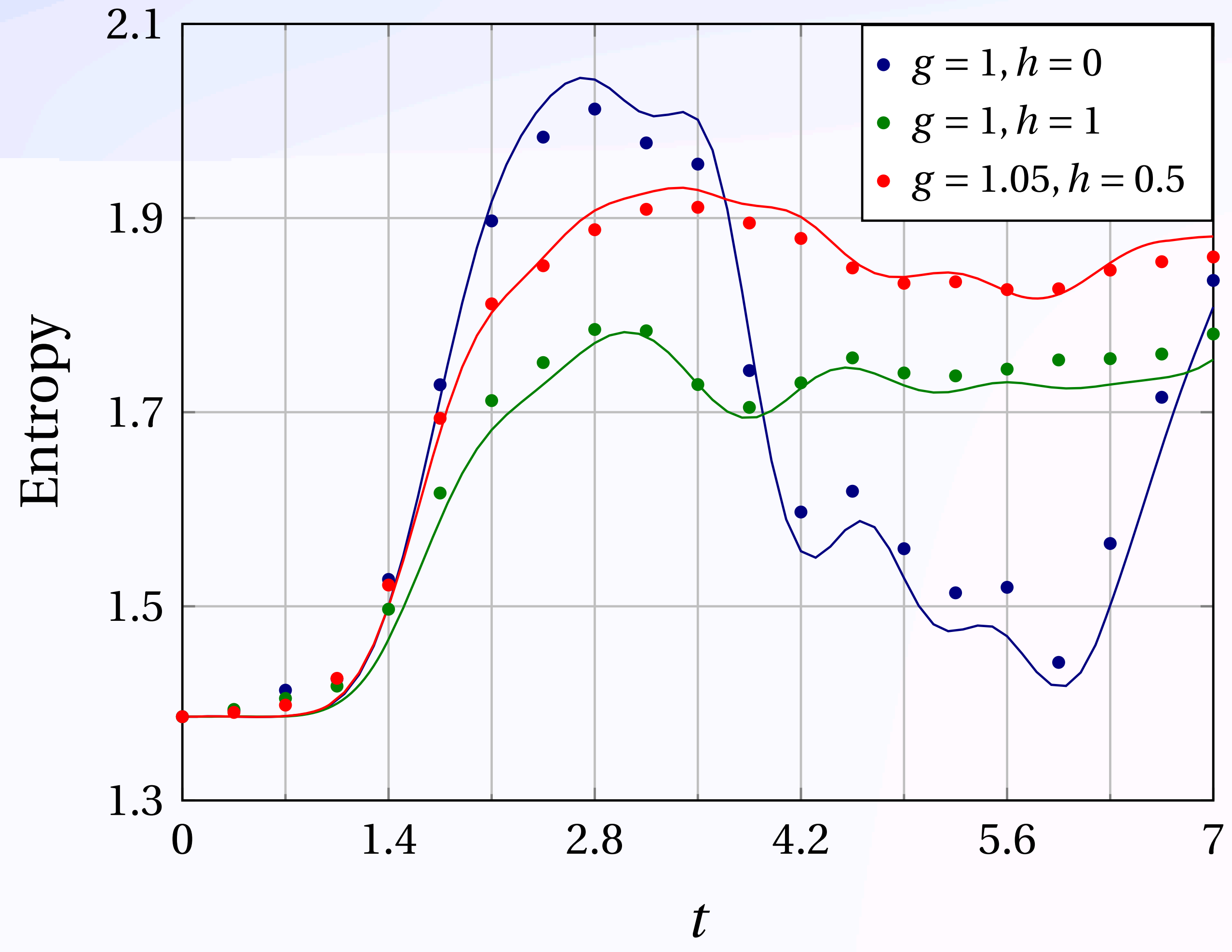


$$\exp(-S_{\mathcal{A}}^{(2)}) = \sum_{\hat{M} \in \mathcal{B}} \langle \hat{M}(t) \hat{V}(0) \hat{M}(t) \hat{V}(0) \rangle_{\beta=0}$$

$$\hat{H} = \sum_i (-\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + g\hat{\sigma}_i^x + h\hat{\sigma}_i^z)$$

First Experimental Measurements of OTOC

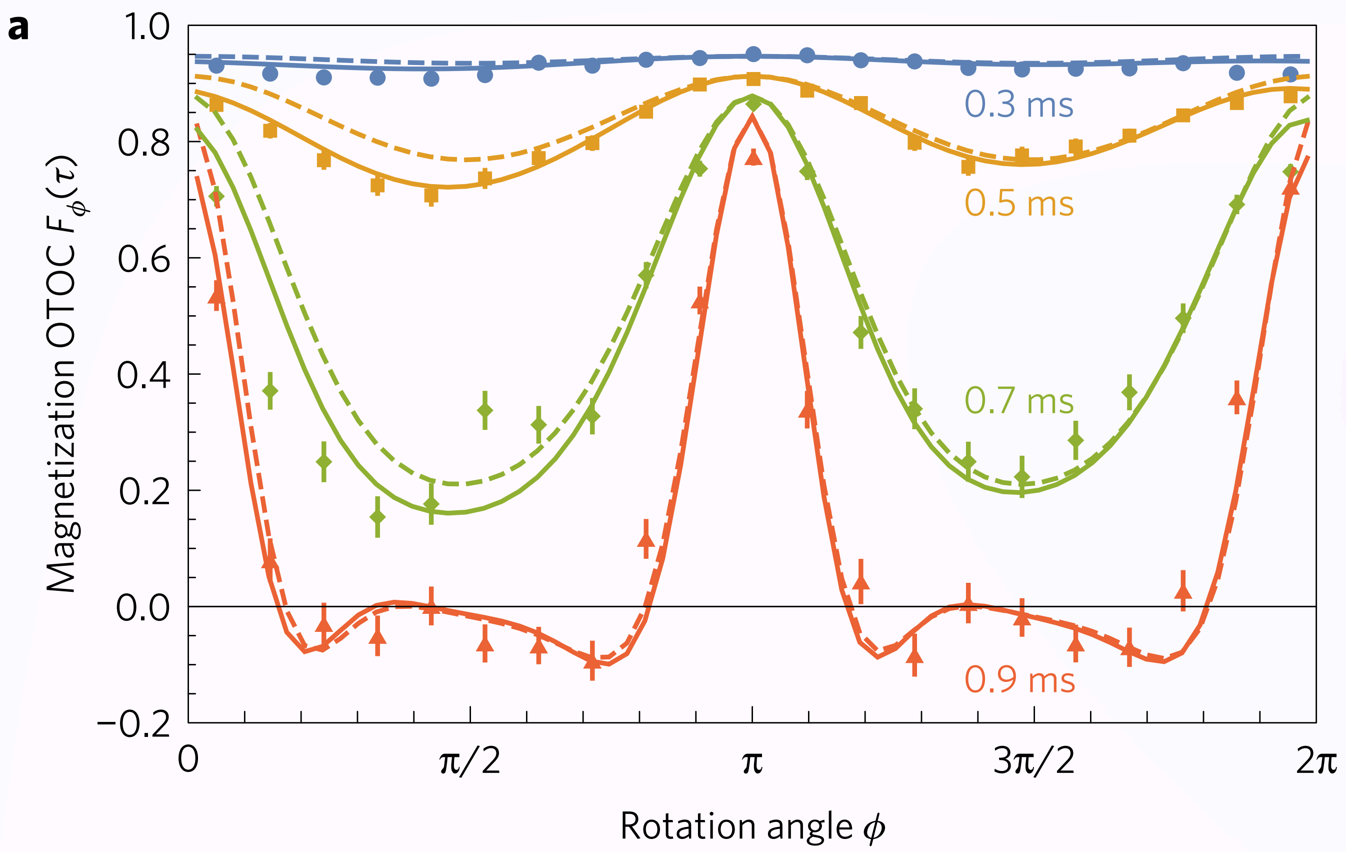
nature physics

ARTICLES

PUBLISHED ONLINE: 22 MAY 2017 | DOI: 10.1038/NPHYS4119

Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet

Martin Gärttner[1†], Justin G. Bohnet[2†], Arghavan Safavi-Naini[1], Michael L. Wall[1], John J. Bollinger[2]
and Ana Maria Rey[1*]



First Experimental Measurements of OTOC

Physics

VIEWPOINT

Seeing Scrambled Spins

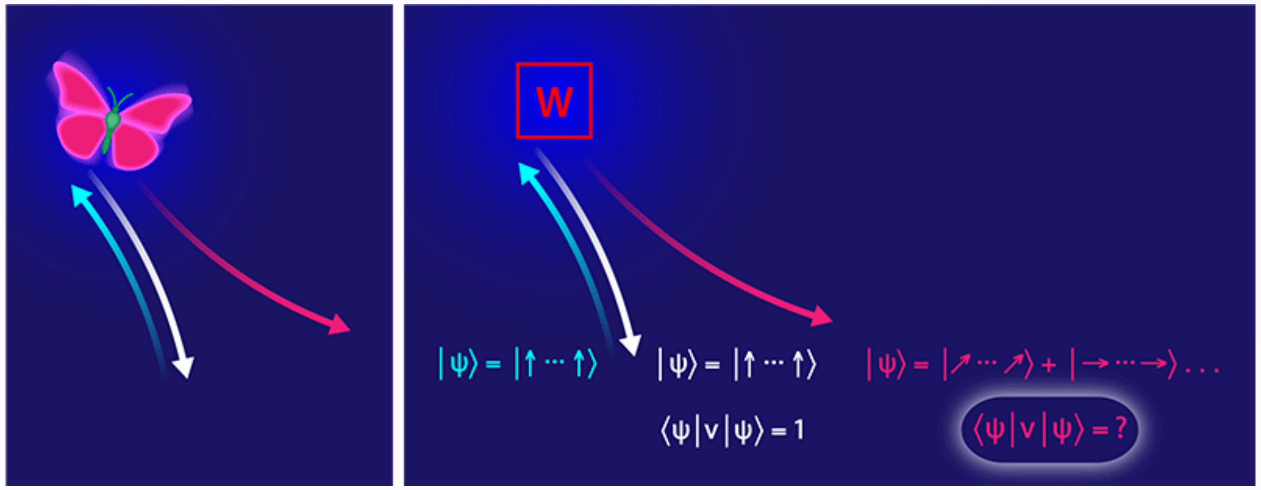
Two experimental groups have taken a step towards observing the "scrambling" of information that occurs as a many-body quantum system thermalizes.

by Brian Swingle* and Norman Y. Yao†

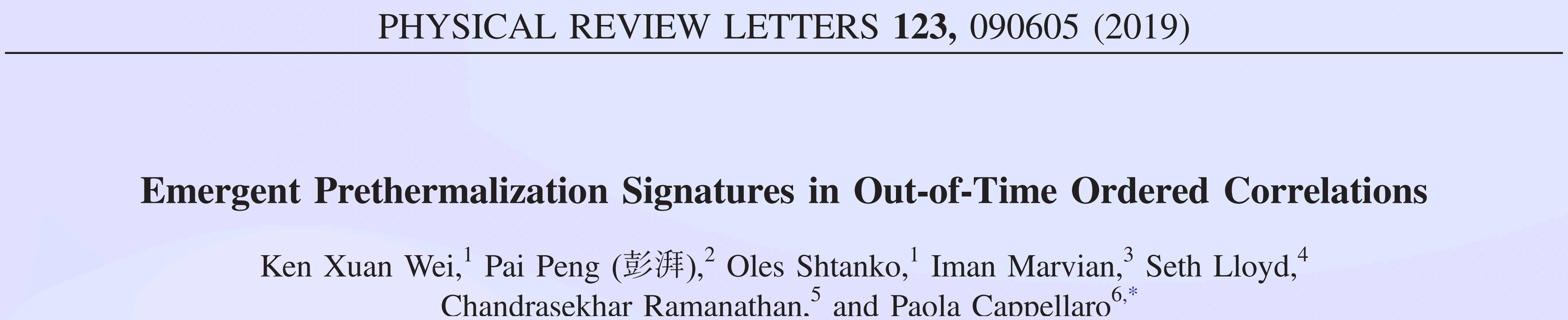
mation. Two groups, one in China [5] and one in the US [6], have taken a step towards tracking this scrambling of information in systems of quantum spins.

[5] J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, "Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator," Phys. Rev. X **7**, 031011 (2017).

[6] M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, "Measuring Out-of-time-order Correlations and Multiple Quantum Spectra in a Trapped-ion Quantum Magnet," Nat. Phys. (2017).



Following-up Experiments on OTOC



PHYSICAL REVIEW LETTERS **123,** 090605 (2019)

Emergent Prethermalization Signatures in Out-of-Time Ordered Correlations

Ken Xuan Wei,[1] Pai Peng (彭湃),[2] Oles Shtanko,[1] Iman Marvian,[3] Seth Lloyd,[4]
Chandrasekhar Ramanathan,[5] and Paola Cappellaro[6,*]

NMR Experiment by MIT group, PRL 2019

LETTER

https://doi.org/10.1038/s41586-019-0952-6

Verified quantum information scrambling

K. A. Landsman[1*], C. Figgatt[1,6], T. Schuster[2], N. M. Linke[1], B. Yoshida[3], N. Y. Yao[2,4] & C. Monroe[1,5]

Trap Ion Experiment by Maryland group, Nature, 2019

RESEARCH

QUANTUM SENSING

Improving metrology with quantum scrambling

Zeyang Li (李泽阳)[1], Simone Colombo[1], Chi Shu[1,2], Gustavo Velez[1,3], Saúl Pilatowsky-Cameo[4], Roman Schmied[5], Soonwon Choi[4], Mikhail Lukin[2], Edwin Pedrozo-Peñafiel[1], Vladan Vuletić[1*]

Cold Atom Experiment by MIT Group, Science 2023

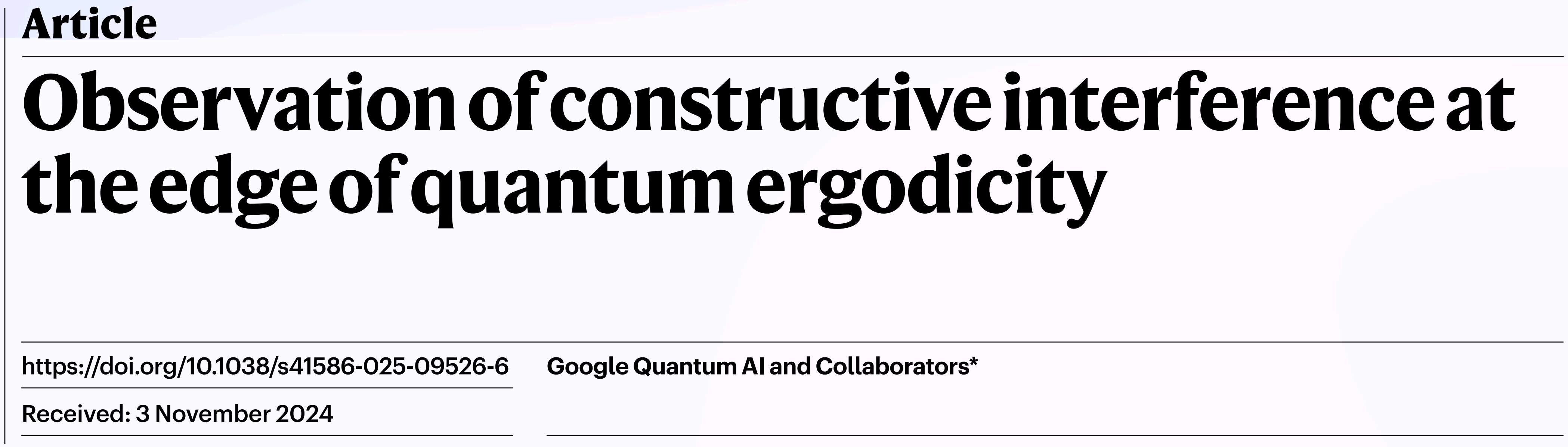
Following-up Experiments on OTOC

RESEARCH

QUANTUM SIMULATION

Information scrambling in quantum circuits

Superconducting Qubit Experiment by Google Group, Science 2021



Article

Observation of constructive interference at the edge of quantum ergodicity

https://doi.org/10.1038/s41586-025-09526-6

Received: 3 November 2024

Google Quantum AI and Collaborators*

Superconducting Qubit Experiment by Google Group, Nature 2025

Comments on Current Experimental Status

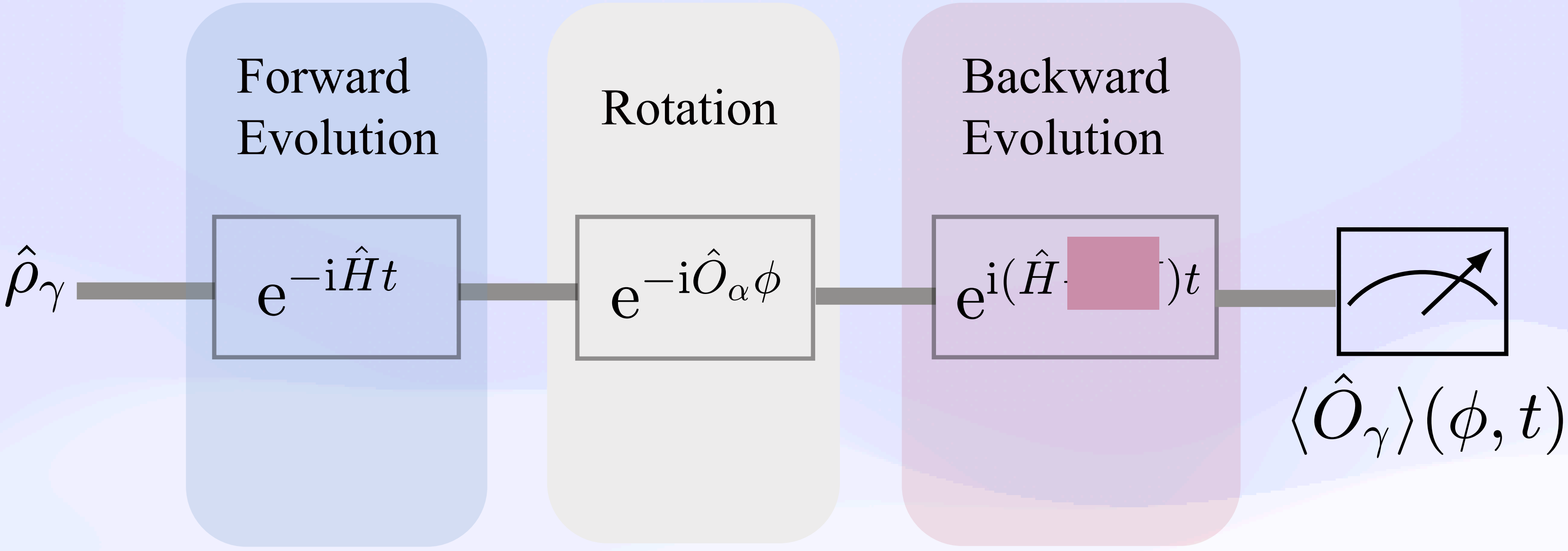
$$Tr[[\hat{W}(0), \hat{V}(t)]^2] \sim e^{\underset{\uparrow}{\lambda} t}$$

Lyapunov exponent

- **None of these experiments sees well-defined exponential behavior of OTOC and is able to extract the Lyapunov exponent.**

- **The reason is that measuring OTOC always involves backward time evolution, which is always imperfect.**

Experimental Protocol



$\hat{\rho}_\gamma$

Forward Evolution

$e^{-\mathrm{i}\hat{H}t}$

Rotation

$e^{-\mathrm{i}\hat{O}_\alpha\phi}$

Backward Evolution

$e^{\mathrm{i}(\hat{H}$ [redacted] $)t}$

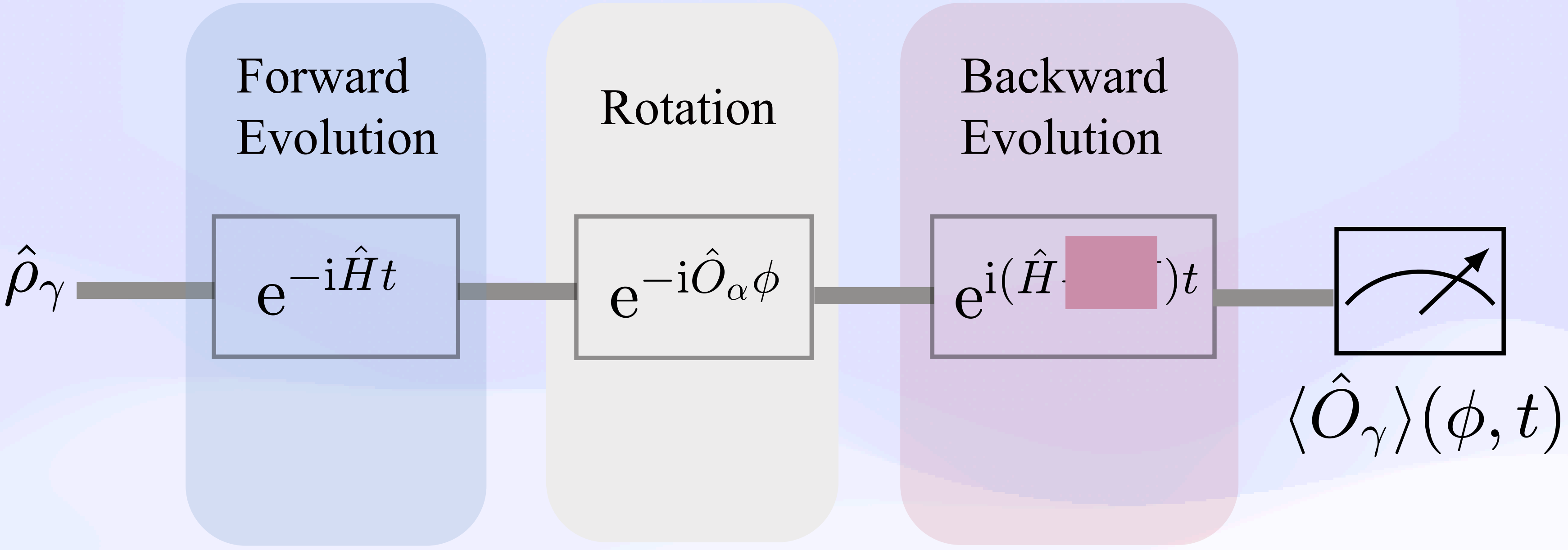
$\langle\hat{O}_\gamma\rangle(\phi, t)$

$$\hat{\rho}_\gamma = 1 + \epsilon\hat{S}_\gamma$$

$$\hat{S}_\gamma = \sum_i s_\gamma^i$$

$$\gamma = x, y, z$$

Experimental Protocol



$$Tr[\hat S_\gamma e^{i\hat H t} e^{-i\hat S_\alpha \phi} e^{-i\hat H t} \hat S_\gamma e^{i\hat H t} e^{i\hat S_\alpha \phi} e^{-i\hat H t}]$$

OTO Correlator

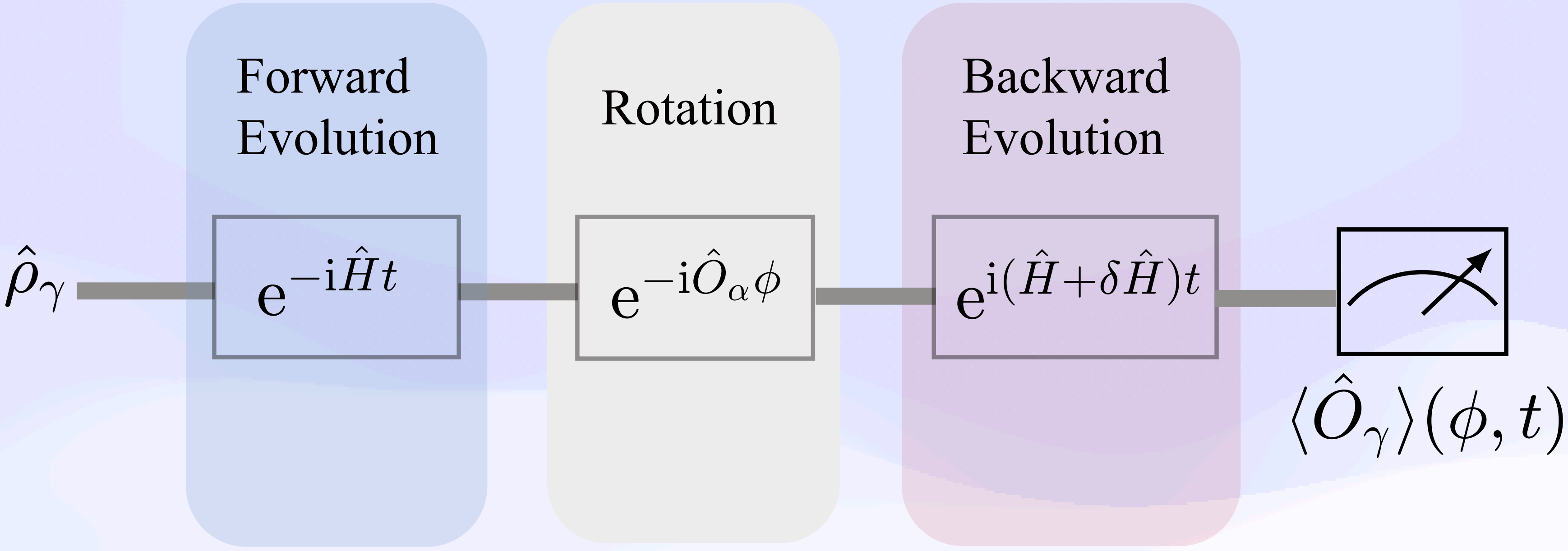
$$F(\phi, t) = Tr[\hat S_\gamma e^{-i\hat S_\alpha(t)\phi} \hat S_\gamma e^{i\hat S_\alpha(t)\phi}]$$

OTO Commutator

$$-\frac{\partial^2 F(\phi, t)}{\partial \phi^2} = Tr[[\hat S_\alpha(t), \hat S_\gamma]^2]$$

Quantum many-body chaos prevents us from observing quantum many-body chaos

Experimental Protocol



$\hat{\rho}_\gamma$

Forward Evolution

$\mathrm{e}^{-\mathrm{i}\hat{H}t}$

Rotation

$\mathrm{e}^{-\mathrm{i}\hat{O}_\alpha\phi}$

Backward Evolution

$\mathrm{e}^{\mathrm{i}(\hat{H}+\delta\hat{H})t}$

$\langle\hat{O}_\gamma\rangle(\phi, t)$

$$F(\phi, t) = Tr[\hat{S}_\gamma \hat{V}(t) e^{-i\hat{S}_\alpha(t)\phi} \hat{S}_\gamma e^{i\hat{S}_\alpha(t)\phi} \hat{V}(t)]$$

$$\hat{V}(t) = e^{-iT\int_0^t dt'\delta\hat{H}(t')}$$

Imperfection in the backward evolution

Consequence: $F(0,t)$ **is not a constant.**

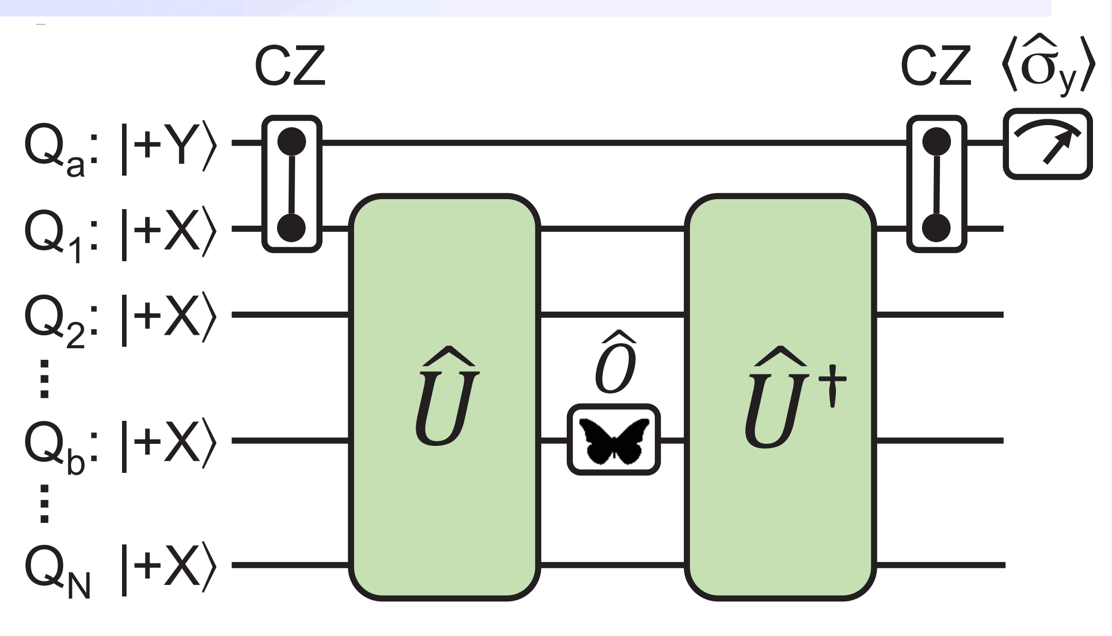
Out-of-Time-Ordered Correlator

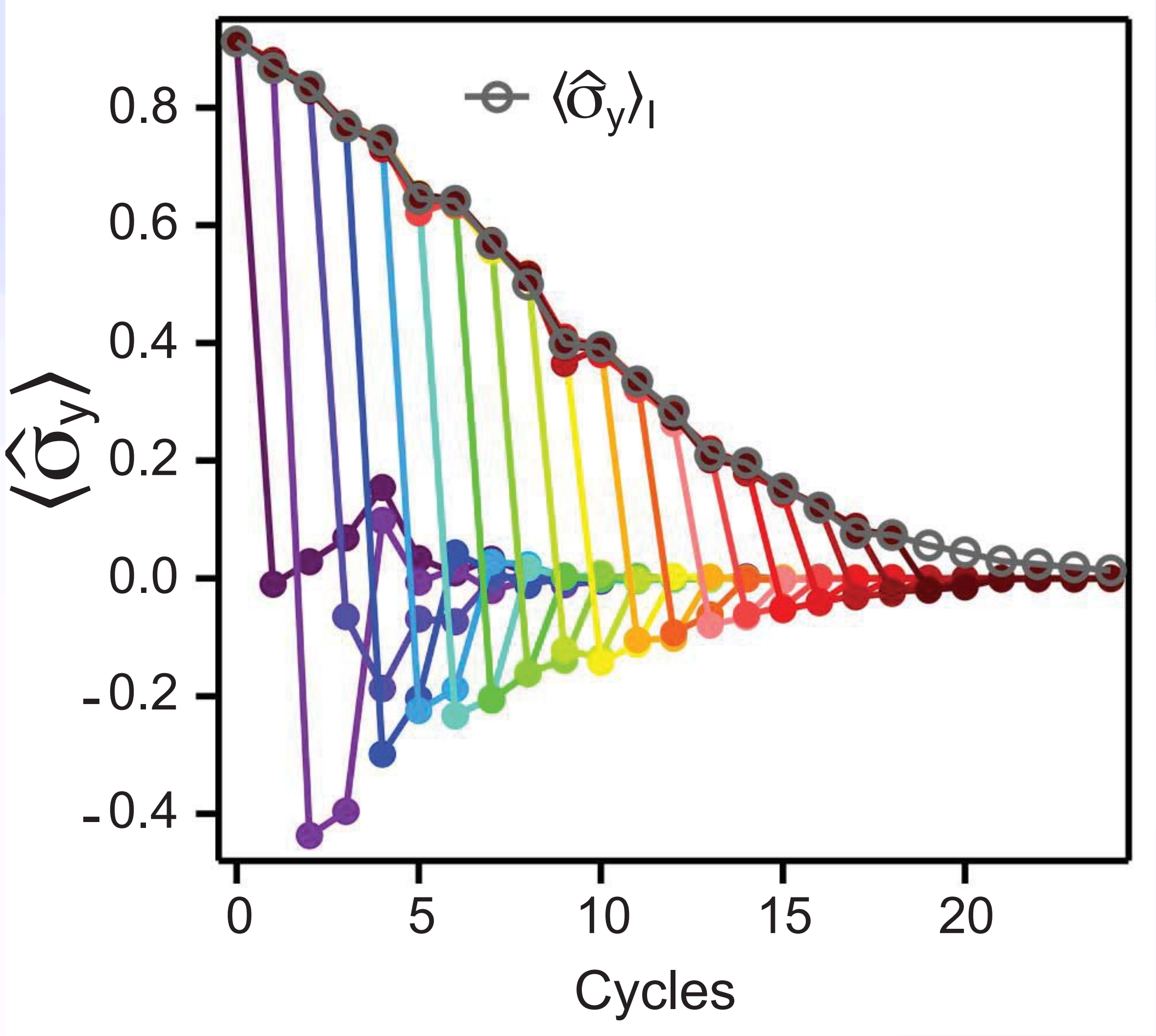
Consequence: $F(0,t)$ **is not a constant.**

RESEARCH

QUANTUM SIMULATION

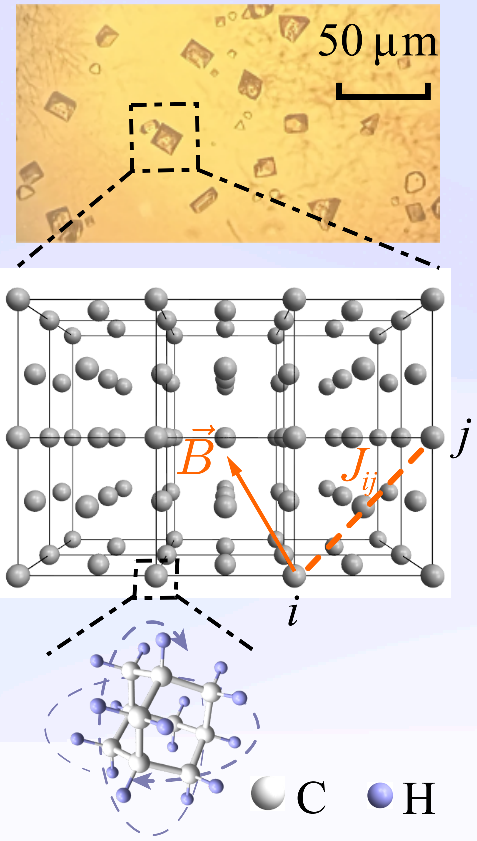
Information scrambling in quantum circuits

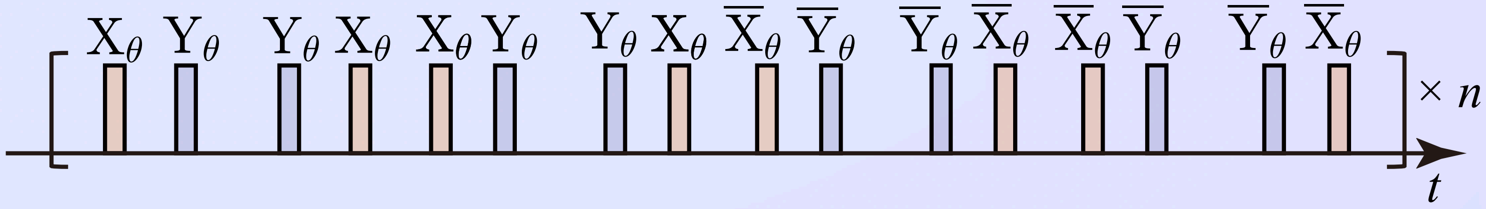




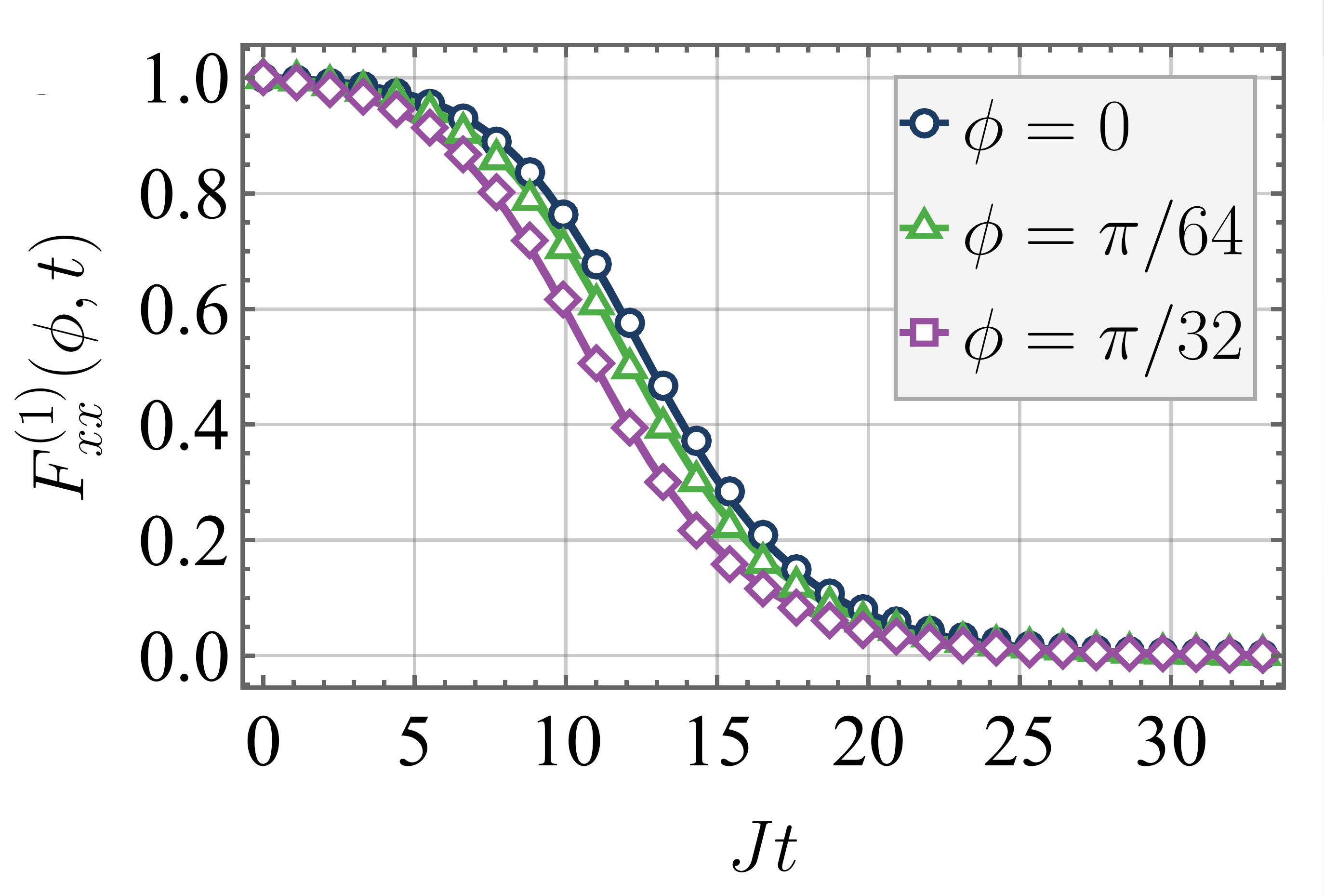
Error-Mitigation: $$\frac{F(\phi,t)}{F(0,t)}$$

Out-of-Time-Ordered Correlator





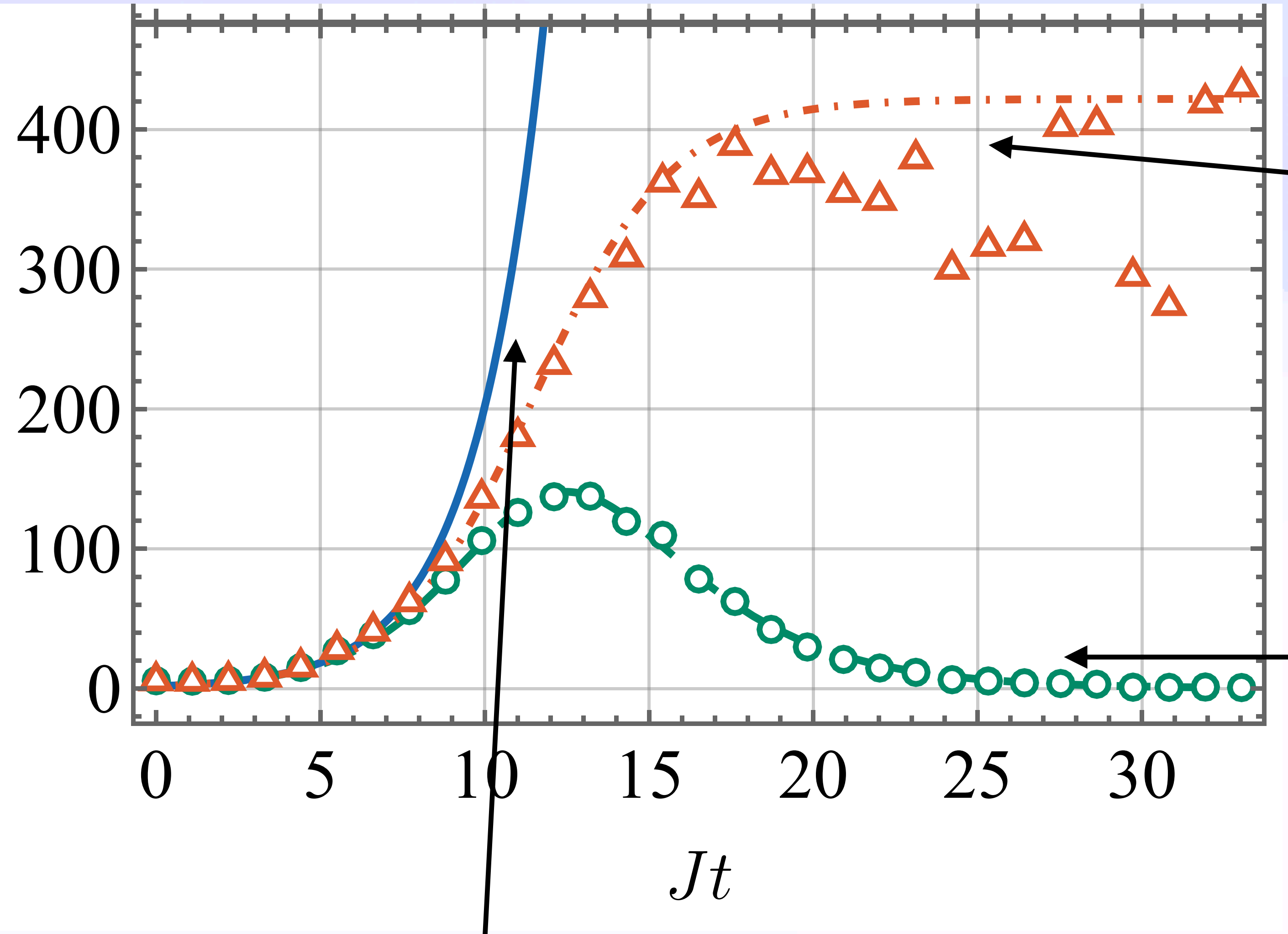
$$\hat{H}_0 = \sum_{i<j,m,n}\sum_{\mu,\nu} J_{ij}\xi_{\mu\nu}\hat{S}^{\mu}_{im}\hat{S}^{\nu}_{jn}$$



Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

Mitigating Error in Reversed Dynamics

OTO Commutator $Tr[[\hat{S}_\alpha(t), \hat{S}_\gamma]^2]$



Error-Mitigation:

$$\frac{F(\phi, t)}{F(0,t)}$$

No Mitigation

How do we obtain this one?

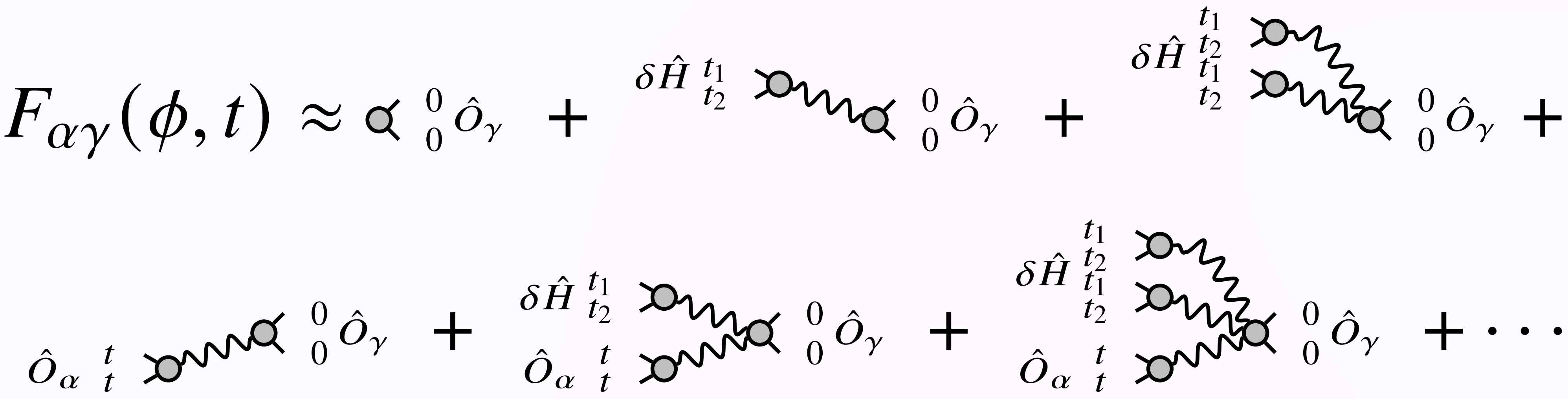
Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

Scramblon Theory for OTOC

$$F(\phi, t) = Tr[\hat{S}_\gamma \hat{V}(t) e^{-i\hat{S}_\alpha(t)\phi} \hat{S}_\gamma e^{i\hat{S}_\alpha(t)\phi} \hat{V}(t)]$$

- $\delta\hat{H} = 0$ **OTOC between** $\hat{S}_\gamma$ **and** $e^{i\phi\hat{S}_\alpha}(t)$
- $\phi = 0$ **OTOC between** $\hat{S}_\gamma$ **and** $\hat{V}(t)$

Scramblon is a new collective mode mediating information scrambling



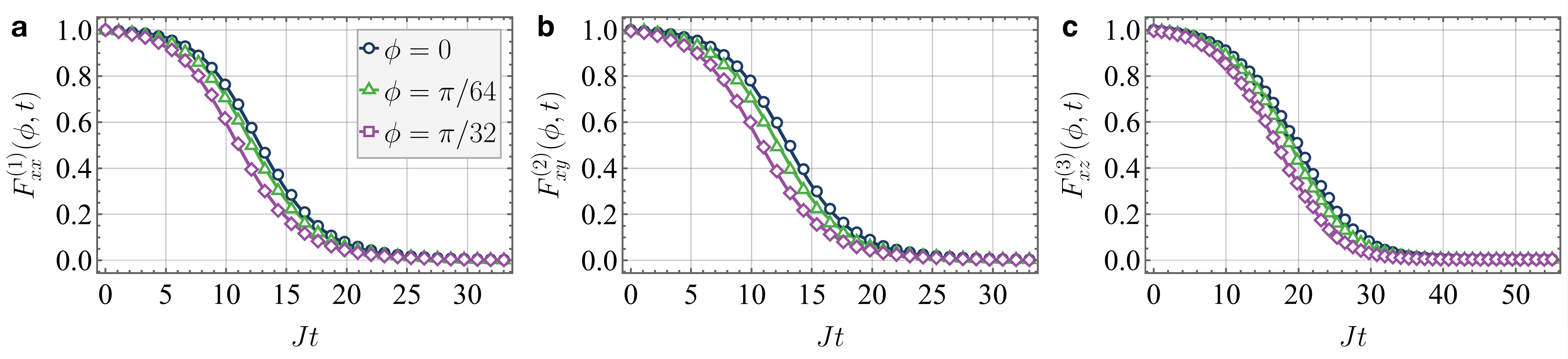
Yingfei Gu, A. Kitaev and Pengfei Zhang, JHEP 2022

Scramblon Ansatz for OTOC with Imperfections

$$F(\phi, t) = Tr[\hat{S}_\gamma \hat{V}(t) e^{-i\hat{S}_\alpha(t)\phi} \hat{S}_\gamma e^{i\hat{S}_\alpha(t)\phi} \hat{V}(t)]$$

Scramblon Ansatz

$$\boxed{F_{\alpha\gamma}(\phi, t) = \frac{1}{\left(1 + a\mathrm{e}^{\varkappa t} + b\phi^2 \mathrm{e}^{\varkappa t}\right)^{2\Delta}}}$$



Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

Verification of the Scarmblon Ansatz

$$F(\phi, t) = Tr[\hat{S}_\gamma \hat{V}(t) e^{-i\hat{S}_\alpha(t)\phi} \hat{S}_\gamma e^{i\hat{S}_\alpha(t)\phi} \hat{V}(t)]$$

Scramblon Ansatz

$$\boxed{F_{\alpha\gamma}(\phi, t) = \frac{1}{\left(1 + a\mathrm{e}^{\varkappa t} + b\phi^2 \mathrm{e}^{\varkappa t}\right)^{2\Delta}}}$$

- κ is independent of α and γ
- a and Δ is independent of α
- $M_{\alpha\gamma} = b_{\alpha\gamma}\Delta_\gamma$ must be symmetric and rank-1

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

Verification of the Scarmblon Ansatz

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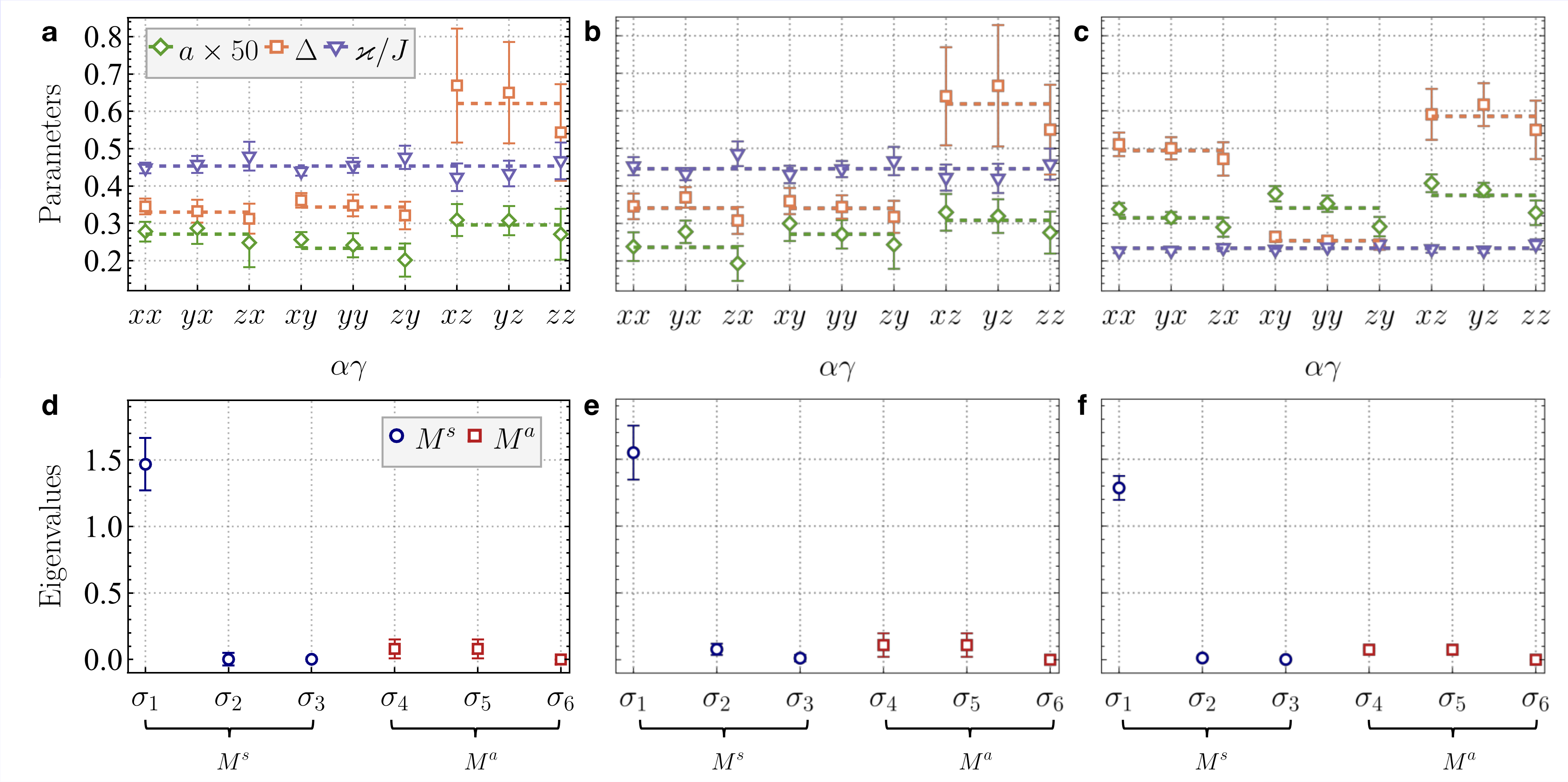

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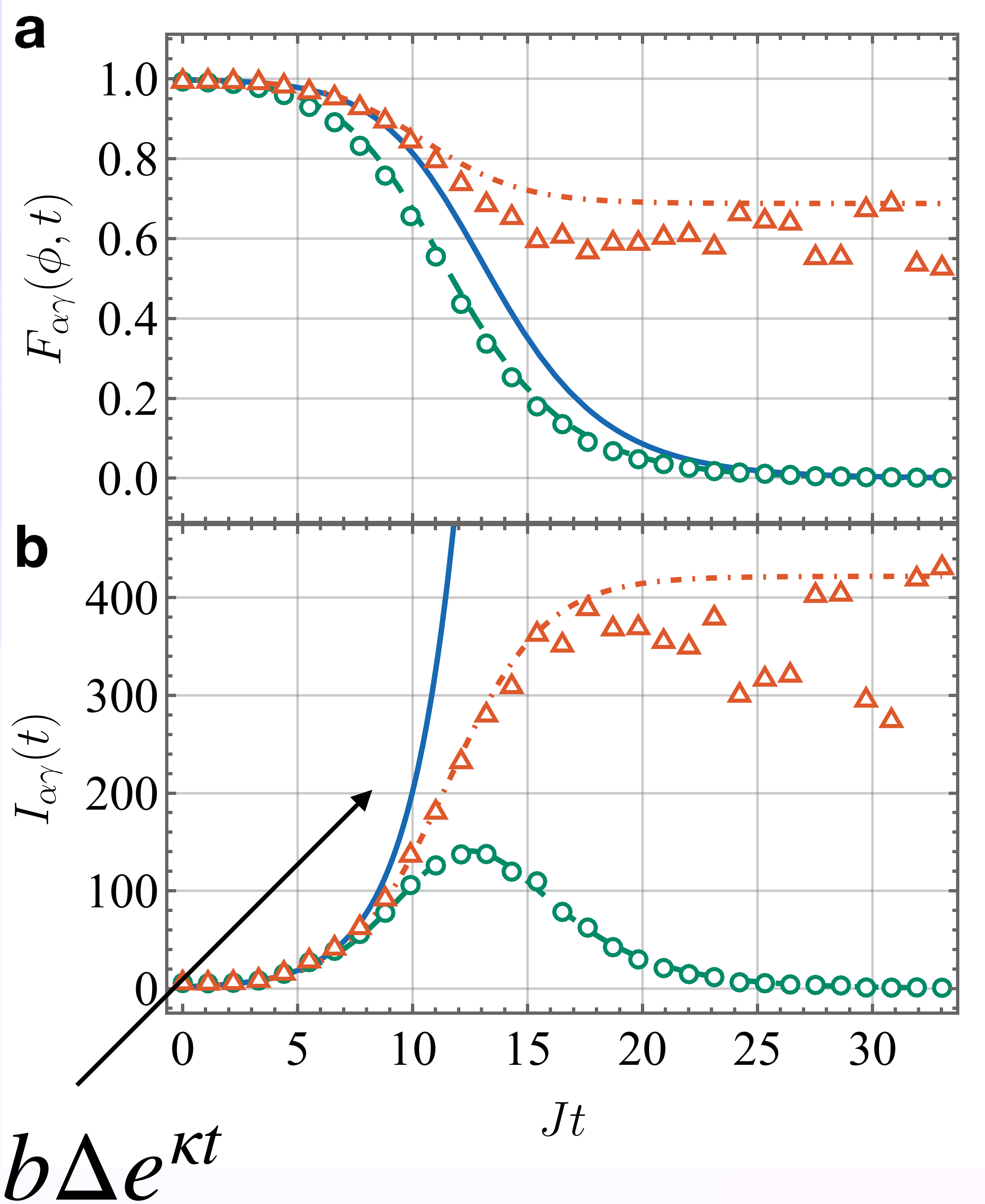
Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

First Experimental Determination of Lyapunov Exponent



$2b\Delta e^{\kappa t}$

Error-Mitigation: $$\frac{F(\phi,t)}{F(0,t)}$$

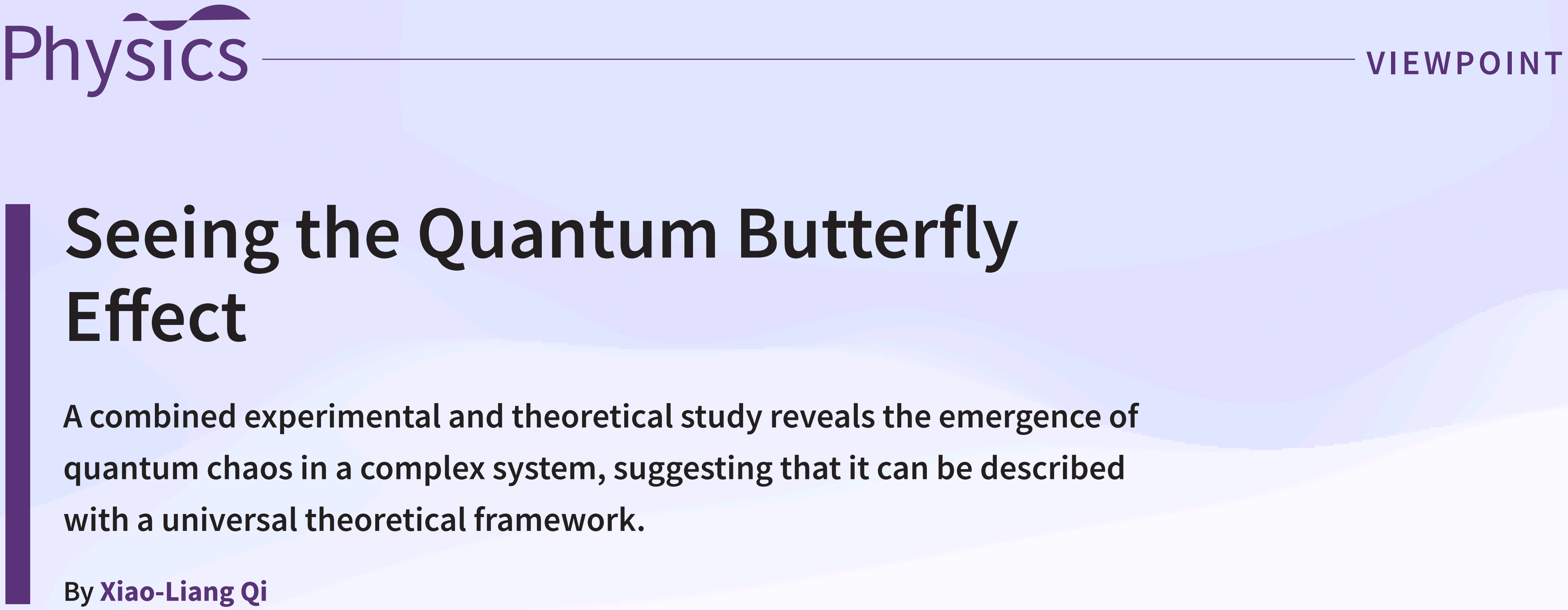
Scramblon Ansatz with

$$a = 0$$

No Mitigation

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, PRL, 2026

First Experimental Determination of Lyapunov Exponent



Physics

VIEWPOINT

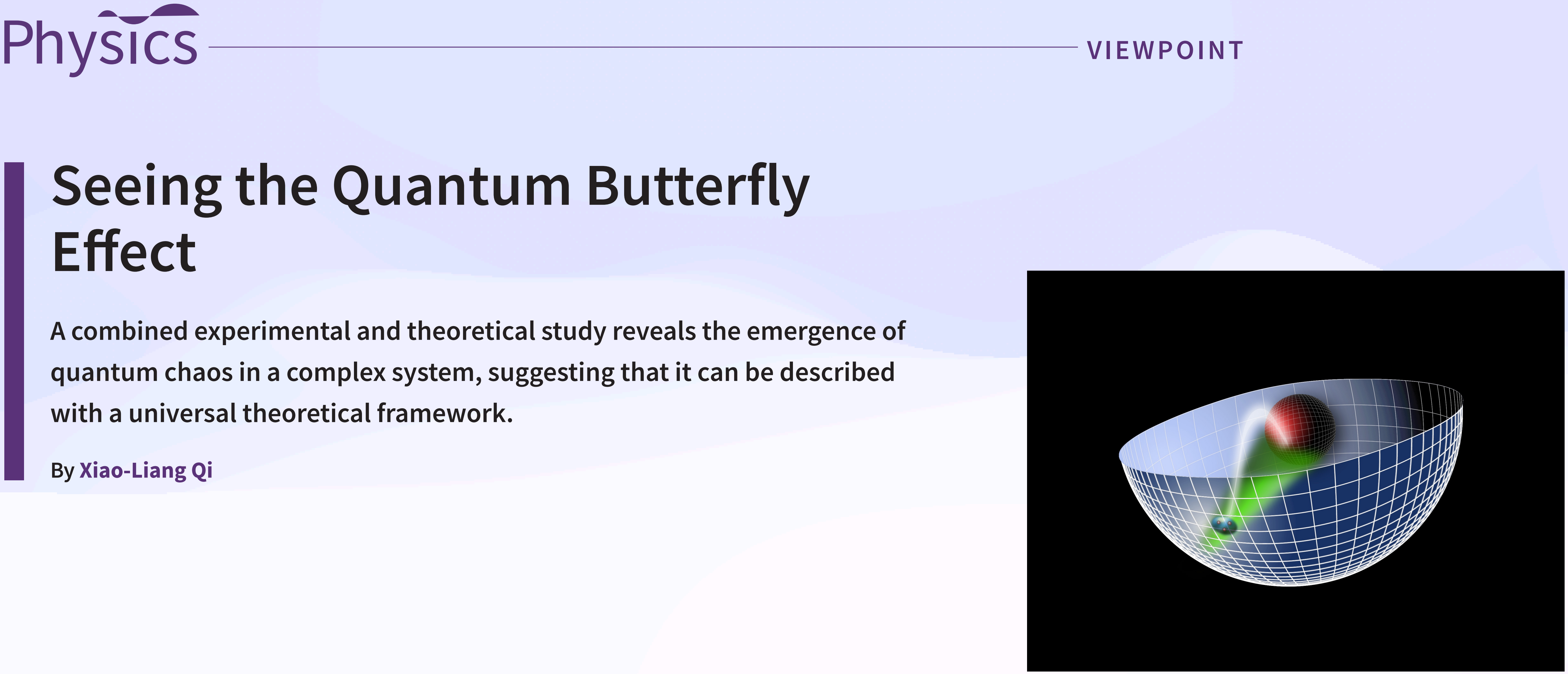
Seeing the Quantum Butterfly Effect

A combined experimental and theoretical study reveals the emergence of quantum chaos in a complex system, suggesting that it can be described with a universal theoretical framework.

By **Xiao-Liang Qi**

This result is significant not only because it provides the first reliable measurement of the quantum Lyapunov exponent in a macroscopic many-body system, but it also verifies that the simplest effective description — involving a single scramble — appears to hold. The finding suggests universality in quantum chaotic systems.

First Experimental Determination of Lyapunov Exponent



Physics VIEWPOINT

Seeing the Quantum Butterfly Effect

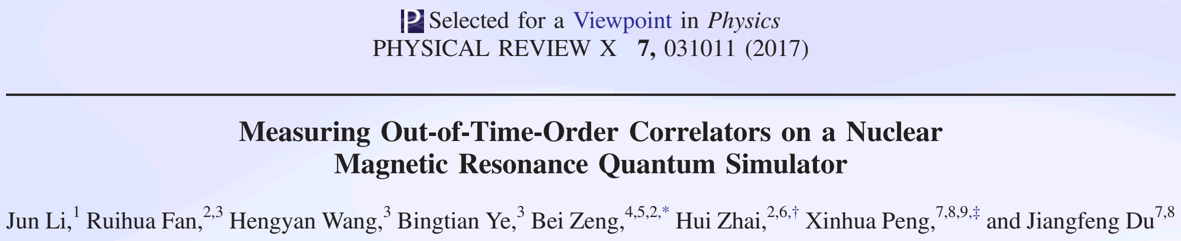
A combined experimental and theoretical study reveals the emergence of quantum chaos in a complex system, suggesting that it can be described with a universal theoretical framework.

By **Xiao-Liang Qi**

This work has also built new links between tabletop experiments and high-energy physics, as the concept of the scramble arises from holographic duality models relating quantum chaos to gravitational dynamics.

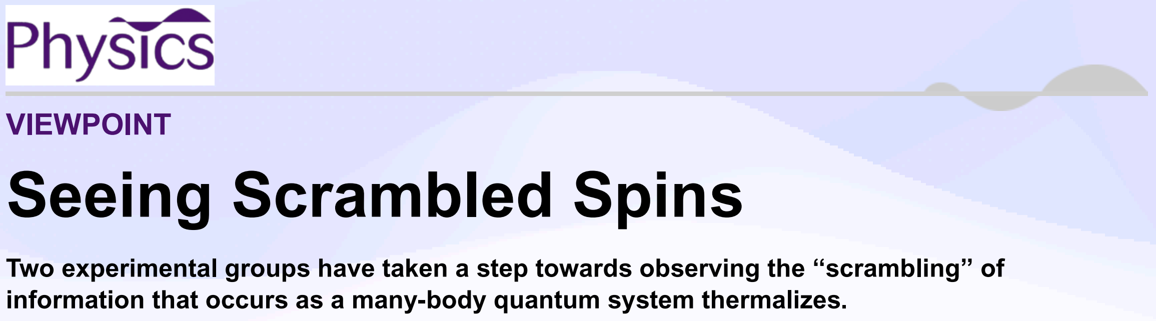
A Journey of Ten Years

2017: The first experimental measurement of OTOC


Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]

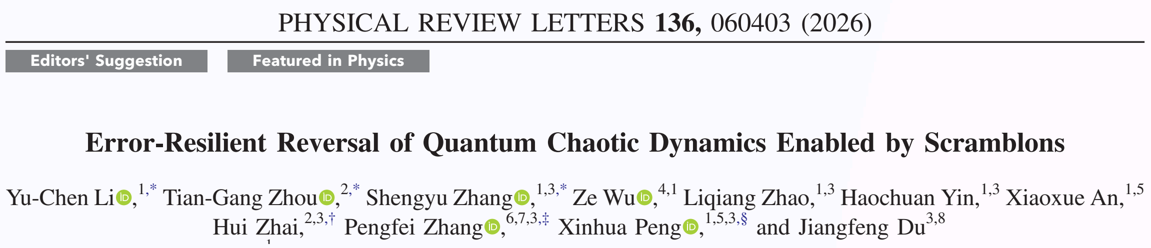

Physics

VIEWPOINT

Seeing Scrambled Spins

Two experimental groups have taken a step towards observing the "scrambling" of information that occurs as a many-body quantum system thermalizes.

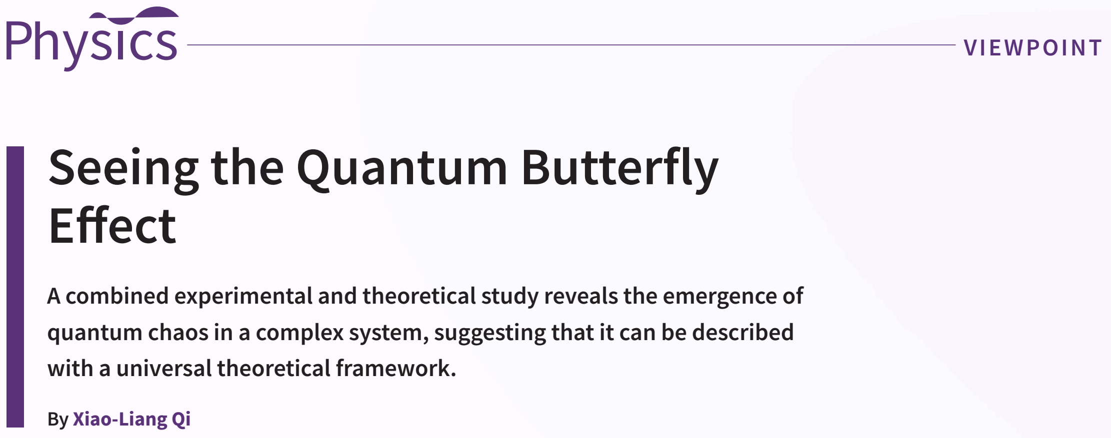
2026: The first experimental determination of Lyapunov experiment from OTOC


PHYSICAL REVIEW LETTERS **136,** 060403 (2026)

Editors' Suggestion | Featured in Physics

Error-Resilient Reversal of Quantum Chaotic Dynamics Enabled by Scramblons

Yu-Chen Li,[1,*] Tian-Gang Zhou,[2,*] Shengyu Zhang,[1,3,*] Ze Wu,[4,1] Liqiang Zhao,[1,3] Haochuan Yin,[1,3] Xiaoxue An,[1,5] Hui Zhai,[2,3,†] Pengfei Zhang,[6,7,3,‡] Xinhua Peng,[1,5,3,§] and Jiangfeng Du[3,8]

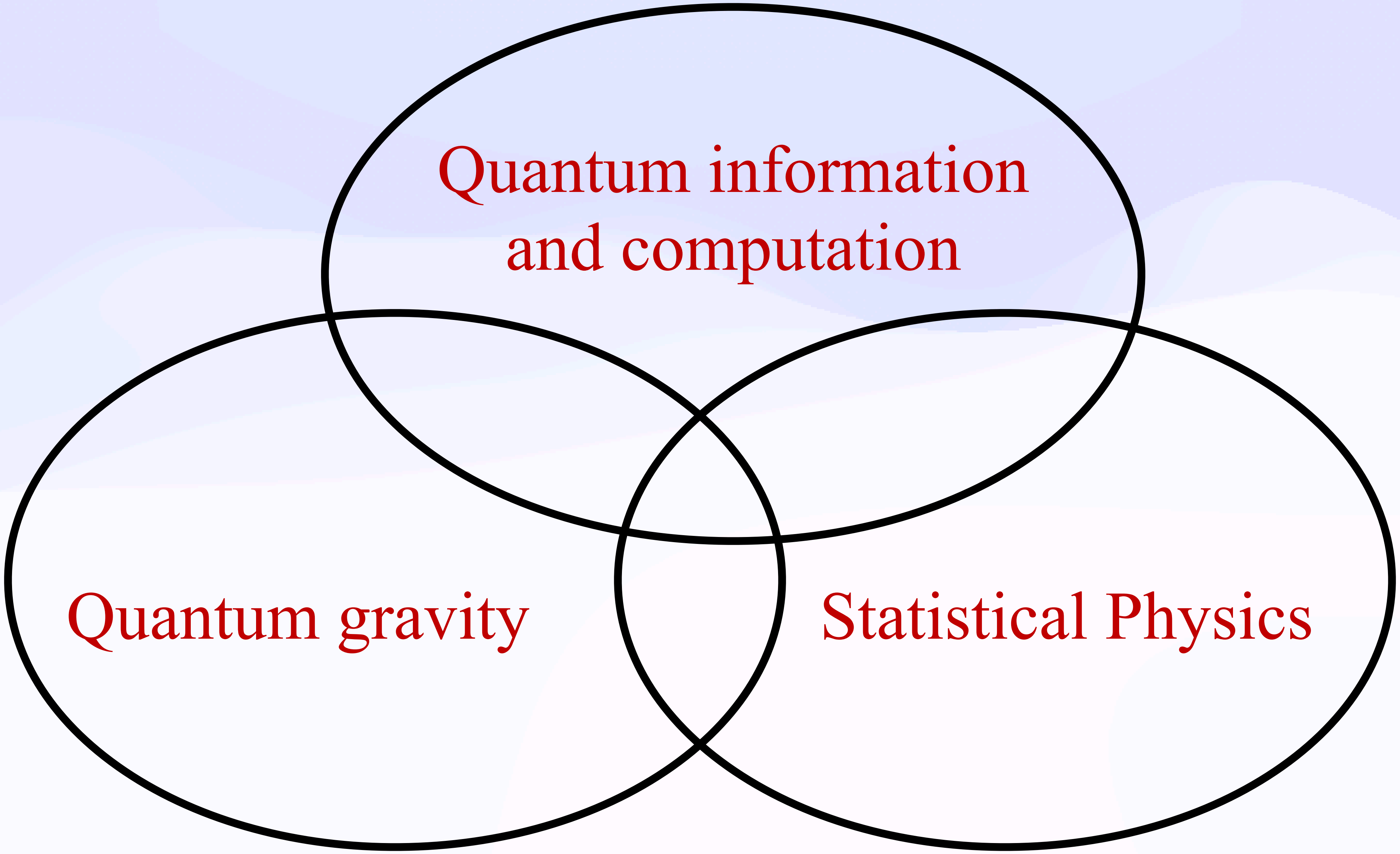

Physics

VIEWPOINT

Seeing the Quantum Butterfly Effect

A combined experimental and theoretical study reveals the emergence of quantum chaos in a complex system, suggesting that it can be described with a universal theoretical framework.

By Xiao-Liang Qi



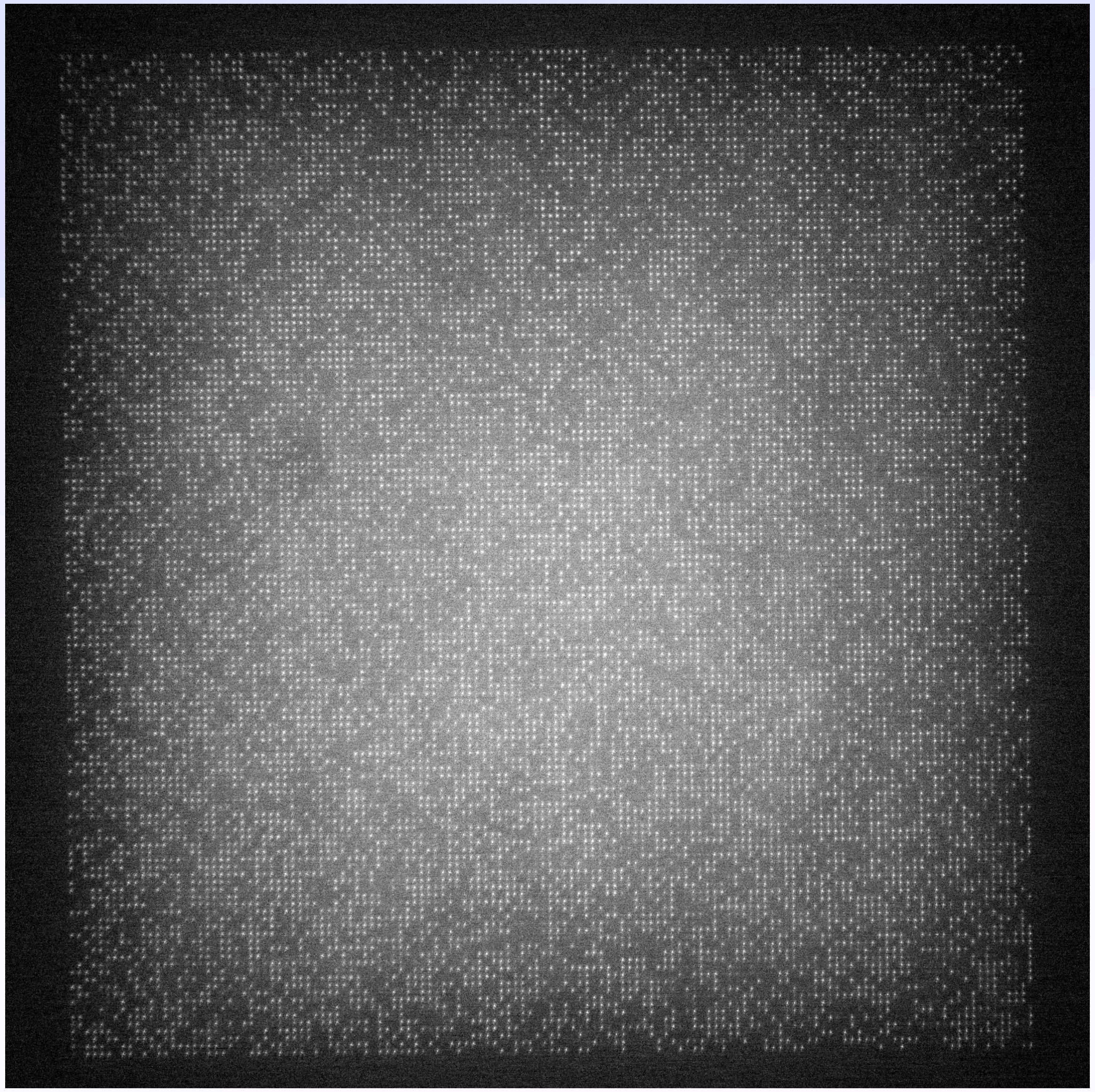
Quantum information and computation

Quantum gravity

Statistical Physics

New Progress on Atom Array Experiments in Tsinghua

~11000 atoms in tweezer array



Yuqing Wang, Zhongchi Zhang, Tao Zhang, Yuxuan Liao, Hanteng Wang, Ye Tian, Binjie Ji, Yujia Wu, Luming Ma, Chen Qing, Chengshu Li, Wei Zhang, Yidong Huang, Wenjun Zhang, Xue Feng, Wenlan Chen, Hui Zhai, arXiv: 2606.02715

See Wenlan’s talk in ICAP

Thank you very much for your attention !